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INTRODUCING ALTERNATIVES RANKING WITH ELECTED NOMINEE (ARWEN) METHOD: A CASE STUDY OF SUPPLIER SELECTION

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Abstract. Supply chain management (SCM) has gradually evolved beyond the straightforward logic of benefits and economic viewpoints. Supplier selection and performance evaluation are the crucial strategic components of any SCM system with a substantial economic impact and risk reduction. Several conflicting factors make supplier selection a challenging multi-criteria decision-making problem. This paper introduces a method called alternative ranking with the elected nominee (ARWEN) to select suppliers in Iran's dairy product chain store. The primary principle of ARWEN is to choose the best alternative based on the lowest change rate rather than the elected nominee. Four extensions of the ARWEN method are proposed depending upon the nature and level of information available to the decision-makers. A fifth extended version termed E-ARWEN is also recommended to consider the negative form of the elected nominee. Two novel statistical tools, the ranking performance index and the Zakeri-Konstantas distance product correlation coefficient, are also put forth to validate the ARWEN extensions' outcomes. The results and verification of this new method are carried out through two supplier selection case examples. Comprehensive comparisons were carried out to explore the new methods' behaviors, indicating ARWEN III and E-ARWEN have similar behavior to VIKOR, SAW, and EDAS in generating rankings.

Keywords: multi-criteria decision-making, ARWEN, ranking performance index, Zakeri-Konstantas distance product correlation coefficient, criteria performance index, supplier selection.

JEL Classification: C02, C44, D70, D81, O14.

Introduction

Supply chain management (SCM) is a complex network of suppliers, manufacturers, distributors, and retailers, set on an integrated flow of resources and information with one main goal: fulfilling the needs of customers (Dutta et al., 2022). One strategic SCM area that

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has a significant economic impact is the supplier selection process. A significant number of financial resources are expended during the supplier selection process. An inconsistent supplier frequently results in economic setbacks and additional costs for acquiring resources. For instance, if the supplier has not been sufficiently assessed beforehand, selection based only on the lowest cost may ultimately result in quality problems, delivery delays, or even supply shortages. However, selecting a supplier is a very complicated process that involves input from numerous departments, persons, and organizations. Reduced economic risk, increased buyer value, and establishment of a covalent association between buyers and suppliers are the primary goals of the supplier selection process. The purpose of supplier selection, according to (Tong et al., 2022; Suraraksa & Shin, 2019), is to assist organizations to gain sustainable advantages in the market al.ng with lowering and conserving costs and reducing manufacturing hazards. Supplier selection is a complex decision-making problem, comprising several suppliers as the candidates and a set of criteria that often have conflicts. To solve this problem and achieve optimized solutions, various multi-criteria decision-making (MCDM) methods are vastly employed (Paldrak et al., 2022; Resende et al., 2021; Schramm et al., 2020; Chai & Ngai, 2020; Konys, 2019; Aouadni et al., 2019; Wetzstein et al., 2016). Strong supplier management maximizes cost-reduction opportunities, value-driven services, and overall systemic efficiencies of the organizations.

In order to rank the alternatives in a decision-making problem, MCDM methods turn the problem matrices that are designed on a number of criteria and alternatives. A typical decision-making process begins with the decision-makers (DMs) stating goals and concludes with the alternative being chosen. The process where MCDM methods work starts with examining the criteria of the problem and ends with the selection of an alternative. In Figure 1, the decision-making process and the area where MCDM methods are used to evaluate the possible alternatives are depicted.

Alternatives are evaluated using MCDM methods within a multi-step framework. According to Opricovic and Tzeng (2004), the main steps of MCDM process are as follows: (i) Creating system evaluation criteria that link system capabilities to objectives; (ii) Generating alternatives for reaching the objectives; (iii) Evaluating alternatives in terms of criteria (values of criteria functions); (iv) Applying a normative MCDM method; (v) Accepting one alternative as "optimal" (preferred); (vi) If the final solution is not acceptable, obtain additional data and go into the next iteration of multi-criteria optimization. Haddad et al. (2020) described MCDM methods as a sequential process where DMs might iterate the process. Formisano and Mazzolani (2015) asserted that the following elements are considered in the evaluation of all decision situations involving a multi-attribute evaluation: (i) A DM or a group of DMs involved in the selection process who are responsible for the evaluation procedure, (ii) A group of alternatives for making decisions, which serve as the building blocks of the evaluation and selection process, (iii) An evaluation set, used by DMs to evaluate the performance of the alternatives, (iv) The preferences of DMs, which are typically expressed in terms of criteria weights and (v) A set of scores, expressing the values of alternatives (i) with respect to different criteria (j).

