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PORTFOLIO ROBUSTNESS EVALUATION: A CASE STUDY IN THE ELECTRICITY SECTOR

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Abstract. Managers continually face the task of allocating resources to projects when there is not enough money to fund them all. Portfolio Robustness Evaluation (PROBE) is a multicriteria decision support system developed to help managers to perform that difficult task. This paper presents a PROBE model, developed for an electricity distribution company, to select the best portfolio of projects, subject to budget constraints for different types of projects and various organisational units in multiple time periods. Projects requiring large-scale investments are analysed separately from the small-scale projects. The robustness of the selected portfolio of large-scale projects is analysed in an iterative process where broader uncertainty ranges are considered for the values of the projects, and also when an environmental impact criterion is added to the evaluation model.

Keywords: portfolio decision analysis, portfolio robustness evaluation, resource allocation, multicriteria analysis, electricity sector.

JEL Classification: C61, D81, G11, G31, L94.

Introduction

The scope of portfolio selection, given its widest interpretation, is extraordinarily broad (Salo *et al.* 2011b). In this paper, the focus is directed at real assets and at capital budgeting, more precisely at the efficient allocation of financial resources to indivisible investment projects and its enhancement with modern portfolio decision analysis tools (Salo *et al.* 2011a).

The use of linear programming in capital budgeting has been suggested since the mid 1950s (Gunther 1955; Lorie, Savage 1955; Markowitz, Manne 1957; Asher 1962). Companies and organisations faced with a number of investment projects that exceed their available

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resources, in general, aim at finding the subset of projects (the portfolio) that maximizes the value created. Assuming that projects are indivisible, the problem leads to a binary mathematical programming formulation subject to a budget constraint (the knapsack problem – Kleinmuntz 2007), which can be extended, when appropriate, with several other types of linear constraints. Interdependencies among projects may also lead to non-linear objective functions (Dickinson *et al.* 2001) and the formulation can also accommodate multiple objectives (see, e.g., Golabi 1987; Ringuest, Graves 1990; Stummer, Heidenberger 2003; Ewing *et al.* 2006; for a more global overview of multicriteria methods used in economics see Zavadskas, Turskis 2011, and in the electricity sector see Atici, Ulucan 2011 and Ertay *et al.* 2013).

The selection of multiple projects under scarcity of resources may also be made by means of a prioritisation approach. Projects are prioritised by their benefit-to-cost ratios and selected until the available budget is exceeded (Edwards 1977; Sharpe, Keelin 1998; Buede, Bresnick 2007; Phillips, Bana e Costa 2007). This approach is appealing in the context of strategic decision-aiding processes, as it permits straightforward interaction with the decision-makers; however, it is not so suitable for handling programming and multi-period constraints. This is the case of the portfolio decision analysis presented in this paper and developed for EDPD (EDP Distribuição – Energia, S.A.), the main distributor of electricity in Portugal.

The paper analyses the heuristic prioritisation procedure used by EDPD's managers and presents an alternative modelling of the problem using the PROBE (Portfolio Robustness Evaluation) decision support system (Lourenço *et al.* 2012). PROBE enables several types of linear constraints to be considered and, given the costs and benefits of the projects, identifies all convex and non-convex efficient portfolios. It also permits the robustness of a chosen portfolio to be analysed, given uncertainty ranges on project benefits, by searching for competitor portfolios that may provide more overall benefits without increasing the total cost. An alternative approach for sensitivity analysis on benefits' uncertainties can be found in Beaujon *et al.* (2001).

The robustness analysis implemented in PROBE allows performing an *a posteriori* sensitivity analysis on several inputs simultaneously, which was a missing feature in the commercial packages for multicriteria resource allocation analysed in Lourenço *et al.* (2008). The non-commercial software RPM-Decisions (http://www.rpm.tkk.fi/rpm-software.html) implements the "Robust Portfolio Modeling" (RPM) approach (Liesiö *et al.* 2007, 2008), which follows a different path. Contrary to PROBE, the robustness analysis provided by RPM is not concerned with analyzing the stability of a portfolio. Instead, it reverses the *ex-post* sensitivity analysis perspective, by incorporating uncertainty *a priori* as "incomplete information" in the formulation of the problem and looking for non-dominated portfolios (Lourenço *et al.* 2012). In RPM "loose preference statements and wide score intervals typically result in a large number of non-dominated portfolios" (Liesiö *et al.* 2008: 682). The decision-maker is then invited to narrow the initial uncertainty domain, therefore reducing the number of non-dominated portfolios, and decision rules can be applied to eventually select one non-dominated portfolio. In contrast, PROBE supposes the decision-maker can use best-guess parameter values to find an attractive proposed portfolio, and then offers a form of robustness analysis in which the proposed portfolio is compared to competitors in its neighbourhood (Lourenço *et al.* 2012). Liesiö and Salo (2012) developed an approach similar to RPM for portfolio selection in problems where a decision-maker faces incomplete information about scenario probabilities and risk preferences. However, given that in the EDPD case there are no probabilities associated with the uncertain parameters, we do not discuss this approach further. A different approach consists in selecting "robust" projects, i.e. the projects more often included in optimal portfolios obtained by using different combinations of the uncertain parameters (Bryan 2010).

Fasth and Larsson (2013) apply "interval contractions" to iteratively reduce the uncertainty (defined under the form of intervals) upon the benefits and the costs of the projects. In an *a priori* approach, Fasth and Larsson (2013) find one minimax optimal portfolio at each step (e.g. by contracting the intervals by 20%) until reaching point values for the parameters of the projects. Then, they analyse in how many portfolios the projects were included. The projects not included in any portfolio, or included in all portfolios, are not analysed further, being discarded, or selected, respectively. The projects that are included in some but not all of the portfolios are further analysed using an *a posteriori* analysis. One variant of the *a posteriori* approach consists in stepwise contracting the intervals of the parameters of the model and in each step calculating the minimum and maximum difference in expected benefit between two portfolios until a dominance relationship appears (see other variants in Fasth, Larsson 2013). However, contrary to RPM and PROBE, Fasth and Larsson's (2013) approach pairwise compares all portfolios, which may result in an unrealistic number of combinations to handle for a large number of projects.

The remainder of this paper is organised as follows. Section 1 introduces the EDPD problem and discusses the drawbacks of the project selection procedure in use at the company. Section 2 develops the alternative PROBE model. There is a budget for projects requiring large-scale investments and another budget for small-scale projects. Therefore, each type of projects is subject to a separate portfolio decision analysis with PROBE. Robustness analysis is focused on the optimal portfolio of large-scale projects, in an iterative process where broader uncertainty ranges are considered for the values of the projects. EDPD has been using only one selection criterion, the maximization of *NPV*, but the company has also other concerns, namely the environmental impacts of large-scale projects. Section 3 shows the extent to which the introduction in the model of an additional evaluation criterion, allowing to take into account the environmental impacts of the projects, would affect the stability of the selected portfolio. The last Section presents some final remarks.

