EUROPEAN UNION MEMBER STATES PREPARING FOR EUROPE 2020. AN APPLICATION OF THE MULTIMOORA METHOD

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Abstract. It is the intention of the European Union to create a growing and sustainable European economy by 2020, a much more moderate target than the 2010 target of becoming the most competitive and dynamic knowledge-based economy in the world. This intention has to be supported by an adequate Optimization and Decision Support System.
Therefore, MULTIMOORA is proposed. MULTIMOORA is a quantitative method, which compares multiple and optimum objectives, expressed in different units, as much as possible on a non-subjective basis. In opposition to similar methods MULTIMOORA does not need normalization, being based on dimensionless measures. Importance of an objective can eventually be given by the stakeholders concerned. MULTIMOORA is composed of three approaches: Ratio System, Reference Point and Multiplicative Form Methods, all of the same importance and each controlling each other. Twenty two objectives, 10 originating from statistics and 12 from statistics and forecasts, important for the future, characterize the 27 EU-Countries economies as a preparation for 2020. Which of these countries are the best prepared for 2020? A Dominance Theory, summarizing the three obtained ordinal numbers per country, ranks the 27 countries for that purpose.

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1. Introduction

The Lisbon Strategy (European Council 2000) was adopted in 2000 and focused on turning the European Union (EU) into "the most competitive and dynamic knowledge–based economy in the world" by 2010.

Today it is obvious that many goals of the Lisbon Strategy for 2010 were not met. Therefore the new strategy Europe 2020 was initiated in 2010 (European Commission 2010). The new strategy aims to the creation of a growing and sustainable European economy. Furthermore, greater policy coordination between national governments and the EU is an additional focal point of the Strategy Europe 2020.

Twenty two objectives, 10 originating from statistics and 12 from statistics and forecasts, important for the future, will characterize the 27 EU-Countries economies as a preparation for 2020. Nevertheless these data concern only the economic guidelines of Strategy Europe 2020 and not the climate and energy targets.

As all these data are expressed in different units the exercise needs an adequate Decision Support System for optimization. For the researcher in multi-objective decision support systems the choice between many methods is not very easy. Indeed numerous theories were developed since the forerunners: Condorcet (the Condorcet Paradox, against binary comparisons, 1785, LVIII), Gossen (law of decreasing marginal utility, 1853) Minkowski (Reference Point 1896, 1911) and Pareto (Pareto Optimum and Indifference Curves analysis 1906, 1927) and pioneers like Kendall (ordinal scales, since 1948), Miller and Starr (Multiplicative Form 1969), Roy *et al.* (ELECTRE, since 1966), Hwang and Yoon (TOPSIS 1981), Brans *et al.* (PROMETHEE since 1984), Saaty (AHP, since 1988) and Opricovic, Tzeng (VIKOR 2004; Fouladgar *et al.* 2012). Already in 1983 at least 96 methods for Multi-Objective Optimization existed (Despontin *et al.* 1983). Since then numerous other methods appeared. Therefore, we only cited the probably most used methods for Multi-Objective Optimization.

Twenty two objectives from 27 countries result in 594 data, which means that methods of partial aggregation can not be used. Schärlig (1985, 1996) gave the name of partial aggregation to: the Electre Group (Electre I, Electre Iv, Electre Is, Electre TRI, Electre II, Electre III and Electre IV) and Prométhée.

The Analytic Hierarchy Process (AHP) of Saaty (1988) uses weights. The use of weights was introduced by Churchman and Ackoff (1954) and Churchman *et al.* (1957). This Additive Weighting Procedure (MacCrimmon, 1968: 29–33) was called SAW, Simple Additive Weighting Method, by Hwang and Yoon (1981: 99). Weights defined in that way for using in Multi-Objective Optimization have the disadvantage to possess a double content namely normalization on the one side and giving importance on the other. In addition, for the European example, 22 weights have to be defined or 44 normalization and importance exercises.

Also the methods of partial aggregation use weights. Even more, all these methods are expert oriented with qualitative statements as a basis. As conclusion one may say that for this application preference has to be given to a method of total aggregation and a method not using weights.

The article is organized in the following way. Section 2 deals with the objectives characterizing the economies of the EU Countries for the present and for the future. Section 3 focuses on further considerations on methods for Multi-Objective Optimization. The MOORA

method is explained in section 4 and MULTIMOORA in section 5, both methods satisfying the conditions of total aggregation and of not using weights. Section 6 describes the numerical example where the European Union States are compared on basis of structural indicators and of the new method. Finally conclusions are drawn: which of these countries are the best prepared for 2020 from the economic point of view? A Dominance Theory, summarizing the three obtained ordinal numbers per country, ranks the 27 countries for that purpose.

2. Choice of objectives characterizing the economies of the EU countries in the present and in the future

The Choice of Objectives characterizing the Economies of a Country for the Present and for the Future is restricted to the available data. Afonso *et al.* (2011) mention that four variables have short-run impact on ratings namely the level of GDP per capita, real GDP growth, the public debt level and the government balance. Government effectiveness, the level of external debt and external reserves are important long-run determinants.

The national economies to be studied concern the Economies of the European Union Member States. We selected twenty-two objectives, ten originating from statistics and twelve from statistics and forecasts important for the future, in order to characterize the 27 EU-Countries economies as shown in the following Table 1.