MCDM methods have been grouped under a number of different categories. The most comprehensive categorization has segregated MCDM methods into two groups consisting

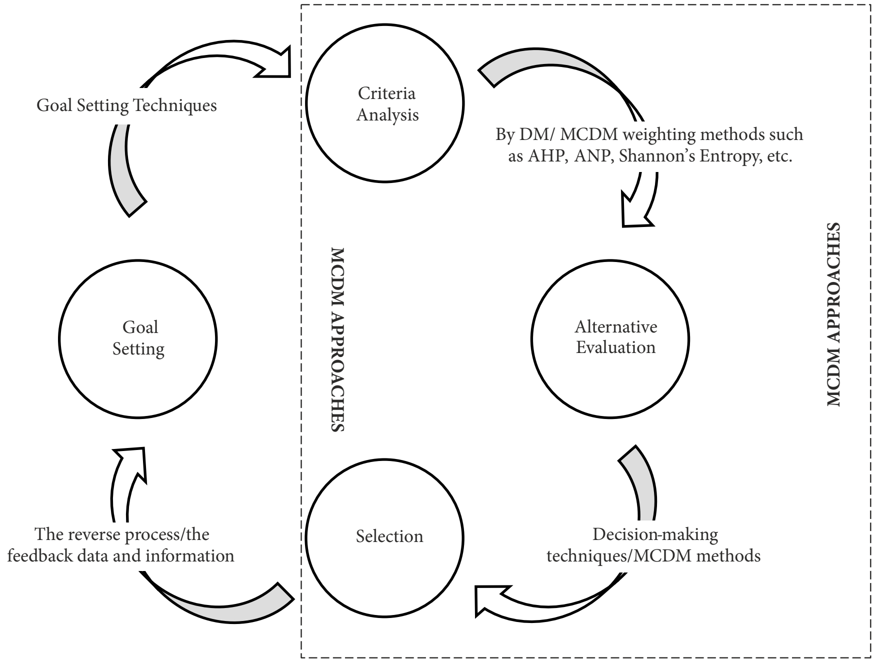

Figure 1. Decision-making circle and territory of MCDM methods

of alternative ranking and criteria weighting methods. MCDM ranking methods themselves have experienced different categorizations (Gupta & Ilgin, 2017; Elhassouny & Smarandache, 2016; Velasquez & Hester, 2013; Smarandache, 2016; Ricci et al., 2011). Regarding the performance of MCDM methods, they can be divided into four categories: (i) Outranking methods like ELimination and Choice Expressing REality (ELECTRE) and Preference ranking for organization method for enrichment evaluation (PROMETHEE); (ii) Compromise ranking like Grey relational analysis (GRA); (iii) Distance-based like Višekriterijumsko kompromisno rangiranje (VIKOR) (Opricovic, 1998; Opricovic & Tzeng, 2002), Technique for order of preference by similarity to ideal solution (TOPSIS) (Hwang & Yoon, 1981), and (iv) Pairwise comparison like analytic hierarchy process (AHP) (Saaty, 1971, 1988). The weighing methods, however, can be divided into two major groups, including subjective weighting methods such as AHP and vital-immaterial mediocre method (VIMM) (Zakeri et al., 2021); and objective weighting such as Shannon's entropy (Zakeri et al., 2019) and CRITIC method (Wang & Zhao, 2016). IVEP method (Zakeri & Konstantas, 2022), a new interesting MCDM objective weighting method called LOPCOW and a novel ranking method DOBI introduced by (Ecer & Pamucar, 2022), FUCOM (Pamučar et al., 2018; Ecer, 2021; Ecer & Torkayesh, 2022), and FUCOM developed by (Žižović & Pamucar, 2019) are the recent development in MCDM methods area.

One of the most common ways to aid DMs make the best decision conceivable is the distance-based method (Sałabun, 2015). In reality, distance is a frequently applied measure in MCDM methods which might be able to prevent compensation between an alternative's best and worst performance under various sets of criteria (Wang et al., 2017). The most widely used distance measures are Hamming distance, Euclidean distance, and Hausdorff distance along with their extensions (Zhou et al., 2018). Additionally, distance measuring has been successfully used in a multitude of areas and is a crucial tool for calculating the deviation and closeness degrees of distinct arguments. The family of distance-based methods includes Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), COmbinative Distance-based ASsessment (CODAS), Vlse Kriterijumska Optimizacija Kompromisno Resenje (VIKOR), Evaluation Based on Distance from Average Solution (EDAS), Multi-Attributive Border Approximation Area Comparison (MABAC) and Measurement of Alternatives and Ranking according to Compromise Solution (MARCOS).

In this research, an innovative and straightforward MCDM method termed alternative ranking with the elected nominee (ARWEN), is introduced, which takes into account the expertise levels of the DMs. ARWEN method is a distance-based approach that handles both positive and negative ideal solutions, just like TOPSIS and VIKOR methods. Based on the knowledge that DM holds regarding the decision-making process, ARWEN is made up of four distinct members: ARWEN I, ARWEN II, ARWEN III, and ARWEN IV. The key contributions of ARWEN are its ease of use for addressing MCDM problems and the coverage of various ranges of information to which DMs have access. These contributions complement the benefits of distance-based MCDM methods. Another aspect that has been examined in this study is the efficiency of an MCDM method for ranking alternatives. The ranking performance index (RPI), a unique statistical measure, is introduced as a solution for the shortcomings of the existing methods for assessing the quality of results derived from MCDM methods. RPI evaluates the efficacy of MCDM methods in order to evaluate their reliability. The methodology is based on the assumption that the weights assigned to the criteria are determined precisely. While addressing these shortcomings, RPI includes the sensitivity analysis and Spearman's rank correlation coefficient theories within its algorithm. RPI analyzes variations in rankings analogous to sensitivity analysis by using criteria weights while preserving original values. In contrast to Spearman's rank correlation coefficient, it also computes the similarity between rankings and operates on a single application and MCDM method.