1. The EDPD project selection problem and procedure

EDP S.A. is the largest industrial group in Portugal and a leader in the energy sector, spreading its activities across several countries in Europe and America. EDPD is the EDP distributor of electricity in mainland Portugal. Each year, EDPD chooses a portfolio of projects among several hundred indivisible projects, each one requiring investment for one to three years, which in total by far exceeds the available budget. There are large-scale projects (e.g. new electric power substations) and small-scale ones (e.g. new low-voltage power lines) from six organisational units, which correspond to six geographical zones, Z1 to Z6. EDPD limits large-scale investments to 75% of each year's budget. This paper analyses the investment decision taken by EDPD for the first of three years. Budgetary data are detailed in Table 1, including data related to commitments with ongoing projects and compulsory investments in the period that should be deducted before proceeding with the resource allocation. There are 372 new projects in the first year; 28 are large-scale ones, with an average cost of € 2 463.36×10^3 whereas the average cost of the small-scale ones is only € 18 93×10^3. Financing all the projects would not be possible because their total cost (€ 75 487.44×10^3) exceeds the total amount available (€ 48.552×10^3) for funding new projects during the period.

Table 1. EDPD's budget (in € 10^3)

	Year 1	Year 2	Year 3	Total
(a) Total budget	20 000	20 000	20 000	60 000
For large-scale projects	15 000	15 000	15 000	45 000
For small-scale projects	5 000	5 000	5 000	15 000
(b) Ongoing investments	6 359	4 451	0	10 810
On large-scale projects	6 234	4 451	0	10 685
On small-scale projects	125	0	0	125
(c) Compulsory investment (on small-scale projects)	552	86	0	638
(d) = (a) – (b) – (c) Available budget for new projects	13 089	15 463	20 000	48 552
For new large-scale projects	8 766	10 549	15 000	34 315
For new small-scale projects	4 323	4 914	5 000	14 237
Generic	2 500	4 914	5 000	12 414
Zone Z1	255			
Zone Z2	296			
Zone Z3	387			
Zone Z4	70			
Zone Z5	281			
Zone Z6	534			

Let c_j, PV_j and NPV_j (= $PV_j - c_j$) be the investment cost, present value and net present value of the candidate project j ($j = 1, \ldots, n$), respectively. The selection procedure used by EDPD can be described by the following six prioritisation steps: (i) List the candidate projects; (ii) Determine the value of the profitability index (or benefit-to-cost ratio) for each project j by NPV_j/c_j (or PV_j/c_j, which will not alter the prioritisation because $PV_j/c_j = NPV_j/c_j + 1$); (iii) Order the projects from most to least profitability; (iv) Go down the list, choosing projects as long as there are available financial resources for each one of the two types of projects, in each one of the six zones and in each one of the three years; (v) When at least one of these budget constraints cannot be satisfied, discard the project under consideration; (vi) Repeat steps (iv) and (v) until all budgets are simultaneously satisfied.

Unfortunately, this prioritisation procedure does not always select the portfolio of projects that maximizes total *NPV*. Section 2.1 shows that this drawback is due to the fact that the prioritisation procedure ignores non-convex efficient portfolios and proposes overcoming this problem by using mathematical programming.

2. An alternative approach to the EDPD problem

2.1. Introducing PROBE to EDPD's managers

Our first suggestion to the EDPD's managers was to split the resource allocation into two sequential problems, focusing firstly on the large-scale projects only, as recommended for other similar resource allocation contexts with projects of significantly different costs (Kleinmuntz, Kleinmuntz 1999). Let us then consider the 28 large-scale projects. Given the global budget available for this type of projects (€ 34 315×10^3), and ignoring for now all other budgetary constraints, the prioritisation approach selects the first 14 projects in Table 2, with a cumulative cost of € 30 065.55×10^3 and a cumulative *NPV* of € 84 759.33×10^3. Alternatively, one can find the optimal solution to the knapsack problem (1), where *B* represents the available budget.

Table 2. Prioritisation of the large-scale projects (in € 10^3)

Priority order	Project	*NPV*	Cost	*NPV*/Cost	Cumulative *NPV*	Cumulative Cost
1	P10	12 016.67	602.38	19.95	12 016.67	602.38
2	P16	9 741.32	700.00	13.92	21 757.99	1 302.38
3	P15	11 447.06	2 023.48	5.66	33 205.05	3 325.86
4	P17	9 741.32	1 773.83	5.49	42 946.37	5 099.69
5	P23	3 896.55	1 436.11	2.71	46 842.92	6 535.80
6	P05	11 411.82	5 221.40	2.19	58 254.74	11 757.20
7	P03	7 511.86	3 683.78	2.04	65 766.60	15 440.98
8	P18	5 971.11	3 144.54	1.90	71 737.71	18 585.52
9	P19	8 181.24	5 375.40	1.52	79 918.95	23 960.92
10	P14	411.00	306.82	1.34	80 329.95	24 267.74
11	P11	220.43	174.57	1.26	80 550.38	24 442.31
12	P02	148.97	126.00	1.18	80 699.35	24 568.31
13	P13	1 735.00	2 112.67	0.82	82 434.35	26 680.98
14	P06	2 324.98	3 384.57	0.69	84 759.33	30 065.55
15	P04	2 993.71	4 964.12	0.60	87 753.04	35 029.67
16	P07	1 467.35	3 528.12	0.42	89 220.39	38 557.79
17	P09	732.40	1 788.49	0.41	89 952.79	40 346.28
18	P01	232.61	587.13	0.40	90 185.40	40 933.41
19	P12	487.14	1 318.71	0.37	90 672.54	42 252.12
20	P28	508.86	1 550.00	0.33	91 181.40	43 802.12
21	P08	549.83	2 902.53	0.19	91 731.23	46 704.65
22	P21	466.13	2 482.62	0.19	92 197.36	49 187.27
23	P22	535.50	3 084.23	0.17	92 732.86	52 271.50
24	P27	898.95	6 229.88	0.14	93 631.81	58 501.38
25	P25	1 24	1 199.68	0.00	93 633.05	59 701.06
26	P26	1 28	1 914.76	0.00	93 634.33	61 615.82
27	P24	1 00	3 452.94	0.00	93 635.33	65 068.76
28	P20	0 46	3 905.33	0.00	93 635.79	68 974.09

Maximize

$$\sum_{j=1}^{28} NPV_j x_j$$

subject to:

$$\sum_{j=1}^{28} c_j x_j \leq B\,, \tag{1}$$

$$x_j \in \{0,1\}, \quad j = 1,\ldots,28\,,$$

with $x_j = 1$ if project j is included in the optimal portfolio and $x_j = 0$ otherwise.

Although not highlighted in Lourenço *et al.* (2012), PROBE tests if there are other optimal solutions and identifies the one with minimal cost by solving problem (2):

Minimize

$$\sum_{j=1}^{28} c_j x_j$$

subject to:

$$\sum_{j=1}^{28} NPV_j x_j = v^p\,, \tag{2}$$

$$x_j \in \{0,1\}, \quad j = 1,\ldots,28\,,$$

where v^p is the *NPV* of the optimal portfolio *p*, being *p* the optimal solution of problem (1).