No. 1, 17 and 21 Government Budget Deficit

The IMF correlates the Government Budget Deficit to GDP for a given year and not to GNP. Let us remember that the Gross Domestic Product (GDP) is a territorial concept, the Value Added realized on the territory of a country during a certain year. On the contrary the Gross National Product (GNP) is a personal concept related to the citizens and the permanent residents of a country[1]. In fact the Government Budget Deficit is a part of the yearly Government Budget and rather has to be related to that budget and not to GDP, otherwise some countries are punished and others are rewarded. Indeed at the time of the EU 15 the government budget of Sweden compared to 56.6% of its GDP (Belgian Federal Department of Finance, Documentatieblad 2008, 46), which means that for instance a government deficit of 3% of GDP corresponds to a deficit of 5.3% of its government budget. At the other side Ireland with 34% government expenses compared to its GDP (Belgian Federal Department of Finance, op.cit. 46) may run a deficit of 8.8% of its government budget. One could conclude that a country with high government expenses compared to GDP is punished in that way.

Interesting to know is that the government budget deficit is linked to the government debt. Indeed the deficit and the increase in the interest payments on the government debt determine an increase in the government debt. In this way the government debt is composed of primary expenditures and of interest payments. Consequently, the impact of the government on the government debt runs through the budget deficit and the obtained rates of interest on the government debt.

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[1]   Also the Treaty of Maastricht, the starting point of the EURO, correlates the Government Budget Deficit to GDP (Belgian Federal Department of Finance, Documentatieblad 2008, 29).

Table 1. The 22 objectives characterizing the Economies of the European Union Member States

No.	Indicator	Period	Dimension	Optimum
	A. Actual			
1	Government Budget Deficit (a)	2010	% of GDP	MIN
2	Government Debt (a)	2010	% of GDP	MIN
3	Current account deficit (BoP) (a)	2010	% of GDP	MIN
4	GDP per capita in PPP (a)	2010	in current international dollar	MAX
5	GDP growth rate (a)	2010	per cent of GDP of previous year	MAX
6	Inflation (a)	2010	per cent compared to a reference period	MIN
7	Government bond yields (a)	2010	per cent	MIN
8	Employment rate (b)	2010	per cent of age group 15–64	MAX
9	Unemployment rate (b)	2010	per cent of the professional population	MIN
10	Tertiary education (b)	2010	per cent of age group 30–34	MAX
	B. Prospective			
11	Median Age (b)	2010	Median of total population	MIN
12	Proportion of population aged 0–14 (b)	2010	per cent of total population	MAX
13	Proportion of population aged 15–64 (b)	2010	per cent of total population	MAX
14	Proportion of population aged 65 and over (b)	2010	per cent of total population	MIN
15	GDP per capita (EU-15 = 100) (c)	2011	index number with EU-15 = 100	MAX
16	GDP growth rate (c)	2011	% of GDP of previous year	MAX
17	Government Budget Deficit (c)	2011	% of GDP	MIN
18	Government consolidated gross debt (c)	2011	% of GDP	MIN
19	GDP per capita (EU-15 = 100) (c)	2012	index number with EU-15 = 100	MAX
20	GDP growth rate (c)	2012	per cent of GDP of previous year	MAX
21	Government Budget Deficit (c)	2012	% of GDP	MIN
22	Government consolidated gross debt (c)	2012	% of GDP	MIN

(a) The data come from IMF and World Bank (2011).
(b) EUROSTAT (2011).
(c) European Commission (2011).

No. 2, 18 and 22 Government Debt

The International Monetary Fund (IMF) once again relates Government Debt to Gross Domestic Product. Government Debt is distinct from the debt of the households, which depends on household expenditures and savings. In Europe the savings ratio of the households is the highest in Spain, Germany and Belgium, translated in Belgium of a participation ratio of the Belgians in their own government budget of around 46%. The International Monetary Fund does not separate domestic from external financing for the government debt, though an important issue. Related to GDP Japan has one of the highest government debts of the world, probably around 200% of GDP, but mainly financed by the own population. As a consequence the interest rate on the government debt is extremely low, sometimes 1% on 10-year govern-

ment bonds. On the contrary Greece with a government debt of 142.8% in 2010; estimated at 157.7% in 2011 and at 166.1% in 2012, had to pay an interest rate of 13.2% in 2010 increased to 16.69% on June 11, 2011 (European Central Bank 2011). The reason is that around 71% of the Greek government debt is in the hands of foreigners, mainly foreign banks.

Once the foreign part of government debt is very high it is difficult to bring it down. Previously in Europe devaluation of the national currencies was helpful, which is impossible with the Euro, unless as a function of a basket of the main other currencies in the world, comparable to the system of drawing rights of the International Monetary Fund. Otherwise, one has to foresee international transfer payments or reduction or complete remission of the government debt in the hands of foreigners. Nevertheless, one has to question the possibility to foresee a fiscal outlook until 2035 as has been done by Gagnon (2011). Any way we stopped our forecasts until the end of 2012.

No. 3 Current Account Deficit in the Balance of Payments (BoP)

The Current Account Balance is the sum of net exports of goods, services, net income, and net current transfers. Once again the IMF correlates the Current Account Deficit in the Balance of Payments to GDP and not as a part of the total of the Current Account of the Balance of Payments for a certain period[2]. Nevertheless it may happen that in typical export oriented countries like the Benelux countries the total of their Current Account of the Balance of Payments is larger than their GDP.

No. 4 GDP per Capita in Purchasing Power Parity (PPP)

Purchasing Power Parity means that the GDP per country is not compared on basis of exchange rates but on basis of the consumer basket with its prices of a country compared with the consumer basket with its prices, the purchasing power, of a standard country, namely the United States. In principle the GDP per Capita in a Purchasing Power Parity (PPP) would measure the real purchasing power of the consumer in a country.