The remaining sections of the paper are organized as follows: the discovered gaps have been addressed in the second section, the literature review; RPI is introduced in the third section as a new method for the validation of the MCDM methods results; in the fourth section, various forms of ARWEN algorithms are introduced; and in the fifth, the various forms of ARWEN are applied to a numerical example of a supplier selection problem; The discussion is included in section six, and the conclusion and recommendations for research are covered in section seven.

1. Literature review

Due to engaging several criteria with different natures and also various candidates, the supplier evaluation process is a generic MCDM problem that has been investigated extensively throughout the past five decades. Different MCDM methods have been applied to find an optimized solution for the mentioned problem, which each added different advantages and shortages to the current course. The first part of this two-part section explores the close relationship between MCDM methods and supplier evaluation problems. We also discuss the different approaches to formulating incomplete information in the related studies. The second part is associated with reviewing differentiation of the different MCDM methods results and the approaches studies employed to deal with this conflict.

The literature review section aims to conduct readers to the following sections, where the paper's contributions are represented. Each part aims to show the related gap in which the solutions have been addressed in the following sections.

1.1. MCDM and supplier evaluation

Numerous studies have employed MCDM methods to select the best supplier in different areas, such as sustainable supplier selection, green supplier selection, supplier selection in the circular economy, Etc. In this part, the recent studies have been reviewed in four categories based on the MCDM methods' different categories. The MCDM method has been categorized into two primary categories of weighting and ranking methods. The MCDM weighting methods category includes the subjective and objective weighting methods, and the MCDM ranking methods encompass the distance-based methods, outranking methods, and compromise ranking methods.

Amongst MCDM methods, TOPSIS is most likely the most popular distance-based MCDM method for solving supplier selection problems. Over the years, various extensions of TOPSIS have been developed to solve decision-making problems (Madi et al., 2016). TOPSIS extracts two sets of positive and negative optimum scores, termed the positive and negative ideal solutions. Its process focuses on finding the alternative located at the closest distance from the positive optimums and farthest distance from the negative optimums. Some recent examples of the application of TOPSIS in solving supplier selection could be found in (Li et al., 2019; Rouyendegh et al., 2020; Çalık, 2021; Haddad et al., 2021; Kahraman & Alkan, 2021; Sun & Cai, 2021; Aouadni & Euchi, 2022). Introduced by (Keshavarz Ghorabaee et al., 2016), the combinative distance-based assessment (CODAS) method evaluates the decision's alternatives through defining a space bounded between two norms, CODAS uses the Euclidean and Taxicab distances. The CODAS application for the selection of the best supplier could be found (Ramírez-Ochoa et al., 2022; Pamucar et al., 2022; Wei et al., 2021a, 2021b; Bolturk, 2018; Badi et al., 2018). Along with TOPSIS, VIKOR (Gao et al., 2020), and COPRAS (Chatterjee & Kar, 2018; Chatterjee et al., 2011) are the most popular distance-based methods for solving supplier selection problems. Some examples of the application of VIKOR could be found in (Abdel-Baset et al., 2019; Salimian et al., 2022; Karami et al., 2021; Fei et al., 2019; Peng et al., 2020). According to EDAS method, the desirability of alternatives is assessed by how far they are from the average solution. EDAS method has a substantial application in supplier selection problems (Stević et al., 2017; Yazdani et al., 2020; Huang et al., 2021; Göçer, 2022). MABAC METHOD solves complicated and ambiguous decision-making concerns by determining the distance between each alternative and the boarder approximation area, while MARCOS method is focused on measuring options and ranking them as a compromise solution. Some applications of MABAC can be found in (Matić et al., 2019; Mishra et al., 2022;

Ghadikolaei et al., 2022), whereas MAFRCOS has applications in (Badi & Pamucar, 2020; Stević et al., 2020; Yazdani et al., 2022). Some examples of the MCDM methods' applications are illustrated in Tables 1–3.

Table 1. Solving supplier selection problems using MCDM objective weighting methods

Authors	The problem	MCDM method	Combination with other MCDM methods	Uncertainty
Alipour et al., 2021	fuel cell combined with hydrogen (FCH) technology components supplier selection problem	Entropy	SWARA and COPRAS	Pythagorean fuzzy
Chen, 2021	Building Material Supplier Selection for a venture capital company	Entropy	ANP and TOPSIS	
Shang et al., 2022	Supplier selection of a forklift trucks and warehouse equipment company	Entropy	BWM and MULTIMOORA	Extended fuzzy reference point approach
Zhang et al., 2022	Emergency supplies for the event of public emergencies	CRITIC	GRA	The spherical fuzzy sets
Liaqait et al., 2022	A supplier selection of a firm selling split air-conditioning units	CRITIC	TOPSIS	Triangular fuzzy sets
Lu et al., 2021	A green supplier selection problem	CRITIC	COPRAS	The picture fuzzy sets

Table 2. Solving supplier selection problems using MCDM objective weighting methods

Authors	The problem	MCDM method	Combination with other MCDM methods	Uncertainty
Liao et al., 2019	Supplier selection problem of a private limited liability company that manufactures biscuits, cakes, and bread located in China	BWM	ARAS	The hesitant fuzzy linguistic
Liu et al., 2022	Sustainable medical supplier selection	BWM	EDAS, ELECTRE	The probabilistic linguistic term set
Menon & Ravi, 2022	A supplier selection problem of an electronics company producing electronic components located in India	AHP	TOPSIS	
Çalık, 2021	A supplier selection problem of an agricultural tool manufacturer located in Turkey	AHP	TOPSIS	The interval-valued Pythagorean Fuzzy
Tavana et al., 2021	A supplier evaluation of Technoron Electronics1, a company of consumer electronic goods located in New Jersey	AHP	MULTIMOORA	Fuzzy sets