Note also that, contrary to using the prioritisation procedure, the efficient solution found could be different if PV_j was used instead of NPV_j (Dantzig 1957) and could result in a lower total *NPV*.

The optimal (and efficient) portfolio for EDPD is formed by 15 projects and is better than the prioritisation portfolio in € 1 745.85×10^3 and costs more € 4 170.16×10^3. The first 13 projects in Table 2 are common to both portfolios, which differ only because project P06 is replaced by projects P04 and P12.

The graph in Figure 1 is a display of the PROBE decision support system that presents all the efficient portfolios when the budget (*B*) varies from zero to the sum of the costs of all large-scale projects. In Figure 1 the convex-efficient portfolios, which correspond to those selected by a prioritisation procedure based on the ratio *NPV*/cost, are shown in light gray and are linked with a dotted line.

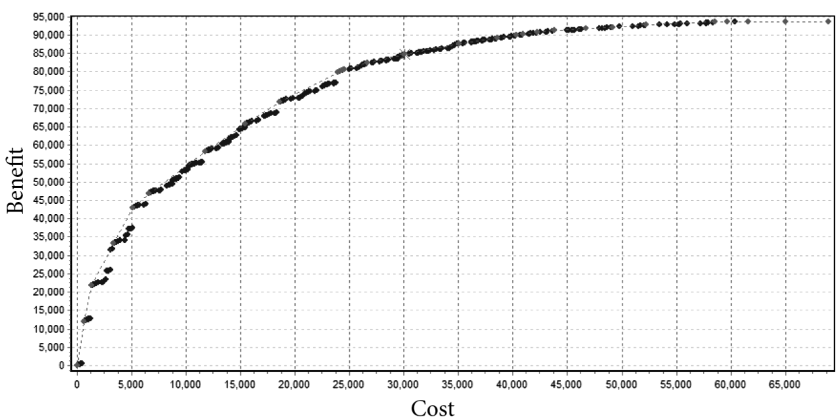


Fig. 1. PROBE display of the efficient frontier (in € 10^3)

The graph in Figure 2 is a zoom of the portion of the graph in Figure 1 between the portfolio chosen by prioritisation (A) and the next convex efficient portfolio (C), showing 22 (non-convex) efficient portfolios that exhibit, without exceeding the budget, higher total *NPV* than portfolio A. The best of them is portfolio B, signalled with a star dot in Figure 2, which is the optimal solution to problem (2).

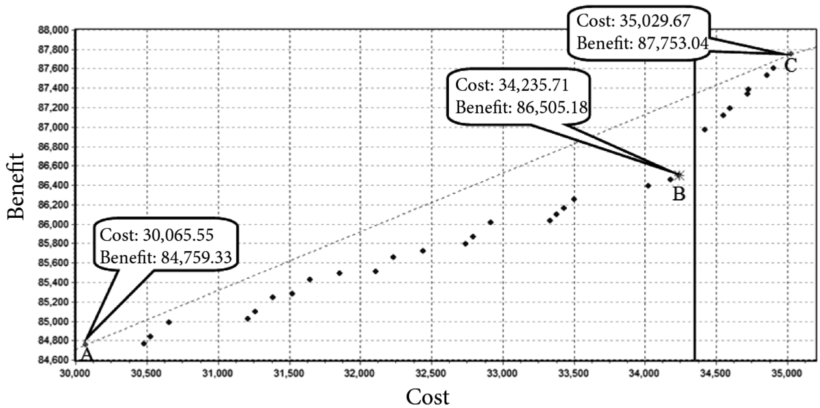


Fig. 2. Zoom of efficient frontier between the convex efficient portfolio chosen by prioritisation (A) and the next convex efficient portfolio (C). The optimal portfolio (B) is signalled with a star dot and the vertical line indicates the available budget (in € 10^3)

When presented with the graph in Figure 2, the head of the EDPD Department of Network Planning noted it makes evident that portfolio B not only offers a better use of the budget than the prioritisation choice A, but also it implies a smaller additional investment to get the benefit of portfolio C. This is related to the usual separation between hard and soft budgetary constraints (hard and soft capital rationing) done in financial literature (see, e.g., Brealey, Myers 2003: 108–109). Budgetary constraints are considered to be soft if it is possible for a company or a department to expand its budgetary limits in the presence of profitable investment projects, financing their activities through the capital markets or the banking system. However, after the 2008 financial crisis the credit limits are in several cases very tight.

2.2. Finding the best portfolio of large-scale projects with multi-period budget constraints

Having introduced the PROBE mathematical programming approach to the EDPD investment problem, it is now interesting to use it to highlight the benefit of having discarded the three annual-budget constraints. For this purpose, one first needs to solve the following new maximization problem (3), which results from problem (1) by replacing the global budget constraint with the three annual-budget constraints, where c_{jy} is the cost of large-scale project j on year y ($y = 1, \ldots, 3$).

Maximize:

$$\sum_{j=1}^{28} NPV_j x_j$$

subject to:

$$\sum_{j=1}^{28} c_{j1} x_j \le 8{,}766, \sum_{j=1}^{28} c_{j2} x_j \le 10{,}549, \sum_{j=1}^{28} c_{j3} x_j \le 15{,}000, \tag{3}$$

$$x_j \in \{0,1\}, \quad j = 1,\ldots,28.$$

Subsequently, problem (4) must be solved to find the least costly portfolio that meets the constraints of problem (3) and provides the same *NPV* (v^p) as the optimal portfolio p previously found.

Minimize:

$$\sum_{j=1}^{28} c_j x_j$$

subject to:

$$\sum_{j=1}^{28} c_{j1} x_j \le 8{,}766, \sum_{j=1}^{28} c_{j2} x_j \le 10{,}549, \sum_{j=1}^{28} c_{j3} x_j \le 15{,}000, \tag{4}$$

$$\sum_{j=1}^{28} NPV_j x_j = v^p,$$

$$x_j \in \{0,1\}, \quad j = 1,\ldots,28.$$

Portfolio B, found in Section 2.1, requires investments in the first two years that exceed the available budgets (respectively, € 6 985.77×10^3 and € 1 621.7×10^3). Therefore, portfolio B is not a feasible solution to the EDPD problem. The optimal solution is now portfolio O (formed by 11 projects: P10, P16, P15, P17, P23, P05, P03, P18, P19, P01 and P28) which offers less 6.8% (€ 5 844.76×10^3) of total *NPV* than portfolio B. This is the consequence of imposing rigid annual budgets. Indeed, as highlighted by Lorie and Savage (1955: 233), "the imposition of additional restrictions upon the freedom of action of any agency can obviously never increase the value of the best opportunity available to that agency".