No. 5, 16 and 20 GDP Growth Rate

GDP Growth Rate represents the per cent of GDP growth compared to previous year in constant prices.

No. 6 Inflation

Inflation is expressed in averages for the year, not end-of-period data. A consumer price index (CPI) measures changes in the prices of goods and services that households consume. Such changes affect the real purchasing power of consumer income. As all the prices of different goods and services do not change at the same rate, a price index can only reflect their average movement. A price index is assigned a value compared to some reference period. Price indices can be used to measure differences in price levels between different countries at the same point in time. For Euro Countries, consumer prices are calculated based on harmonized prices.

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[2]   Also the World Bank correlates the Current Account Deficit in the Balance of Payments to GDP (World Bank 2011).

No. 7 Government Bond Yields

Yields of Government Bonds concern Government bond yields with maturity close to 10 years.

No. 8 Employment Rate

The Employment Rate is the rate of people employed as a percentage of the age group 15–64.

No. 9 Unemployment Rate

The Unemployment rate is the rate of people unemployed as a percentage of the professional population (people disposed to work in the age group 15–64).

No. 10 Tertiary Educational Attainment

Tertiary Educational Attainment is the rate of people who finished any form of higher education as a percentage of the age group 30–34.

No. 11 Median Age

Median Age is the median of the ages of the entire population.

No. 12 Proportion of population aged 0–14

Population aged 0–14 is compared to total population.

No. 13 Proportion of population aged 15–64

Population aged 15–64 is compared to total population.

No. 14 Proportion of population aged 65 and over

Population aged 65 and over is compared to total population.

No. 15 and 19 GDP per capita (EU-15 = 100)

GDP per capita is a percentage compared to the GDP per capita with EU-15 = 100. In the year 2000 the European Union was composed of the original 15 member states. In 2004 the 10 new countries joined and finally in 2007 Romania and Bulgaria. In 2013 Croatia will join.
    The data for the 22 objectives are given in the following Table 2.

3. Choice of a method for multi-objective optimization

Based on previous analysis, methods of partial aggregation and methods based on weights are excluded for large decision matrices.
    How is it possible to exclude Weights? Therefore it is necessary to read the decision matrix, in the European example composed of 594 elements, not in the horizontal but in the vertical way. By making averages per column of the decision matrix dimensionless measures are obtained.