Table 3. Solving supplier selection problems using MCDM objective weighting methods

Authors	The problem	MCDM method	Combination with other MCDM methods	Uncertainty
Wie et al., 2021	A green supplier selection in China	EDAS	The multiple attribute group decision making (MAGDM)	The probabilistic linguistic term sets
Liu et al., 2021	Selection of the best sustainable circular supplier in the manufacturing sector	EDAS	WASPAS	The Pythagorean fuzzy sets
Göçer, 2022	A supplier selection of the optimal limestone suppliers	EDAS	SAW	The Pythagorean fuzzy sets
Wei et al., 2020	A green supplier selection problem	MABAC	MAGDM, Entropy	uncertain probabilistic linguistic term sets
Mishra et al., 2022	A sustainable supplier selection of an auto-manufacturing company in Iran	MABAC	DEA, FOCUM	The hesitant fuzzy sets
Salimian et al., 2022	A supplier selection for medical purposes	MARCOS	E-VIKOR, IVIF-Entropy	The interval-valued intuitionistic fuzzy sets

Zakeri and Konstantas (2022) addressed three primary sources of uncertainty in solving MCDM problems, including (1.) the uncertainty of inputs of MCDM algorithms, (2.) the uncertainty generated from the MCDM methods due to employing different philosophies and different normalization processes, and (3.) the uncertainty in the algorithms' outputs where could be observed in the different results for the same problem. They also mentioned that "DMs' expectations, judgments, interpretations, different levels of knowledge/expertise, and different levels of access to the sources of information" are the main components that fabricate the first source. In the above tables, probability functions and different extensions of fuzzy math have been employed to deal with this type of uncertainty generator. Furthermore, many studies used the grey systems theory and the rough set theory.

One of the major sources of uncertainty in the inputs of MCDM algorithms is DM's access to information regarding the MCDM problems' elements, the importance of criteria, and the scores of each alternative against criteria. This generator also fashions and amplifies other sources of uncertainty. Dealing with this type of uncertainty is not predicted in the existing MCDM methods, requiring a supplement to formulate it. This paper aims to address this gap by introducing a new MCDM method.

1.2. The MCDM results validation

In contrast to subjective weighting methods, which derive criteria weights through DMs' opinions, arguments, and judgments, objective weighing methods generate weights from the decision matrix based on the interactions of the alternatives and criteria. Both subjective and objective weighting methods involve different justifications to calculate criteria weights which frequently yields different weights for the same set of criteria and different solutions of the same problem is achieved. According to Zanakis et al. (1998), the following elements contribute to the discrepancy that MCDM methods reveal when generating different outputs: (i) How the methods use weights in their calculations is one way that they differ from one another, (ii) How the MCDM methods select the best alternative is another distinction between them and (iii) Many methods attempt to scale the objectives, changing the previously established weights. Since using the wrong MCDM method could have resulted in unforeseen problems with decision-making, it is necessary to evaluate the obtained results of MCDM methods and validate the outcomes. The comparison of their application results is typically the fundamental approach for validating MCDM methods (Sałabun & Urbaniak, 2020). Another widely used technique for validating the results generated by MCDM methods and demonstrating their reliability is the use of sensitivity analysis (Mukhametzyanov & Pamucar, 2018), although applying sensitivity analysis has also some drawbacks (Saltelli et al., 2019) including the alteration of criteria weights based on the subjective opinions. Spearman's rank correlation coefficient, which is the third widely used method for validating MCDM methods, faces some serious problems (Kou et al., 2012). Ishizaka and Nemery (2013) stated that at least three MCDM methods must be considered in order to unveil their performance and establish their superiority. In light of this fact, it might be claimed that more case studies/examples are required in order to confirm the global reliability of a particular MCDM method. In addition to sensitivity analysis and Spearman's rank correlation, Pearson's product-moment correlation coefficient and Kendall's coefficient of concordance could be used to assess the results of MCDM methods (Qaradaghi & Deason, 2018).

2. Ranking performance index (RPI)

Ranking performance index (RPI) uses the final results provided by an MCDM method to evaluate its performance while assuming that the priority weights are consistent. The action is driven by a simple query to determine how much the ranking deviates from the initial result when one of the predetermined criteria is removed. The three main steps in the PRI computing process are: 1. Computing the performance of each criterion; 2. Computing the final ranking in accordance with the criteria performance; and 3. Computing the performance of the MCDM method by comparing the alternative ranks against the criteria weights and performance of the criteria. The RPI computation procedure is described using the following simple steps:

Step 1. Computing criteria performance.

Step 1.1. To compute criteria performance, at first, they are required to be in descending order, as shown by Eq. (1), with the consideration of Eq. (2), (3), where C_j stands for jth criterion.

$$w_{C_{j_{min}}} \to w_{C_z} \to w_{C_{j_{max}}}, \ j = \{1,\ldots,n\}, \ z \in n, \ w_{C_z} > w_{C_{j_{min}}}. \tag{1}$$

If $w_{C_z} = w_{C_l}$.