2.3. Portfolio robustness evaluation

Different methods can lead to different estimates of the benefits of the projects, as remarked by Beaujon *et al.* (2001). Therefore, it is important to explore the consequences for portfolio selection of having those uncertain estimates. The EDPD investment problem is affected by the uncertainty of the expected cash flows of the projects, namely, the uncertainty of their *PV* values. The core feature of PROBE is the ability to evaluate the robustness of the selected portfolio considering several sources of uncertainty simultaneously. Since the investment in large-scale projects can absorb 75% of the total budget, it is wise to analyse the extent to which the choice of the optimal portfolio O, found in Section 2.2, is robust.

In the particular case of a single benefit criterion, like the *NPV*, we define robustness as follows. Let us denote by p the optimal portfolio with total benefit $v^p = \sum_{j \in p} v_j$, where v_j indicates the benefit value of project j, and total cost $c^p = \sum_{j \in p} c_j$, and by d another portfolio with $v^d = \sum_{j \in d} v_j$ and $c^d = \sum_{j \in d} c_j$. Suppose that the benefits of p and d are affected by uncertainty within $\underline{v}^p \leq v^p \leq \overline{v}^p$ and $\underline{v}^d \leq v^d \leq \overline{v}^d$, respectively, where $\underline{v}^p = \sum_{j\in p} \underline{v}_j$, $\overline{v}^p = \sum_{j\in p} \overline{v}_j$, $\underline{v}^d = \sum_{j\in d} \underline{v}_j$, $\overline{v}^d = \sum_{j\in d} \overline{v}_j$ and $\underline{v}_j \leq v_j \leq \overline{v}_j$ for all j. Portfolio d is a competitor of the optimal portfolio p if, and only if, (i) $\sum_{j\in d\setminus p} \underline{c}_j \leq \sum_{j\in p\setminus d} \overline{c}_j$ and (ii) there exists a combination of feasible project benefit values such that $\sum_{j\in p\setminus d} v_j - \sum_{j\in d\setminus p} v_j < 0$. The choice of portfolio p will undoubtedly be robust when p has no competitors. (Portfolio robustness evaluation in problems with multiple benefit criteria requires a different definition, which is presented in Lourenço *et al.* 2012).

In the EDPD case the decision support system PROBE was used to evaluate the robustness of portfolio O for variations of ±α% (1 ≤ α ≤ 20, increasing 1% at each iteration) affecting the *PV* of each project, simultaneously. There is no competitor for a level of uncertainty α ≤ 5%. Table 3 shows that the first competitor portfolio appears for α = 6% and the number of competitors rises until nine for α = 20%.

Table 3. Differences between portfolio O and its competitors

Detected for the first time when…	Number of competitor portfolios	Projects exclusive to the optimal portfolio	Projects exclusive to the new competitor portfolio
α = 6%	1	P28	P02, P11
α = 11%	2	P01, P28	P02, P11
α = 12%	3	P28	P11
α = 16%	5	P01, P28	P11
		P28	P02
α = 19%	7	P01, P28	P02
		P01, P19, P28	P02, P04, P09
α = 20%	9	P01, P19, P28	P02, P04, P11, P14
			P04, P09

The differences in the large-scale projects that compose portfolio O and its competitors are also shown in Table 3. It can be observed that: when α = 6% there is only one competitor portfolio, which includes projects P02 and P11 instead of project P28; and when α = 11% there are two competitor portfolios, one detected for the first time when α = 6% and a new one that includes P02 and P11 instead of projects P01 and P28.

Table 4 shows that most of the projects that form portfolio O also belong to the competitor portfolios, for example, 91% of the projects are kept with α = 10%, and 73% when α = 20%, which indicates a significant level of stability.

Table 4. Analysis of the stability of portfolio O

Uncertainty on the *PV*	No. of stable projects	Percentage of stable projects
1% ≤ α ≤ 5%	11	100
6% ≤ α ≤ 10%	10	91
11% ≤ α ≤ 18%	9	82
19% ≤ α ≤ 20%	8	73

Another relevant analysis is the regret evaluation. For an optimal portfolio p and a competitor portfolio d, the maximal regret (i.e. loss) in *NPV* caused by selecting p instead of d corresponds to the absolute value of $\sum_{j \in p \backslash d} \underline{v}_j - \sum_{j \in d \backslash p} \overline{v}_j$. For k competitors (with $k > 1$) the maximal regret of selecting p corresponds to the maximum of the k maximal (pairwise) regrets. Figures 3 and 4 show, in absolute and relative terms, the maximal regret in the overall *NPV* of portfolio O, for different levels of uncertainty. It can be observed in Figure 4 that the maximal regret in *NPV* for $\alpha = 6\%$ represents only 0.03% of the minimum *NPV* of portfolio O; for $\alpha = 20\%$ the maximal regret in *NPV* is € 406.3×10^3, which is only 0.69% of the minimum *NPV* of portfolio O. These small potential losses were considered irrelevant by EDPD's managers and confirmed the robustness of selecting portfolio O.

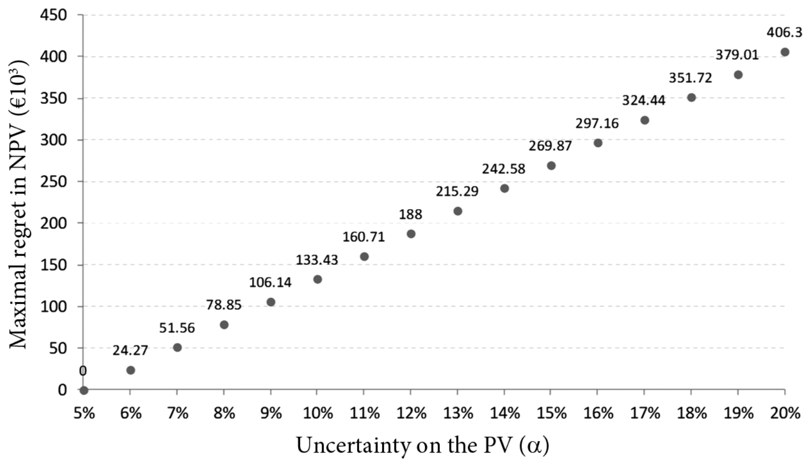


Fig. 3. Uncertainty on the *PV* vs. maximal regret in *NPV*

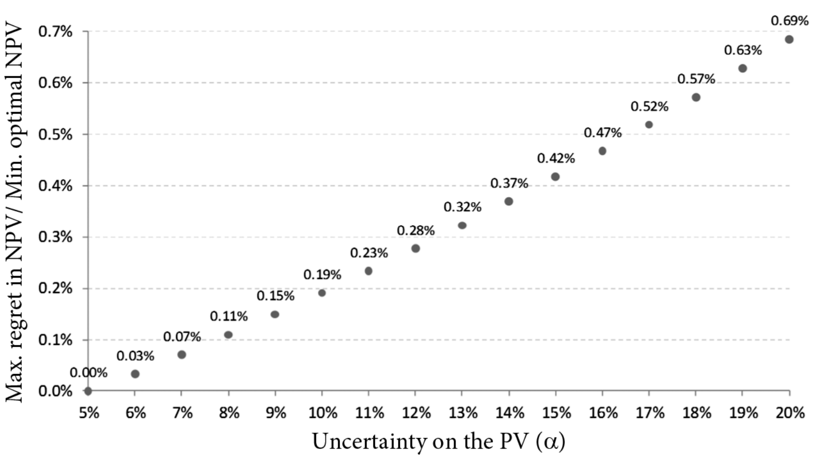


Fig. 4. Uncertainty on the *PV* vs. max. regret in *NPV* / min. *NPV* of portfolio O

2.4. Finding the best portfolio of small-scale projects

Let us now turn the analysis to the 344 small-scale projects and look for the best portfolio under the three annual-budget constraints and the six zone constraints concerning the first year of investment (the data for the 344 small-scale projects is presented in the Appendix, Table 6). The optimal portfolio found by solving problem (5) includes 265 projects, which cost € 4 637.09×10^3 and provide a total *NPV* of € 23 554.7×10^3.