Table 2. Data for the 22 objectives for the European Union Member States (2010–2012)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
	MIN	MIN	MIN	MAX	MAX	MIN	MIN	MAX	MIN	MAX	MIN	MAX	MAX	MIN	MAX	MAX	MIN	MIN	MAX	MAX	MIN	MIN
Belgium	4.1	96.8	–1.04	32,600	2.2	2.3	3.9	62.0	8.3	44.4	40.9	16.9	65.9	17.2	106	2.4	3.7	97	106	2.2	4.2	97.5
Bulgaria	3.2	16.2	0.99	4,800	0.2	3.0	5.5	59.7	10.2	27.7	41.4	13.6	68.9	17.5	40.5	2.8	2.7	18	41.6	3.7	1.6	18.6
Czech R.	4.7	38.5	3.84	14,200	2.3	1.2	3.8	65.0	7.3	20.4	39.4	14.2	70.6	15.2	75	2	4.4	41.3	76.1	2.9	4.1	42.9
Denmark	2.7	43.6	–5.48	42,200	1.7	2.2	2.9	73.4	7.4	47.0	40.5	18.1	65.6	16.3	110	1.7	4.1	45.3	109	1.5	3.2	47.1
Germany	3.3	83.2	–5.71	30,300	3.7	1.2	2.8	71.1	7.1	29.8	44.2	13.5	65.8	20.7	109	2.6	2	82.4	110	1.9	1.2	81.1
Estonia	–0.1	6.6	–3.76	10,700	2.3	2.7	5.4	61.0	16.9	40.0	39.5	15.1	67.8	17.1	60.6	4.9	0.6	6.1	62	4	2.4	6.9
Ireland	32.4	96.2	0.71	34,900	–0.4	–1.6	9.2	60.0	13.7	49.9	34.3	21.3	67.4	11.3	111	0.6	10.5	112	111	1.9	8.8	118
Greece	10.5	142.8	10.5	20,100	–4.5	4.7	13.2	59.6	12.6	28.4	41.7	14.4	66.7	18.9	75.9	–3.5	9.5	158	75.5	1.1	9.3	166
Spain	9.2	60.1	4.56	23,100	–0.1	2.0	5.1	58.6	20.1	40.6	39.9	14.9	68.3	16.8	91	0.8	6.3	68.1	90.7	1.5	5.3	71
France	7	81.7	1.74	29,800	1.5	1.7	3.3	64.0	9.8	43.5	39.8	18.5	64.9	16.6	97.2	1.8	5.8	84.7	97.2	2	5.3	86.8
Italy	4.6	119	3.29	25,600	1.3	1.6	4.7	56.9	8.4	19.8	43.1	14.1	65.7	20.2	92.8	1	4	120	92.3	1.3	3.2	120
Cyprus	5.3	60.8	7.74	21,600	1.0	2.6	5.0	69.7	6.3	45.1	36.2	16.9	70.0	13.1	88	1.5	5.1	62.3	88.1	2.4	4.9	64.3
Latvia	7.7	44.7	–3.58	8,000	–0.3	–1.2	7.0	59.3	18.7	32.3	40.0	13.8	68.8	17.4	47.6	3.3	4.5	48.2	49.2	4	3.8	49.4
Lithuania	7.1	38.2	–1.85	8,400	1.3	1.2	5.1	57.8	17.8	43.8	39.2	15.0	68.9	16.1	52.7	5	5.5	40.7	54.8	4.7	4.8	43.6
Luxemburg	1.7	18.4	–7.82	79,500	3.5	2.8	3.1	65.2	4.5	46.1	38.9	17.7	68.3	14.0	248	3.4	1	17.2	251	3.8	1.1	19
Hungary	4.2	80.2	–2.06	9,700	1.2	4.7	7.3	55.4	11.2	25.7	39.8	14.7	68.7	16.6	59.7	2.7	–1.6	75.2	60.6	2.6	3.3	72.7
Malta	3.6	68.0	4.17	14,800	2.7	2.0	4.4	56.0	6.9	18.6	39.2	15.6	69.6	14.8	75	2	3	68	75.2	2.2	3	67.9
Netherlands	5.4	62.7	–7.16	35,400	1.8	0.9	3.1	74.7	4.5	41.4	40.6	17.6	67.1	15.3	119	1.9	3.7	63.9	118	1.7	2.3	64
Austria	4.6	72.3	–2.71	34,100	2.3	1.7	3.3	71.7	4.4	23.5	41.7	14.9	67.5	17.6	114	2.4	3.7	73.8	114	2	3.3	75.4
Poland	7.9	55.0	4.47	9,300	3.8	2.7	5.9	59.3	9.6	35.3	37.7	15.2	71.3	13.5	57.6	4	5.8	55.4	58.8	3.7	3.6	55.1
Portugal	9.1	93.0	9.88	16,200	1.3	1.4	8.4	65.6	12.0	23.5	40.7	15.2	66.9	17.9	70	–2.2	5.9	102	67.8	–1.8	4.5	107
Romania	6.4	30.8	4.06	5,700	–1.3	6.1	7.1	58.8	7.3	18.1	38.3	15.2	69.9	14.9	40.9	1.5	4.7	33.7	41.9	3.7	3.6	34.8
Slovenia	5.6	38.0	0.84	17,300	1.4	2.1	4.3	66.2	7.3	34.8	41.4	14.0	69.5	16.5	79.6	1.9	5.8	42.8	80.2	2.5	5	46
Slovakia	7.9	41.0	3.44	12,100	4.0	0.7	4.2	58.8	14.4	22.1	36.9	15.3	72.4	12.3	69.3	3.5	5.1	44.8	71.2	4.4	4.6	46.8
Finland	2.5	48.4	–3.09	33,600	3.6	1.7	3.1	68.1	8.4	45.7	42.0	16.6	66.4	17.0	105	3.7	1	50.6	106	2.6	0.7	52.2
Sweden	0	39.8	–6.26	37,000	5.7	1.9	3.0	72.7	8.4	45.8	40.7	16.6	65.3	18.1	114	4.2	–0.9	36.5	115	2.5	–2	33.4
U. K.	10.4	80.0	3.18	27,400	1.4	3.3	3.3	69.5	7.8	43.0	39.6	17.5	66.0	16.5	101	1.7	8.6	84.2	101	2.1	7	87.9

Reference Point Methods like TOPSIS (Hwang, Yoon 1981) and VIKOR (Opricovic, Tzeng 2004) do not use weights but rather dimensionless measures, but they are overtaken by MOORA which is composed of two different dimensionless based methods, each controlling each other.

Chakraborty (2011) for decision making in manufacturing compared MOORA to TOPSIS and VIKOR concerning: computational time, simplicity, mathematical calculations and stability. For all these characteristics MOORA was superior to TOPSIS and VIKOR.

Later (Brauers, Zavadskas 2010) MULTIMOORA overtook MOORA by assembling three instead of two methods and, in fact, by assembling all possible methods with dimensionless measures.

The Ratio System is the first part of MOORA. In MOORA the decision matrix is read not in the horizontal but in the vertical way. The obtained ratios are used as inputs in the Reference Point Method where they are compared with the coordinates of a Reference Point.

In order to make the series of dimensionless measures methods complete a Full Multiplicative Form is added, whereby its factors loose their identity and ipso facto become dimensionless.

4. Multi-objective optimization by ratio analysis (MOORA)

4.1. The two parts of MOORA

The method starts with a matrix of responses of different alternatives on different objectives:

$$(x_{ij}),$$

with: x_{ij} as the response of alternative j on objective i

$i = 1, 2,…, n$ as the objectives.

$j = 1, 2,…, m$ as the alternatives.

MOORA goes for a ratio system in which the response of an alternative on an objective is compared to a denominator, which is representative for all alternatives concerning that objective.

If the sum of each alternative per objective is chosen for this denominator, the traditional formula of averages is obtained:

$$x_{ij}^o = \frac{x_{ij}}{\sum\limits_{j=1}^{m} x_{ij}}. \tag{1}$$

This formula may lead to unexpected results. Indeed, for instance in the case of productivity growth some sectors, regions or countries may show a decrease instead of an increase in productivity i.e. a negative number. In this way the sum in the denominator could become negative and ipso facto all ratios become negative. Even worse the denominator can become zero and division by zero means a senseless operation.

Brauers, Zavadskas (2006) still tested eight other formulas. They all had main disadvantages with exception of the formula where for the denominator the square root of the sum of squares of each alternative per objective was chosen:

$$x_{ij}^* = \frac{x_{ij}}{\sqrt{\sum\limits_{j=1}^{m} x_{ij}^2}}, \tag{2}$$

with: x_{ij} = response of alternative *j* on objective *i*.