Then

$$\left(\sum_{j=l}^{n} r_{ij}\right)^{w_{C_l}} > \left(\sum_{j=z}^{n} r_{ij}\right)^{w_{C_z}} \Rightarrow w_{C_l} > w_{C_z},\ z,l \in n,\ i = \{1,\ldots,m\}; \tag{2}$$

$$\left(\sum_{j=l}^{n} r_{ij}\right)^{w_{C_l}} < \left(\sum_{j=z}^{n} r_{ij}\right)^{w_{C_z}} \Rightarrow w_{C_l} < w_{C_z},\ z,l \in n,\ i = \{1,\ldots,m\}, \tag{3}$$

where r_{ij} indicates the normalized performance of i^{th} alternative against j^{th} criterion (cost criteria need to be converted to benefit criteria).

Eqs (4) and (5) are the most popular normalizing processes that are employed by various MCDM methods.

$$r_{ij} = x_{ij}\left(\sum_{j=1}^{n} x_{ij}\right)^{-1}; \tag{4}$$

$$r_{ij} = x_{ij}^{2}\left(\sum_{j=1}^{n} x_{ij}^{2}\right)^{-1}; \tag{5}$$

$$X = x_{ij}. \tag{6}$$

In Eq. (6), where x_{ij} indicates elements of the decision matrix.

Step 1.2. Initiating the elimination process from $w_{C_{j_{min}}}$ according to Eq. (7), where E_j shows elimination of the j^{th} criterion. In this step, each criterion and its corresponding alternative performance is eliminated, and new weights assign to the remained criteria, where $\sum w_j = 1$. The number of the elimination process equals n, where n expresses number of criteria.

$$N_{E_j} = n,\ n \in j,\ j = \{1,\ldots,n\}. \tag{7}$$

Step 1.3. Ranking the alternatives against the remaining criteria using MCDM method which has been employed to solve the decision-making problem.

Step 1.4. Computing the performance of each criterion according to the following steps:

Step 1.4.1. Establishing the following comparison matrix (Table 4), where R_{C_j} is the alternative rankings without the impact of j^{th} criterion and R_{A_j} is the ranking derived by the MCDM method.

Table 4. The ranking comparison matrix

		R_{A_i}		
		A_1	$\cdots$	A_m
R_{C_j}	A_1	v_{11}		
	$\cdots$		$\ddots$	
	A_m			v_{mm}

Step 1.4.2. Computing the criterion performance index (CPI) using Eqs (8) and (9), where P_{C_j} indicates the CPI values.

$$v_{iiC_j} = R_{C_{jA_i}} R_{A_i}{}^{-1},\ i = \{1,2,\ldots,m\}; \tag{8}$$

$$P_{C_j} = \sum v_{iiC_j} \left(\sum\sum v_{iiC_j} \right)^{-1}. \tag{9}$$

There is another index which is extracted from Eq. (8), called the criteria impact index (CII), which can be estimated using Eq. (10), where m is the number of alternatives. It demonstrates the relative impact of each criterion on the ranking of an alternative.

$$CII_{c_j} = m^{-1} \sum v_{iiC_j}. \tag{10}$$

Step 2. Rank the alternatives with the new set of weights (Eq. (11)) using the same MCDM method. In the following equation, w'_j represents criteria weights.

$$w'_j = P_{C_j},\ j = \{1,2,..,n\}. \tag{11}$$

Step 3. The final step is the computation of RPI. To compute RPI, the new ranks with impact of P_{C_j} need to be compared with the ranking results with impact of criteria weights. Following steps show the computing process of RPI.

Step 3.1. Developing the second ranking comparison matrix (see Table 5), where Γ_i^* represents the weighted proximity coefficient that comes when the MCDM method is applied with the original weights, $\Gamma_i^{*\prime}$ stands for the weighted proximity coefficient with impact of CPI.

Table 5. The ranking comparison matrix

	A_1	...	A_m
Γ_i^*	x_1	...	x_m
$\Gamma_i^{*\prime}$	y_1	...	y_m

Step 3.2. The final step is the computation of *RPI* using Eq. (12).

$$RPI = \sum_{i=1}^{m} \left| \Delta_{y_i, x_i} \right|,\ i = \{1,2,\ldots,m\}. \tag{12}$$

The numerical intervals that can be assigned to *RPI* are in accordance with Eqs (13) and (14) respectively:

– If the number of alternatives is an odd number, then the interval is in accordance with Eq. (13).

$$0 < RPI \leq {}^{801m\left(\left(m \times \left(m - \frac{1}{2}\right)\right) + \left(m - \left(m + \frac{1}{2}\right)\right)\right)}\Big/_{800m}. \tag{13}$$

– If the number of alternatives is an even number, then the interval is in accordance with Eq. (14).

$$0 < RPI \leq {}^{m^2}\!/_{80}. \tag{14}$$

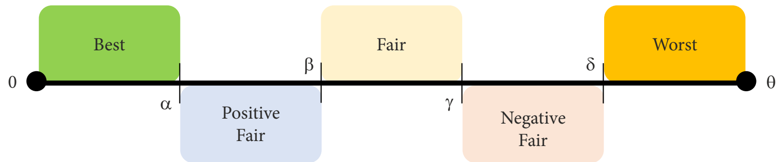

Figure 2. The five spectrums of *RPI*

The most important part in computing *RPI* is its interpretation. The interpretation is based on five spectrums (see Figure 2), including 1. The best spectrum; 2. The positive fair spectrum; 3. The fair; 4. The fair negative spectrum; and 5. The worst spectrum.