Maximize:

$$\sum_{j=1}^{344} NPV_j x_j$$

subject to:

$$\begin{gathered}
\sum_{j=1}^{344} c_{j1} x_j \le 4{,}323\,,\; \sum_{j=1}^{344} c_{j2} x_j \le 4{,}914\,,\; \sum_{j=1}^{344} c_{j3} x_j \le 5{,}000\,,\\
\sum_{j\in Z1} c_{j1} x_j \ge 255,\; \sum_{j\in Z2} c_{j1} x_j \ge 296,\\
\sum_{j\in Z3} c_{j1} x_j \ge 387,\; \sum_{j\in Z4} c_{j1} x_j \ge 70,\\
\sum_{j\in Z5} c_{j1} x_j \ge 281,\; \sum_{j\in Z6} c_{j1} x_j \ge 534,\\
x_j \in \{0,1\},\quad j = 1,\ldots,344,
\end{gathered} \tag{5}$$

where: c_{jy} is the investment cost of project j in year y ($y = 1,\ldots, 3$), Zn denotes the subset of small-scale projects that belong to zone n ($n = 1,\ldots, 6$), $x_j = 1$ if the small-scale project j is included in the optimal portfolio and $x_j = 0$ otherwise.

The optimal solution of problem (5) is confirmed to be also efficient by solving problem (6).

Minimize:

$$\sum_{j=1}^{344} c_j x_j$$

subject to:

$$\begin{gathered}
\sum_{j=1}^{344} c_{j1} x_j \le 4{,}323,\; \sum_{j=1}^{344} c_{j2} x_j \le 4{,}914,\; \sum_{j=1}^{344} c_{j3} x_j \le 5{,}000,\\
\sum_{j\in Z1} c_{j1} x_j \ge 255,\; \sum_{j\in Z2} c_{j1} x_j \ge 296,\\
\sum_{j\in Z3} c_{j1} x_j \ge 387,\; \sum_{j\in Z4} c_{j1} x_j \ge 70,\\
\sum_{j\in Z5} c_{j1} x_j \ge 281,\; \sum_{j\in Z6} c_{j1} x_j \ge 534,\\
\sum_{j=1}^{344} NPV_j x_j = 23{,}554.7,\\
x_j \in \{0,1\},\quad j = 1,\ldots,344.
\end{gathered} \tag{6}$$

Note that the constraint with the budget limit on year 3 is not active in problems (5) and (6), because the 344 small-scale projects do not have investment costs in that year. However, we opted to keep that constraint in these formulations to clearly express the concern expressed by EDPD's managers.

2.5. Comparing the results of the two approaches

Table 5 presents the results of portfolio decision analyses of the EDPD problem done with the PROBE mathematical programming formulation and the EDPD prioritisation procedure. The portfolio found with the former provides a global *NPV* greater than that of the latter in € 421.53×10^3, for an investment cost greater in € 1 915.09×10^3 and closer to the budget. Most of that additional benefit is due to large-scale projects, precisely € 372.07×10^3, for an additional investment of € 1 836.56×10^3.

Table 5. Benefits and costs of the solutions found (in € 10^3)

Type of project / Approach	*NPV*	Costs			
		Year 1	Year 2	Year 3	Total
Large-scale projects					
(a) Budget for new large projects		8 766.00	10 549.00	15 000.00	34 315.00
(b) PROBE portfolio	80 660.42	8 741.63	9 828.18	7 528.24	26 098.05
(c) EDPD portfolio	80 288.35	8 684.56	9 273.69	6 303.24	24 261.49
(d) = (b) – (c)	372.07	57.07	554.49	1.225	1 836.56
Small-scale projects					
(e)Budget for new small projects		4 323.00	4 914.00	5 000.00	14 237.00
(f) PROBE portfolio	23 554.70	4 322.76	314.33	0	4 637.09
(g) EDPD portfolio	23 505.24	4 320.58	237.98	0	4 558.56
(h) = (f) – (g)	49.46	2.18	76.35	0	78.53
Total					
(i) Budget for new projects		13 089.00	15 463.00	20 000.00	48 552.00
(j) PROBE portfolio	104 215.10	13 064.39	10 142.51	7 528.24	30 735.10
(k) EDPD portfolio	103 793.60	13 005.14	9 511.67	6 303.24	28 820.10
(l) = (l) – (k)	421.53	59.25	630.84	1 225.00	1 915.09

3. Multicriteria portfolio analysis of the effect of adding an environmental impact criterion

As mentioned in the introduction, and detailed in Lourenço *et al.* (2012), the PROBE decision support system allows a multicriteria portfolio decision analysis to be performed (Salo *et al.* 2011a). This was used to take into consideration EDPD's concern with the environmental impacts (*EI*) of large-scale projects, and to observe the extent to which the stability of the portfolio of large-scale projects that maximizes profitability, found in Section 2.2, would be affected by an increasing importance given to minimizing environmental impacts as well. For this purpose, two evaluation criteria, *NPV* and *EI*, were considered and, within a valid range of the latter, it was assumed that EDPD decision-makers' preferences would be such that (i) the company would be willing to accept trading off less

NPV with less *EI*, and vice versa, and (ii) *NPV* and *EI* are additive independent (Keeney, Raiffa 1993). Under these compensatory working hypotheses, an additive value model, defined as follows (7), was built:

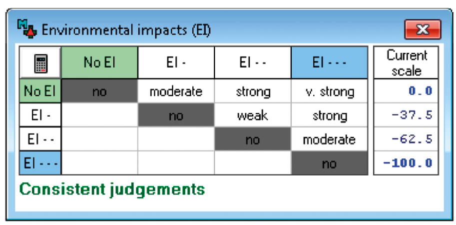
$$V_j = k_1 v_1(NPV_j) + k_2 v_2(EI_j) \text{ with } k_1 > 0 \text{ , } k_2 > 0 \text{ and } k_1 + k_2 = 1 \text{ ,} \tag{7}$$

where: V_j is the overall value of large-scale project *j*; k_1 and k_2 are the scaling constants ("relative weights") of the *NPV* and *EI* criteria, respectively; and $v_1(NPV_j)$ and $v_2(EI_j)$ are the partial value scores of project *j* calculated by the respective value functions – built in such a way that the overall value of the "do nothing" project, i.e., a project with €0 of *NPV* and no *EI*, should result equal to 0 (Clemen, Smith 2009; Morton 2010). The value function over *NPV* was assumed to be linear and was anchored on $v_1(€\ 0) = 0$ and $v_1(€\ 6\ 000 \times 10^3) = 100$. In this exploratory multicriteria analysis, the EDPD's environmental manager first considered four plausible scenarios of increasing environmental impact, ranging from "the project has no significant environmental impact (No EI)" to "the project has significant impact in a protected area (EI ---)" (see Fig. 5a). Then, the MACBETH method (Bana e Costa *et al.* 2012) was applied to build a value function for the *EI* criterion. For this purpose, the EDPD's manager judged the difference in attractiveness between each two of the four *EI* scenarios, shown in the MACBETH judgements matrix of Figure 5b. For example, the difference in attractiveness between "No EI" and "EI -" was judged as *moderate*, whereas the difference between "No EI" and "EI --" was judged as *strong*.

The project has…

… no significant environmental impact	(No EI)
… significant noise or visual impact	(EI -)
… significant noise and visual impact	(EI --)
… significant impact in a protected area	(EI ---)

(a)



	No EI	EI -	EI - -	EI - - -	Current scale
No EI	no	moderate	strong	v. strong	0.0
EI -		no	weak	strong	-37.5
EI - -			no	moderate	-62.5
EI - - -				no	-100.0

(b)

Fig. 5. Environmental impact criterion: (a) descriptor of performance and (b) MACBETH matrix of judgements and value function

The "current scale" column in Figure 5b shows the proposed value scale that was generated by MACBETH, using the linear programming problem described by Bana e Costa *et al.* (2005), which was validated by the EDPD's environmental manager. (The *NPV* scores, *EI* scores and annual costs of the large-scale projects are presented in the Appendix, Table 7).

Under these modelling conditions, and with V_j replacing NPV_j in (3) and (4), the first project to leave the portfolio previously selected would be P01, when k_2 is raised to 5.9%, which means that an *NPV* of € 232.61×10^3 would not be enough to compensate a significant noise and visual impact.

Conclusions

This paper describes a case study in the domain of capital budgeting involving different types of projects and budget constraints in multiple time periods. Two approaches for project portfolio selection were discussed: a heuristic prioritisation procedure followed by the electricity distribution company EDPD, which proved to be suboptimal, and the alternative PROBE approach based on 0–1 linear programming. Large-scale and small-scale projects were analysed separately, for there is a fixed budget for each type. The uncertainty involving the expected cash flows of the large-scale projects was also analysed in terms of its potential consequences for the selection of the portfolio that, respecting all the constraints, could offer the highest total *NPV* to the company. With the support of the PROBE software, several measures and scenarios of uncertainty were built in order to reassure the decision-makers about the robustness of the final investment decision. Besides being concerned with the *NPV*, EDPD also wants to consider other aspects when evaluating the benefits of the projects. A criterion related to environmental impacts of large-scale projects was incorporated in the initial model and a new multicriteria portfolio decision analysis revealed that a small trade-off between *NPV* and environmental impact could alter the composition of the best portfolio.

Research underway aims at incorporating in the PROBE resource allocation model probabilities associated with uncertain input data, which is particularly relevant in assessing environmental impacts. Besides *NPV* and environmental impact, quality of service, appraised by the number of consumers' complaints, is currently being considered in a new multicriteria benefit evaluation model. It is important to remark that the PROBE software runs a mono-objective mathematical programming model. A future line of research is the extension of PROBE to consider multiple objective functions, one for each benefit dimension, instead of optimising an aggregated overall benefit. Last but not least, besides the classic approach to address uncertainty on the current mono-objective model, the extension of portfolio decision analysis to the fuzzy methodological framework opens an interesting path for multi-objective allocation of resources, as explored by Ekel *et al.* (2006, 2008).

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APPENDIX

Table 6. Small-scale projects data (in € 10^3)

Project	Zone	*NPV*	Cost on year 1	Cost on year 2	Project	Zone	*NPV*	Cost on year 1	Cost on year 2
1	2	3	4	5	6	7	8	9	10
P001	Z1	101.18	67.06	0	P036	Z1	40.81	12.86	0
P002	Z1	183.02	57.69	0	P037	Z1	35.09	12.03	0
P003	Z1	40.19	53.98	0	P038	Z1	66.02	12	0
P004	Z1	0.9	14.75	0	P039	Z1	135.43	15.74	0
P005	Z1	133.21	53.97	0	P040	Z1	18.95	10.06	0
P006	Z1	124.46	14.85	26.41	P041	Z1	32.25	12.07	0
P007	Z1	60.06	20.15	0	P042	Z1	167.1	7.75	0
P008	Z1	57.12	13.16	0	P043	Z1	0	10.09	0
P009	Z1	1.19	38.38	0	P044	Z1	1.6	3.55	0
P010	Z1	142.43	37.66	0	P045	Z2	34.9	7.98	0
P011	Z1	47.51	30.55	0	P046	Z2	12.27	7.16	0
P012	Z1	47.26	28.43	0	P047	Z2	5.29	10.57	0
P013	Z1	295.71	6.51	27	P048	Z2	186.84	5.21	0
P014	Z1	135.42	25.04	0	P049	Z2	3.49	22.32	0
P015	Z1	94.77	24.94	0	P050	Z2	52.27	6.32	22.42
P016	Z1	31.71	6.55	0	P051	Z2	25.19	6.58	0
P017	Z1	91.06	23	0	P052	Z2	22.38	8.1	0
P018	Z1	53.98	23.87	1.12	P053	Z2	84.94	13.51	0
P019	Z1	13.72	23.15	0	P054	Z2	222.7	8.4	0
P020	Z1	41.76	22.98	0	P055	Z2	18.01	4.66	0
P021	Z1	0.02	10.15	0	P056	Z2	22.86	4.89	0
P022	Z1	1.54	2.5	0	P057	Z2	26.52	3.44	13.15
P023	Z1	23.35	11.57	13.92	P058	Z2	2.18	1.38	0
P024	Z1	16.61	17.55	0	P059	Z2	0.14	3.77	5.26
P025	Z1	8.9	15.41	0	P060	Z2	105.51	10.83	0
P026	Z1	1.14	21.2	0	P061	Z2	141.47	15.42	0
P027	Z1	3.31	16.65	0	P062	Z2	77.63	15.16	0
P028	Z1	47.31	21	0	P063	Z2	0	45.45	0
P029	Z1	75.59	12.66	0	P064	Z2	184.56	18.23	0
P030	Z1	19.58	6.84	0	P065	Z2	52.59	11.07	0
P031	Z1	3.15	3	0	P066	Z2	91.59	5.34	0
P032	Z1	73.98	14.98	0	P067	Z2	132.12	23.81	0
P033	Z1	61.59	13.91	0	P068	Z2	79.06	5.01	0
P034	Z1	84.48	13.38	0	P069	Z2	57.92	24.67	0
P035	Z1	60.59	13.28	0	P070	Z2	121.84	10.94	0