$j = 1, 2,..., m; m$ the number of alternatives.

$i = 1, 2,...n; n$ the number of objectives.

x_{ij}^* = a dimensionless number representing the response of alternative j on objective *i*.

Dimensionless Numbers, having no specific unit of measurement, are obtained for instance by deduction, multiplication or division. The normalized responses of the alternatives on the objectives belong to the interval [0; 1]. However, sometimes the interval could be [−1; 1]. Indeed, for instance, in the case of productivity growth some sectors, regions or countries may show a decrease instead of an increase in productivity i.e. a negative dimensionless number.

For optimization these responses are added in case of maximization and subtracted in case of minimization:

$$y_j^* = \sum_{i=1}^{i=g} x_{ij}^* - \sum_{i=g+1}^{i=n} x_{ij}^*, \tag{3}$$

with: $i = 1, 2,…, g$ as the objectives to be maximized.

$i = g + 1, g + 2,…, n$ as the objectives to be minimized.

y_j^* as the assessment of alternative *j* with respect to all objectives.

An ordinal ranking in a descending order of the y_j^* shows the final preference.

For the second part of MOORA the Reference Point Theory is chosen with the *Min-Max Metric* of Tchebycheff as given by the following formula (Karlin, Studden 1966: 280):

$$\min_j \left\{ \max_i |r_i - x_{ij}^*| \right\}, \tag{4}$$

with $|r_i - x_{ij}^*|$ the absolute value if x_{ij}^* is larger than r_i for instance by minimization.

This reference point theory starts from the already normalized ratios as defined in the MOORA method, namely formula (1). Preference is given to a reference point possessing as co-ordinates the dominating co-ordinates per attribute of the candidate alternatives and which is designated as the *Maximal Objective Reference Point*. This approach is called realistic and non-subjective as the co-ordinates, which are selected for the reference point, are realized in one of the candidate alternatives. The alternatives A (10; 100), B (100; 20) and C (50; 50) will result in the Maximal Objective Reference Point R_m (100; 100).

Given the composition of equation (4) the results are ranked in an ascending order.

4.2. The Importance given to an objective by the attribution method in MOORA

It may look that one objective can not be much more important than another one as all their ratios are smaller than one (see formula 2) Nevertheless it may turn out to be necessary to stress that some objectives are more important than others. In order to give more importance to an objective its ratios could be multiplied with a *Significance Coefficient*.

In the Ratio System in order to give more importance to an objective its response on an alternative under the form of a dimensionless number could be multiplied with a Significance Coefficient:

$$\ddot{y}_j^* = \sum_{i=1}^{i=g} s_i x_{ij}^* - \sum_{i=g+1}^{i=n} s_i x_{ij}^* , \tag{5}$$

with: $i = 1, 2,\ldots,$ g as the objectives to be maximized.

   $i = g + 1, g + 2,\ldots, n$ as the objectives to be minimized.

   s_i = the significance coefficient of objective i.

   $\ddot{y}_j^*$ = the total assessment with significance coefficients of alternative j with respect to all objectives.

For the Reference Point Approach the place of the significance coefficient would be:

$$\min_j \left\{ \max_i \left| s_i r_i - s_i x_{ij}^* \right| \right\} .$$

The *Attribution of Sub-Objectives* represents another solution. Take the example of the purchase of fighter planes (Brauers 2002). For economics, the objectives concerning the fighter planes are threefold: price, employment and balance of payments, but also there is military effectiveness. In order to give more importance to military defense, effectiveness is broken down in, for instance, the maximum speed, the power of the engines and the maximum range of the plane. Anyway, the *Attribution Method* is more refined than a significance coefficient method could be as the attribution method succeeds in characterizing an objective better. For instance, for employment two sub-objectives replace a significance coefficient of two and in this way characterize the direct and indirect side of employment separately.

**4.3. The importance given to an objective by the attribution method
   in the application for the economies of the EU-countries**

After the given military example the importance given to an objective in the application for the Economies of the EU-countries is not done by the introduction of significance coefficients but by sub-objectives. However, the sub-objectives are already present but are depending on a super objective:

   The Super Objective of Economic Importance represented by 10 objectives:
   1. Current account deficit of the Balance of Payments (2010)
   2. GDP per capita in Purchasing Power Parity (2010)
   3. GDP growth rate (2010)
   4. Inflation (2010)

5. Employment rate (2010)
6. Unemployment rate (2010)
7. GDP per capita index number with basis EU-15 = 100 (2011)
8. GDP growth rate (2011)
9. GDP per capita index number with basis EU-15 = 100 (2012)
10. GDP growth rate (2011)

The Super Objective of Public Finance represented by 7 objectives:
1. Government Budget Deficit (2010)
2. Government Debt (2010)
3. Government bond yields (2010)
4. Government Budget Deficit (2011)
5. Government Debt (2011)
6. Government Budget Deficit (2012)
7. Government Debt (2012)

The Super Objective of the Population Pyramid represented by 5 objectives:
1. Tertiary education (2010)
2. Median Age (2010)
3. Proportion of population aged 0–14 (2010)
4. Proportion of population aged 15–64 (2010)
5. Proportion of population aged 65 or more (2010)

5. MULTIMOORA

As MOORA is based on dimensionless measures why not go further by adding the remaining form which uses dimensionless measures namely the Multiplicative Form? We even prefer to speak of the "Full-Multiplicative Form". Otherwise it could refer to a combination with linearity.