Each spectrum is structured between two upper and lower boundaries, and each boundary possesses a different interpretation as follows:

1. if $0 \le RPI \le \alpha$, it signifies that performance of the MCDM method is acceptable, and the results are reliable.
2. if $\alpha \le RPI \le \beta$, it signifies that performance of the MCDM method in ranking is still reliable but not strong as the previous spectrum, thus results validation using another MCDM method is recommended.
3. if $\beta \le RPI \le \gamma$, it means that the performance of the MCDM method, and its results require to be validated.
4. if $\gamma \le RPI \le \delta$, it means that the performance of the MCDM method is not reliable and solving the problem by another MCDM method is strongly recommended.
5. if $\gamma \le RPI \le \delta$, it means that the problem must solve by other MCDM methods, and the results are completely incorrect.

The boundaries are computed using Eqs (15)–(24).

if *m* is an odd number, then:

$$\theta = {\left(m \times \left({m-1}/{2}\right)\right) + \left(m - \left({m+1}/{2}\right)\right)}\Big/{10} + {\alpha}/{20}; \tag{15}$$

$$\delta = {(m-1)^2}/{20} + {\alpha}/{40}; \tag{16}$$

$$\gamma = {\left(m \times \left({m-1}/{2}\right)\right) + \left(m - \left({m+1}/{2}\right)\right) + (m-1)^2}\Big/{40} + {\alpha}/{20}; \tag{17}$$

$$\beta = {\left(m \times \left({m-1}/{2}\right)\right) + \left(m - \left({m+1}/{2}\right)\right)}\Big/{20} - {\alpha}/{10}; \tag{18}$$

$$\alpha = {\left(m \times \left({m-1}/{2}\right)\right) + \left(m - \left({m+1}/{2}\right)\right)}\Big/{40m} \tag{19}$$

if *m* is an even number, then:

$$\theta = {m^2}/{20} + {\alpha}/{40}; \tag{20}$$

$$\delta = {\left((m-1) \times \left({m-2}/{2}\right)\right) + \left((m-1) - \left({m}/{2}\right)\right)}\Big/{10} + {\alpha}/{20}; \tag{21}$$

$$\gamma = {}^{\delta+\beta}\!\big/_{20} + {}^{\alpha}\!\big/_{10}; \tag{22}$$

$$\beta = {}^{m^2}\!\big/_{40} - {}^{\alpha}\!\big/_{20}; \tag{23}$$

$$\alpha = {}^{m^2}\!\big/_{80}. \tag{24}$$

3. Alternatives ranking with the elected nominee method (ARWEN)

Regarding the level of information that is available in relation to the requirements for DMs, ARWEN includes four forms: ARWEN I, ARWEN II, ARWEN III, and ARWEN IV. The elected nominee alternative could be taken from the decision matrix or chosen by DMs depending on the various degrees of information availability. With one exception, if DMs choose to evaluate the alternatives with a different alternative beyond the decision matrix to meet their expectations, the elected nominee alternative theoretically does not have an independent existence. The alternatives are ranked in order of higher levels of similarity to the selected nominee. The basis of the ARWEN's algorithm is grounded on the larger value of Γ_i, as shown in Eq. (25):

$$\Gamma_i = (2n) - \left(\sum_{j=1}^{n} w_j \left(\max_i r_{ij} . \left(r_{ij} \right)^{-1} \right) \right), i = \{1,\ldots,m\}, j = \{1,\ldots,n\}, \tag{25}$$

where n is the number of criteria.

3.1. ARWEN forms

ARWEN incorporate four hypothesizes for evaluation of the alternatives:

When the criteria of the problem are unidentified for DMs. This includes the first two forms of ARWEN I and ARWEN II.

ARWEN I: This member evaluates the decision matrices that are constructed with merely beneficial criteria. In beneficial criteria, higher values indicate better performance of the alternative.

ARWEN II: when composition of the criteria encompasses both benefit and cost criteria, ARWEN II evaluates the alternatives.

The second group of the method is employed when DMs can identify the criteria that embraces ARWEN III and ARWEN IV forms, which have been distinguished by DMs' level of access to information regarding the criteria.

ARWEN III: there is no perfect information available to the DMs about the criteria.

ARWEN IV: DMs has perfect information about the criteria.

The following steps describe the application of hypothesizes in each four sub-methodologies of ARWEN.

3.2. ARWEN steps

The steps of different forms of ARWEN are provided in this section where they have been divided into two groups based on the mentioned categorization in the previous section.

3.2.1. Criteria are unidentified for DM(s)

To calculate the relevance weight of each criterion in this case, DMs do not have access to or possess flawless data/information about the criteria; instead, only imprecise data is available. In a real situation, a DM deals with decision-making challenges where s/he lacks comprehensive facts or perfect information about the challenge, necessitating the reduction of decision risks. When all of the criteria are benefits and when some criteria are costs, ARWEN proposes two different extensions. These extensions are developed based on the lowest changing range of alternatives than the elected nominee.

3.2.2. Steps of ARWEN I

ARWEN's algorithms do not use normalization of the alternative scores against the criteria.