Continued Table 6

1	2	3	4	5	6	7	8	9	10
P071	Z2	0.06	31.5	0	P110	Z2	29.48	15.5	0
P072	Z2	39.57	1.67	0	P111	Z2	35.56	13.93	26.86
P073	Z2	0.01	20.53	0	P112	Z2	194.79	15.41	0
P074	Z2	14.37	7.34	6.5	P113	Z2	119.95	14.67	0
P075	Z2	63.39	32.23	0	P114	Z2	24.89	14.55	0
P076	Z2	123.42	39.43	0	P115	Z2	31.98	14.44	0
P077	Z2	48.81	29.75	0	P116	Z2	204.09	22.56	0
P078	Z2	34.26	38.89	0	P117	Z3	58.39	23.1	0
P079	Z2	20.93	9.75	0	P118	Z3	42.58	38.54	0
P080	Z2	3.84	15.45	0	P119	Z3	58.2	30.01	0
P081	Z2	28.83	8.04	9.25	P120	Z3	38.37	29.95	0
P082	Z2	72.48	19.95	0	P121	Z3	62.94	27.1	0
P083	Z2	165	7.33	0	P122	Z3	32.8	28.43	0
P084	Z2	154.89	15.23	0	P123	Z3	60.11	32.84	0
P085	Z2	18.52	5.26	0	P124	Z3	83.03	26.99	0
P086	Z2	96.32	19.29	0	P125	Z3	362.81	23.15	0
P087	Z2	11.09	9.51	7.16	P126	Z3	271.75	23.49	0
P088	Z2	45.47	34.64	0	P127	Z3	206.07	16.93	0
P089	Z2	14.07	7.29	0	P128	Z3	358.09	17.1	0
P090	Z2	0.18	6.62	10	P129	Z3	11.42	8.93	0
P091	Z2	45.2	30.13	0	P130	Z3	139.51	17.7	0
P092	Z2	12.97	22.22	0	P131	Z3	6.48	3.64	0
P093	Z2	28.85	20.25	0	P132	Z3	253.23	15.59	1.48
P094	Z2	72.71	10.38	0	P133	Z3	83.6	20.96	0
P095	Z2	102.75	11.71	0	P134	Z3	153.64	36.57	0
P096	Z2	116.23	29.11	0	P135	Z3	85.48	4.88	0
P097	Z2	63.72	23.61	0	P136	Z3	154.34	22.83	0
P098	Z2	11	80	0	P137	Z3	158.67	25.11	0
P099	Z2	80.18	20.84	0	P138	Z3	157.15	4.27	0
P100	Z2	26.34	22.41	0	P139	Z3	124.02	8.51	0
P101	Z2	61.38	15.42	0	P140	Z3	58.01	21.87	0
P102	Z2	34.8	16.74	0	P141	Z3	117.88	5.6	0
P103	Z2	12.43	4.85	0	P142	Z3	299.01	26.68	0
P104	Z2	64.3	27.4	0	P143	Z3	23.35	6.85	0
P105	Z2	115.98	17.55	0	P144	Z3	318.5	23.58	0
P106	Z2	57.76	16.88	0	P145	Z3	47.21	6.69	0
P107	Z2	5.11	20.3	0	P146	Z3	117.57	2.97	0
P108	Z2	26.03	9.82	0	P147	Z3	94.29	11.59	0
P109	Z2	38.05	15.65	0	P148	Z3	26.28	8.94	0

Continued Table 6

1	2	3	4	5	6	7	8	9	10
P149	Z3	88.9	9.8	0	P188	Z3	73.05	25.68	0
P150	Z3	172.81	30.36	0	P189	Z3	72.71	22.85	0
P151	Z3	122.14	8.42	0	P190	Z3	95.22	19.3	0
P152	Z3	123.92	5.91	0	P191	Z3	125.8	22.1	0
P153	Z3	38.48	7.9	0	P192	Z3	70.54	32.7	0
P154	Z3	54.98	8.09	0	P193	Z3	159.37	9.11	0
P155	Z3	144.53	26.96	0	P194	Z3	62.25	26.92	0
P156	Z3	204.29	14.56	0	P195	Z3	4.2	23.32	0
P157	Z3	72.65	3.05	0	P196	Z3	81.28	16.26	25.97
P158	Z3	69.86	8.65	0	P197	Z3	45.54	26.6	0
P159	Z3	42.37	4.14	0	P198	Z3	0.82	18.85	0
P160	Z3	28.94	20.68	0	P199	Z3	14.3	6.78	0
P161	Z3	33.07	15.4	0	P200	Z4	216.29	24.84	0
P162	Z3	41.04	6.77	4.46	P201	Z4	35.78	34.7	0
P163	Z3	9.86	3.65	0	P202	Z4	59.41	13.54	0
P164	Z3	27.9	3.89	0	P203	Z4	24.54	31.76	0
P165	Z3	71.07	12.39	0	P204	Z4	49.69	17.93	0
P166	Z3	0.43	30.29	1.95	P205	Z4	76.68	16.06	0
P167	Z3	41.23	13.67	0	P206	Z4	3.83	24.3	0
P168	Z3	108.87	34.96	0	P207	Z4	12.92	21.07	0
P169	Z3	36.86	17.07	0	P208	Z4	74.53	14.16	0.74
P170	Z3	49.46	23.93	2.68	P209	Z4	14.91	4.25	0
P171	Z3	74.83	37.52	0	P210	Z4	17.51	4.6	0
P172	Z3	81.22	22.89	0	P211	Z4	66.16	13.66	0
P173	Z3	30.6	20.92	0	P212	Z4	51.18	14.52	12.94
P174	Z3	158.4	4.51	0	P213	Z4	83.51	15.99	0
P175	Z3	28.92	4.02	0	P214	Z4	148.52	6.56	0
P176	Z3	44.86	2.73	0	P215	Z4	30.53	10.2	0
P177	Z3	16.85	8.1	0	P216	Z4	35.65	11.11	7.75
P178	Z3	60.83	5.14	0	P217	Z4	0.1	10.26	18.67
P179	Z3	104.88	6.23	0	P218	Z4	37.76	15.04	8.42
P180	Z3	56.07	4.08	0	P219	Z5	8.24	9.43	13.16
P181	Z3	237.64	22.92	0	P220	Z5	0.4	3.66	0
P182	Z3	63.31	17.13	0	P221	Z5	35.08	13.11	2.16
P183	Z3	184.58	32.78	0	P222	Z5	56	13.56	0
P184	Z3	94.47	16.16	0	P223	Z5	8.41	4	0
P185	Z3	127.9	13.41	0	P224	Z5	55.74	3.95	0
P186	Z3	34.16	27.32	0	P225	Z5	6.49	5.17	0
P187	Z3	83.04	22.02	0	P226	Z5	19.9	13.33	0