The following *n*-power form for multi-objectives is called from now on a Full-Multiplicative Form:

$$U_j = \prod_{i=1}^{n} x_{ij} ,$$ (6)

with: j = 1, 2,..., m; m the number of alternatives.

i = 1, 2,…, n; n being the number of objectives.

x_{ij} = response of alternative j on objective i.

U_j = overall utility of alternative j.

The overall utilities (U_j), obtained by multiplication of different units of measurement, become dimensionless. The outcome of this presentation is nonlinear, which presents an advantage, as the utility function of human behavior toward several objectives has to be considered as nonlinear.

Rule
In the full-multiplicative form the relation between the utilities U_j does not change if more importance is given to an objective by multiplying it by a factor. Indeed, at that moment all alternatives are multiplied with that factor.

Consequence 1
In the full-multiplicative form the introduction of weights is meaningless. Indeed weights are here, in fact, multiplying coefficients.

Consequence 2
In the full-multiplicative form an attribute of the size 10, 10^2, 10^3, 10^6, 10^9 etc. can be replaced by the unit size without changing the relationship between the utilities of the alternatives.

This consequence is extremely important for attributes expressed in monetary units. Instead of expressing an attribute in tens, hundreds, thousands, millions, billions, for instance, of dollars, the use of one digit in the integer part is sufficient.

Is it possible to give more importance to an objective? Allocating an exponent to an objective signifies stressing the importance of this objective.

How is it possible to combine a minimization problem with the maximization of the other objectives? Therefore, the objectives to be minimized are denominators in the formula:

$$U_j' = \frac{A_j}{B_j}, \tag{7}$$

with: $A_j = \prod_{i=1}^{g} x_{ij}$,

$j = 1, 2,..., m$; m the number of alternatives;
i as the number of objectives;
g as the number of objectives to be maximized;

with: $B_j = \prod_{i=g+1}^{n} x_{ij}$,

$n-g$ = the number of objectives to be minimized;

with: U_j' the utility of alternative j with objectives to be maximized and objectives to be minimized.

In the Full Multiplicative Form a problem may arise for zero and negative values making the results senseless. Therefore the index number 100 replaces the zero number. At that moment for instance 96.6 substitutes the negative value of minus 3.4. Consequently, 103.4 represents the positive value of 3.4.

Summarizing, one can conclude that the MULTIMOORA method is composed of three parts: the Ratio System, the Reference Point Method and the Full Multiplicative Form. The three methods were already discussed but separately by Brauers (2004), whereas the name of MULTIMOORA was given by Brauers and Zavadskas in 2010.

6. MULTIMOORA as applied on the multi-objective optimization of the economies of the EU Member countries

The initial data were translated in dimensionless ratios according to Eqs. 2 and 3, i.e. the Ratio System of MOORA. Subsequently Eq. 4 used the ratios obtained in Eq. 2 to calculate the distances to the Reference Point of MOORA. Finally, the Full Multiplicative Form used the initial data to rank the Member States according to Eq. 6 and 7. Following Table 3 presents the results of multi-objective optimization.

Table 3. The three Approaches of MULTIMOORA as applied for the Economy of the EU Member States (a)

	Member States	Ratio S.	Ref. Point	Full Multipl. F.	Rank RS	Rank RP	Rank MF
1	Belgium	−0.07166	0.3196606	4182584.6	12	6	10
2	Bulgaria	−0.17729	0.5091396	310541.0781	13	22	16
3	Czech Republic	−0.11831	0.4534052	2039033.258	11	13	15
4	Denmark	0.865113	0.2549106	88764673141	6	3	8
5	Germany	0.373762	0.335337	65568553083	7	8	6
6	Estonia	0.172228	0.4689265	1.05313E+12	4	16	2
7	Ireland	−0.50645	0.7270675	12751986265	25	26	23
8	Greece	−1.1709	0.710736	68949.84482	27	27	27
9	Spain	−0.38842	0.4814092	582713.4039	24	19	25
10	France	0.11367	0.3719685	18481324.78	18	9	18
11	Italy	−0.33378	0.4322718	3849003.582	22	12	22
12	Cyprus	−0.0528	0.6053159	3072201.537	19	24	20
13	Latvia	−0.3024	0.4873291	9.29662E+11	15	21	12
14	Lithuania	−0.16221	0.4846028	1715231781	10	20	11
15	Luxemburg	1.609567	0.5091396	1.45714E+11	1	7	3
16	Hungary	−0.26461	0.4757423	1046439986	20	17	5
17	Malta	−0.21317	0.466408	2143188.821	17	15	17
18	Netherlands	0.75226	0.3005764	1.9199E+11	5	2	7
19	Austria	0.378763	0.3094369	49788752164	8	4	9
20	Poland	−0.33806	0.4784686	419639.2501	16	18	19
21	Portugal	−0.83554	0.6884153	256512.0942	26	25	26
22	Romania	−0.38694	0.5030054	42127.3994	23	23	24
23	Slovenia	0.054059	0.4239422	7002669.453	14	10	14
24	Slovakia	−0.29411	0.4593844	1376975.646	9	14	13
25	Finland	0.567699	0.3128448	70281186108	3	5	4
26	Sweden	1.716535	0.2896712	1.53242E+14	2	1	1
27	United Kingdom	−0.00365	0.427989	4160130.904	21	11	21

(a) The details of the calculations can eventually be given by the authors. Table 2 forms the basis of these calculations.