Step 1. Constructing the decision matrix, where $C_j = \{C_1, \cdots, C_n\}$ denote the set of criteria and $A_i = \{A_1, A_2, \cdots, A_m\}$ expresses the set of alternatives, and (X_{ij}) represents the decision matrix in Eq. (26):

$$X_{ij} = r_{ij}; \tag{26}$$

Step 2. Specifying the elected nominee through the largest value of each criterion (Eq. (27):

$$A_i^+ = \left\{ \max_{1 \le j \le n} r_{ij} \right\},\ i = \{1, \ldots, m\}; \tag{27}$$

Step 3. Transforming the decision matrix into a new decision matrix with respect to Eq. (28):

$$\nu_{ij} = \max_i r_{ij} . \left(r_{ij} \right)^{-1},\ 1 \le \nu_{ij}^+ \le 2; \tag{28}$$

Step 4. Computing the proximity coefficient for each alternative according to Eq. (29), where Γ_i stands for the proximity coefficient:

$$\Gamma_i = (2n) - \left(\sum_{j=1}^{n} \nu_{ij} \right). \tag{29}$$

The proximity coefficient illustrates the similarity of the alternatives to the elected nominee alternative.

Step 5. Ranking alternatives in accordance with the larger value of (Γ_i).

3.2.3. Steps of ARWEN II

The study of the decision matrix, which includes both benefit and cost criteria, is one of the ARWEN II phases. The only difference between the ARWEN I and ARWEN II steps is the transformation of the matrix to the benefits matrix, which is done as follows:

Step 1. Transforming the original decision matrix into benefit decision matrix using Eq. (30) and (31) for the cost criteria, where (r_{ij}^-) stands for the cost criteria.

$$p_{ij} = r_{ij}^- . \left(\sum_{i=1}^{m} r_{ij}^- \right)^{-1}; \tag{30}$$

$$r_{ij}^+ = \left(1 - p_{ij} \right). \tag{31}$$

3.3. Criteria are identified and known for DM(s)

There are two degrees of information access that are covered by ARWEN III and ARWEN IV. Although DMs in ARWEN III have a general knowledge of the problem, they are unable to identify the best solution (See RBOP method by Zakeri, 2019). ARWEN IV, on the other hand, considers an ideal alternative when it has comprehensive understanding of the problem. The opinions, judgement, and expectations of the DMs are used to determine this alternative, albeit it may also result from group consensus based on the organizational strategies. The ARWEN II and ARWEN III steps are described in the following sections.

3.3.1. Steps of ARWEN III

ARWEN III evaluates the alternatives when DMs have no access to perfect information regarding the problem criteria. The following steps show the ARWEN III process in analyzing the decision matrix.

Step 1. Establishing the decision matrix (X_{mn}), where $C_j = \{C_1, C_2, \cdots, C_n\}$ denotes the set of criteria, $A_i = \{A_1, A_2, \cdots, A_m\}$ stands for the set of alternatives, and $W_j = \{w_1, w_2, \cdots, w_n\}$ is the set of criteria weights $\left(\sum_{j=1}^{n} W_j = 1\right)$. Since DMs possess complete information regarding the importance of each criterion, the values of weights are determined by the DMs' decisions.

Step 2. Converting the decision matrix into the benefit decision matrix according to the Eqs (30), (31).

Step 3. Determining the weighted elected nominee with respect to Eq. (28).

Step 4. Computing weighted proximity coefficient with respect to Eq. (32):

$$\Gamma_i^* = (2n) - \left(\sum_{j=1}^{n} \left\langle \max_i r_{ij} \middle/ r_{ij} \right\rangle w_j \right). \tag{32}$$

Step 5. Ranking alternatives in accordance with the largest value of (Γ_i^*).

3.3.2. Steps of ARWEN IV

When DMs have complete knowledge of all alternatives, both inside and outside the decision-making problem and criteria, ARWEN IV offers a solution. As a result, it makes DMs assume certain things from the selected alternative. An alternative is the primary ranking factor in ARWEN IV. This alternative is made out of expectations of the DMs, which may or may not be a simple abstraction of the best qualities that DMs anticipate in a selected alternative (see Zakeri et al., 2022a). The ARWEN IV algorithm is represented by the steps below.

Step 1. Establishing the decision matrix according to the ARWEN III first step.

Step 2. Establishing a new decision matrix with the definition of an elected nominee according to the DM's expectations, in which all criteria are optimum and (A_m^*) is the elected nominee. In the process, A_m^* evaluates against (C_j^*) as criteria (Eq. (33)), where:

$$\forall C_n^* \in C_j^* \rightarrow C_n^* \geq C_n,\ C_j = \{C_1, C_2, \cdots, C_n\}; \tag{33}$$

Step 3. Converting the decision matrix into the benefit decision matrix according to the Eqs (30), (31).

Step 4. The final step is the alternatives ranking with respect to the larger value of (Γ_i) according to the Eq. (32).

3.4. Extended ARWEN (E-ARWEN)

In this section, the extended form of ARWEN, called E-ARWEN is exhibited. The E-ARWEN's evaluation process limits the fluctuation of the alternatives between two bounds of a positive and negative elected nominee. The following section shows the steps of E-ARWEN's algorithm.

3.5. Steps of E-ARWEN

Step 1. Constructing the decision matrix (X_{mn}).

Step 2. Transforming the decision matrix into the benefit decision matrix according to the Eqs (30), (31).