Continued Table 6

1	2	3	4	5	6	7	8	9	10
P227	Z5	22.8	5.61	23.78	P266	Z5	18.52	41.86	0
P228	Z5	62.67	10.05	0	P267	Z5	14.72	33.91	0
P229	Z5	29.08	11.76	0	P268	Z5	111.87	16	0
P230	Z5	30.27	2.46	0	P269	Z5	76.76	20.2	1.12
P231	Z5	105.71	22.18	0	P270	Z5	17.58	24.13	0
P232	Z5	14.23	4.29	0	P271	Z5	73.73	6.51	14.58
P233	Z5	11.16	10.76	0	P272	Z5	58.97	20.34	0
P234	Z5	1.95	20	0	P273	Z5	98.3	18.27	0
P235	Z5	2.44	26.59	0	P274	Z5	0.86	28.5	0
P236	Z5	38.13	25.19	0	P275	Z5	6.71	18.7	0
P237	Z5	0.96	19.95	0	P276	Z5	3.74	20.64	5.91
P238	Z5	39.64	25.43	0	P277	Z5	5.3	11.77	0
P239	Z5	44.83	11.62	0	P278	Z5	176.73	37.75	0
P240	Z5	7.34	34.83	0	P279	Z5	327.91	13.12	0
P241	Z5	18.97	12.25	0	P280	Z5	17.48	4.82	0
P242	Z5	206.61	17.01	0	P281	Z5	58.16	26.03	0
P243	Z5	97.81	9.17	0	P282	Z5	31.86	45.28	0
P244	Z5	43.69	6.35	0	P283	Z5	142.9	17.54	0
P245	Z5	155.52	51.15	0	P284	Z5	58.36	12.07	0
P246	Z5	85.43	5.32	0	P285	Z5	105.57	27.42	0
P247	Z5	156.27	12.3	0.03	P286	Z5	291.64	34.4	0
P248	Z5	48.79	20.93	0	P287	Z5	43.43	14.54	0
P249	Z5	77.82	8.63	0	P288	Z5	40.1	14.91	0
P250	Z5	286.17	100.89	0	P289	Z5	115.91	7.95	0
P251	Z5	34.79	20.3	5.29	P290	Z5	42.27	8.74	0
P252	Z5	64.72	8.05	0	P291	Z5	15.73	14.24	0
P253	Z5	39.02	11.76	0	P292	Z5	43.85	3.54	0
P254	Z5	19.01	31.67	0	P293	Z5	183.72	14.66	2.93
P255	Z5	288.59	16.29	0	P294	Z6	42.45	41.36	0
P256	Z5	71.18	16.34	0	P295	Z6	9.94	13.84	0
P257	Z5	81.61	11.03	0	P296	Z6	30.9	28.67	0
P258	Z5	43.25	17.38	0	P297	Z6	43.78	31.22	0
P259	Z5	281.03	21.32	0	P298	Z6	1.47	18.39	0
P260	Z5	96.18	31.2	0	P299	Z6	25.18	15.63	0
P261	Z5	27.31	31.51	0	P300	Z6	40.47	15.46	0
P262	Z5	68.91	18.14	0	P301	Z6	2.6	22.11	0
P263	Z5	55.67	20.77	0	P302	Z6	26.59	12.25	0
P264	Z5	212.9	25.79	0	P303	Z6	0.76	5.6	0
P265	Z5	146.96	26.26	0	P304	Z6	53.85	19.4	0

End of Table 6

1	2	3	4	5	6	7	8	9	10
P305	Z6	105.63	20.91	0	P325	Z6	13.2	15.06	0
P306	Z6	81.56	23.84	0	P326	Z6	79.24	58.38	0
P307	Z6	24.06	8.77	0	P327	Z6	20.36	3.13	0
P308	Z6	9.07	13.93	0	P328	Z6	0.01	20.05	0
P309	Z6	57.73	7.91	0	P329	Z6	0.1	20.44	0
P310	Z6	6.76	6.06	0	P330	Z6	35.58	6.36	0
P311	Z6	20.29	15.05	0	P331	Z6	114.28	22.88	0
P312	Z6	40.79	9.3	0	P332	Z6	275.55	9.07	0
P313	Z6	31.12	5.57	0	P333	Z6	56.18	19.12	0.23
P314	Z6	79.53	15.2	0	P334	Z6	26.85	8.49	0
P315	Z6	77.72	22.98	0	P335	Z6	76.74	13.42	0
P316	Z6	50.53	11.61	17.01	P336	Z6	83.96	8.21	0
P317	Z6	41.31	20.25	0	P337	Z6	212	6.94	21.75
P318	Z6	88.72	10.45	0	P338	Z6	124.89	36.82	0
P319	Z6	89.51	20.01	0	P339	Z6	98.32	19.9	0
P320	Z6	103.52	8.99	0	P340	Z6	83.72	25.69	0
P321	Z6	109.16	15.7	0	P341	Z6	119.5	8.45	0
P322	Z6	117.24	19.1	0	P342	Z6	30.16	17.6	0
P323	Z6	149.28	6.65	14.38	P343	Z6	0.01	11.13	0
P324	Z6	43.81	13.42	0	P344	Z6	14.8	101.58	0

Table 7. *NPV* scores, *EI* scores and annual costs (in € 10^3) of the large-scale projects

Project	*NPV* score	*EI* score	Cost on year 1	Cost on year 2	Cost on year 3
1	2	3	4	5	6
P01	3.88	–62.5	27.64	559.49	0
P02	2.48	0	126	0	0
P03	125.20	–62.5	268.78	1800	1.615
P04	49.90	0	2 372.47	2591.65	0
P05	190.20	0	997.95	553.67	3 669.78
P06	38.75	0	3 205.39	169.18	10
P07	24.46	0	3 239.98	288.14	0
P08	9.16	–62.5	2 466.87	431.62	4.04
P09	12.21	–62.5	319.88	1 468.61	0
P10	200.28	–37.5	545.53	56.85	0
P11	3.67	0	69.57	105	0
P12	8.12	0	1 189.53	129.18	0
P13	28.92	0	1 373.22	739.45	0
P14	6.85	–37.5	299.82	7	0
P15	190.78	0	1 587.36	436.12	0
P16	162.36	–37.5	300	400	0

End of Table 7

1	2	3	4	5	6
P17	162.36	–37.5	648.38	1 125.45	0
P18	99.52	0	696.08	1430	1 018.46
P19	136.35	–37.5	2 605.13	2 770.27	0
P20	0.01	0	2 033.12	1 872.21	0
P21	7.77	–37.5	2 066.03	416.59	0
P22	8.93	–37.5	35.49	3 048.74	0
P23	64.94	0	839.78	596.33	0
P24	0.02	–37.5	3 070.71	382.23	0
P25	0.02	–37.5	550.03	649.65	0
P26	0.02	–100	907.09	1 007.67	0
P27	14.98	–37.5	1 280.18	757.59	4 192.11
P28	8.48	0	225	100	1225

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