7. The theory of dominance

How to make a synthesis between the results of the three approaches: Ratio System, Reference Point Method, which uses the ratios obtained in the ratio system as coordinates, and the Full Multiplicative Form?

The use of the Rank Correlation Method is misleading, see Appendix A. Instead Brauers *et al.* developed a *Theory of Dominance* (Brauers, Zavadskas 2011).

7.1. Axioms on ordinal and cardinal scales

1. *A deduction of an Ordinal Scale, a ranking, from cardinal data is always possible* (Arrow 1974).
2. *An Ordinal Scale can never produce a series of cardinal numbers* (Arrow).
3. *An Ordinal Scale of a certain kind, a ranking, can be translated in an ordinal scale of another kind.*

In application of axiom 3 we shall translate the ordinal scales of the three methods of MULTIMOORA in another one based on Dominance, being Dominated, Transitivity and Equability.

7.2. Dominance, being dominated, transitiveness and equability

Stakeholders or their representatives may give a different importance to objectives in a multi-objective problem, but this is not the case with the three methods of MULTIMOORA. These three methods represent all possible methods with dimensionless measures in multi-objective optimization and one can not argue that one method is better than or is of more important than the other ones.

Dominance
Absolute Dominance means that an alternative, solution or project is dominating in ranking all other alternatives, solutions or projects which are all being dominated. This absolute dominance shows as rankings for MULTIMOORA: (1-1-1).

General Dominance in two of the three methods with a P b P c P d (P preferred to) is for instance of the form:
(d-a-a) is generally dominating (c-b-b).
(a-d-a) is generally dominating (b-c-b).
(a-a-d) is generally dominating (b-b-c)
and further on transitiveness plays fully.

Transitiveness
If a dominates b and b dominates c than a will also dominate c.

Overall Dominance of one alternative on another
For instance (a-a-a) is overall dominating (b-b-b) which is being overly dominated by (a-a-a).

Equability
Absolute Equability has the form: for instance (e-e-e) for 2 alternatives.
Partial Equability of 2 on 3 exists e. g. (5-e-7) and (6-e-3).

Circular Reasoning

Despite all distinctions in classification some contradictions remain possible in a kind of Circular Reasoning.

We can cite the case of:

Object A (11-20-14) dominates generally Object B (14-16-15).

Object B (14-16-15) dominates generally Object C (15-19-12)

but Object C (15-19-12) dominates generally Object A (11-20-14).

In such a case the same ranking is given to the three objects.

The same rules apply for the three methods of MULTIMOORA with no significance coefficients proposed as the three methods are considered to have the same importance.

In the following Table 4 the final classification of MULTIMOORA is presented.

Table 4. The final classification of MULTIMOORA for the Economies of the EU Countries

MULTIMOORA	Countries	Rank RS	Rank RP	Rank MF
1	Sweden	2	**1**	**1**
2	Luxemburg	**1**	7	**3**
3	Finland	**3**	5	**4**
4	Netherlands	**5**	**2**	7
5	Denmark	**6**	**3**	8
6	Germany	**7**	8	**6**
7	Austria	**8**	4	**9**
8	Estonia	**4**	16	**2**
9	Belgium	12	**6**	**10**
10	Slovakia	**9**	14	**13**
11	Slovenia	14	**10**	**14**
12	Lithuania	**10**	20	**11**
13	Czech Republic	**11**	13	**15**
14	France	**18**	9	18
15	Hungary	20	**17**	**5**
16	Latvia	**15**	21	**12**
17	Malta	**17**	**15**	17
18	Bulgaria	**13**	22	**16**
19	Poland	**16**	**18**	**19**
20	Cyprus	**19**	24	**20**
21	U. K.	**21**	**11**	**21**
22	Italy	**22**	**12**	**22**
23	Romania	**23**	23	**24**
24	Spain	**24**	**19**	25
25	Ireland	**25**	26	**23**
26	Portugal	**26**	**25**	**26**
27	Greece	27	27	27

If there is General Dominance in two of the three methods the domination is indicated with a large bold figure.

7.3. Some special remarks

Overall Dominance of one Alternative on another is noted in the following cases:
- The Netherlands on Denmark
- Poland on Cyprus
- United Kingdom on Italy and Romania
- Italy on Romania
- Portugal on Greece.

The group of ten which only joined the EU in 2004 is doing quite well and especially these countries: Estonia (which even joined the EURO Group), Slovakia, Slovenia, Lithuania, and the Czech Republic.

The so called PIIGS are indeed the last classified together with Romania, but also the United Kingdom is not very well classified.

8. Conclusion

We prefer to estimate the economic worth of the European Union Member States by Multi-Objective Optimization towards 2020. Therefore 22 objectives with an actual and future outlook were selected to characterize each EU Member State.

Next problem was the choice of an effective method of Multi-Objective Optimization. This method has to use complete and not partial aggregation, as an overall view of the countries is needed, and has to avoid the use of weights, being dual on normalization and importance. Therefore methods based on dimensionless measures are preferred. Responding to all these conditions MULTIMOORA was finally chosen. In addition MULTIMOORA is composed of three approaches, each controlling each other. In this way all possible methods based on dimensionless measures are included.

Having the results of the three approaches, Ratio Analysis System, Reference Point Approach and Full Multiplicative Form, the problem remains how to come to a final and unique solution. For that purpose the correlation of ranks is senseless. A Theory of Dominance is rather preferred.

The final results classify Sweden, first followed by Luxemburg, Finland, the Netherlands and Denmark. Some of the ten countries, which joined the EU in 2004, are doing quite well, leaded by Estonia, which even joined the EMU. As expected the PIIGS countries are classified at the bottom, but joined by an unforeseen United Kingdom. In this way we have an idea how the European countries are advancing on economic terms to the European Strategy for 2020.

On basis of the outcomes it would perhaps be possible, eventually with more available data, to come to ratings comparable to the Credit Rating Agencies ratings. Further investigation in this sense could be useful.

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APPENDIX A

The rank correlation method

The method of correlation of ranks consists of totalizing ranks. Rank correlation was introduced first by psychologists such as Spearman (1904, 1906 and 1910) and later taken over by the statistician Kendall in 1948. He argues (Kendall 1948: 1): "we shall often operate with these numbers as if they were the cardinals of ordinary arithmetic, adding them, subtracting them and even multiplying them," but he never gives a proof of this statement. In his later work this statement is dropped (Kendall and Gibbons 1990).

In ordinal ranking 3 is farther away from 1 than 2 from 1, but Kendal (1948: 1) goes too far (Table A1).

For Kendal B is far away from A as it has 7 ranks before and A only 4, whereas it is not true cardinally.

In addition a supplemental notion, the statistical term of Correlation, is introduced. Suppose the statistical universe is just represented by two experts, for us it could be two methods. If they both rank different items in a same order to reach a certain goal, it is said that the correlation is perfect. However, perfect correlation is a rather exceptional situation. The problem is then posited: how is correlation measured in other situations. Therefore, the following Spearman's coefficient is used (Kendall 1948: 8):

$$\rho = 1 - \frac{6\sum D^2}{N(N^2 - 1)}, \tag{A1}$$

where D stands for the difference between paired ranks, and N for the number of items ranked.

According to this formula, perfect correlation yields the coefficient of one. An acceptable correlation reaches the coefficient of one as much as possible. No correlation at all yields a coefficient of zero. If the series are exactly in reverse order, there will be a negative correlation of minus one, as shown in the following example (Table A2).

Table A1. Ordinal versus cardinal: comparing the price of one commodity1

	Ordinal	Cardinal
	1	
	2	
	3	
	4	
A	5	6.03$
	6	6.02$
	7	6.01$
B	8	6$

Table A2. Negative rank order correlations

Items	Expert 1	Expert 2	D	D²
1	1	7	–6	36
2	2	6	–4	16
3	3	5	–2	4
4	4	4	0	0
5	5	3	2	4
6	6	2	4	16
7	7	1	6	36
				–
Σ				112

This table shows that the sum of ranks in the case of an ordinal scale has no sense. Correlation leads to:

$$\rho = 1 - \frac{6 \times 112}{7(49-1)} = -1.$$

However, as addition of ranks is not allowed, also a subtraction, the difference D, is not permitted, nor the multiplication or the division.

Most people will understand the ordinal problem better by the way of a qualitative scale, e. g.:

1st very good;
2nd moderate;
3rd very bad.

But equally one could say:

1st very good;
2nd good;
3rd more or less good;
4th moderate;
5th more or less low;
6th low;
7th very low.

How is the first 2nd comparable with the second 2nd? etc.

Arbitrary Methods to go from an Ordinal Scale to a Cardinal Scale

1) Arithmetical Progression: 1, 2, 3, 4, 5…………
2) A Geometric Progression: 1, 2, 4, 8, 16…………
3) The Fundamental Scale of Saaty (1987): 1, 3, 5, 7, 9
4) The Normal Scale of Lootsma (1987)

   $e^0 = 1$
   $e^1 = 2.7$
   $e^2 = 7.4$
   $e^3 = 20.1…………$

5) The Stretched Scale of Lootsma (1987)

   $e^0 = 1$
   $e^2 = 7.4$
   $e^4 = 54.6$
   $e^6 = 403.4………$

6) The Point of View of the Psychologists (Miller 1965)

   Ordinal Scales: 1, 2, 3, 4, 5, 6, 7.
   After 7 an individual would no more know the cardinal significance compared to the previous 7 ones.

In fact infinite variations are possible. They all stress acceleration or dis-acceleration processes but are not aware of a possible trend break. The full multiplicative method with its huge numbers illustrates this trend break the best as shown in next Table A3.

With the usual Arithmetical Progression: 1, 2, 3, 4, 5, … the distance from the rank 4 to 5 would be the same as from 3 to 4 which is certainly not the case here. In addition, all the other progressions fail to discover a trend break too.

Table A3. Ranking of scenarios for the Belgian regions by the Full-Multiplicative Method at the year 1996

1	Scenario IX	Optimal Economic Policy in Wallonia and Brussels	203,267
2	Scenario X	Optimal Economic Policy in Wallonia and Brussels even agreeing on the Partition of the National Public Debt	196,306
3	Scenario VII	Flanders asks for the Partition of the National Public Debt	164,515
4	Scenario VIII	No Solidarity at all	158,881
5	Scenario II	Unfavorable Growth Rate for Flanders	90
6	Scenario IV	an Unfavorable Growth Rate for Flanders and at that moment asks also for the Partition of the National Public Debt	87
7	Scenario III	Partition of the National Public Debt	54
8	Scenario I	the Average Belgian	51
9	Scenario V	Average Belgian but as compensation Flanders asks for the Partition of the National Public Debt	49
10	Scenario O	Status Quo	43
11	Scenario VI	Flanders asks for the Partition of the National Public Debt	42

Source: Brauers, Ginevičius (2010)

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