Step 3. Determining the elected nominees, (A_j^+) as the positive elected nominee (Eq. (27), and (A_j) as the negative elected nominee (Eq. (34)), where

$$A_j^- = \left\{ \min_{1 \leq j \leq n} r_{ij} \right\},\ i = \{1, \ldots, m\}; \tag{34}$$

Step 4. Converting the decision matrix into two new decision matrices with respect to Eqs (35), (36):

$$v_{ij}^+ = \max_i r_{ij} . \left(r_{ij} \right)^{-1},\ 1 \leq v_{ij}^+ \leq 2; \tag{35}$$

$$v_{ij}^- = r_{ij} . \left(\min_i r_{ij} \right)^{-},\ 1 \leq v_{ij}^- \leq 2; \tag{36}$$

Step 5. Computing the positive and negative proximity coefficients for each alternative in accordance with (Eq. (37), (38)):

$$\Gamma_i^+ = (2n) - \left(\sum_{j=1}^{n} v_{ij}^+ \right); \tag{37}$$

$$\Gamma_i^- = \sum_{j=1}^{n} v_{ij}^-; \tag{38}$$

Step 6. Calculating the percent convergence coefficient according to the (Eq. 39), where (n) stands for the number of criteria.

$$PCC_i = 100 - \left\langle \frac{\Gamma_i^-}{2n\left(\Gamma_i^- + \Gamma_i^+\right)} \times 100 \right\rangle; \tag{39}$$

Step 7. Ranking alternatives according to the larger value of (PCC_i).

4. Supplier evaluation examples

In this section, an application of different forms of ARWEN method is presented to solve a supplier selection example. In this section, different forms of ARWEN are applied to the case and the results are illustrated in different subsections.

4.1. Case study of a dairy product chain store

4.1.1. Data collection

In this section, ARWEN method is applied to select the best dairy product supplier for a chain store in Tehran, Iran. Four suppliers are selected including Kalleh, Pegah, Damdaran, and Haraz, to meet the chain-store needs. To evaluate the suppliers, defect rate (C_1), appropriateness of product price to market price (C_2), promotions "the numbers of promitions per year" (C_3), ability to adapt to increase, decrease, and change of order timing (C_4), after-sales services (C_5), delivery reliability (C_6) are defined as the criteria. The following numerical scales of grading and weighting are used by ARWEN to evaluate the significance of each criterion and rank the alternative against them. The proposed scales take into account three ranges, made up of extremely low, moderate, and very high values, and are inspired by the scale employed in the VIMM approach (Zakeri et al., 2021). The scale enables DMs to choose from a wide range of numbers between very low, moderate, and very high. The suggested scales are depicted in the following pictures (Figures 3 and 4).

The above scales provide a wide range of decision options for DM(s) to translate their judgments and opinions into numbers and with seventeen options, it helps DM(s) to choose the best number, which is the closest to his/her decision. All ARWEN members are applied for the chain store supplier selection problem. Table 6 shows the decision matrix of the dairy product supplier selection, where C_1 pulled out, then we have a full beneficial matrix. The defect rate (C_1) is a non-beneficial criterion.

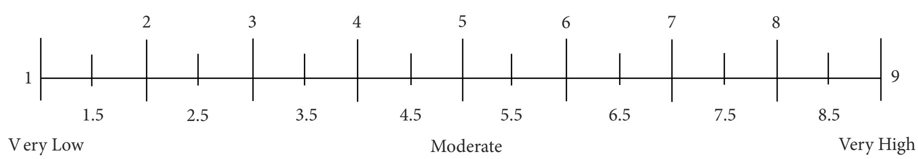

Figure 3. Numerical scale of rating for ARWEN family

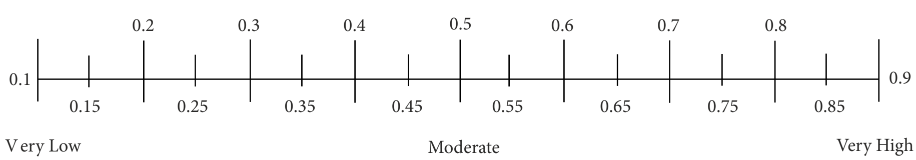

Figure 4. Numerical scale of weighting for ARWEN family

Table 6. Supplier evaluation decision matrix

	C_2	C_3	C_4	C_5	C_6
Haraz	1	6	7	6.5	6.5
Pegah	1.25	8	7	7	7
Damdaran	0.75	8	6.5	7.5	6.5
Kalleh	1.5	8	7	7.5	7.5

4.1.2. ARWEN applications

In this section five different forms of ARWEN, including the E-ARWEN are applied to the considered supplier selection case study. The applications are exhibited in different sub-sections.

4.1.2.1. ARWEN I application

The original supplier evaluation matrix of Table 6 is first transformed using Eqs (30) and (31), as shown in Table 7. The key element for ranking in ARWEN algorithms is the value of the proximity coefficient. The larger value of (Γ_j) reveals the higher priority of the supplier. The value of (Γ_i) for each supplier and its rank are shown in Table 8, where the values are computed using Eq. (29).

Table 7. Transformed decision matrix

Supplier	C_2	C_3	C_4	C_5	C_6
Haraz	1.500	1.333	1.000	1.154	1.154
Pegah	1.200	1.000	1.000	1.071	1.071
Damdaran	2.000	1.000	1.077	1.000	1.154
Kalleh	1.000	1.000	1.000	1.000	1.000

Table 8. Proximity coefficient of each supplier and the suppliers' ranking

Supplier	C_2	C_3	C_4	C_5	C_6	Γ_i	Rank
Haraz	1.500	1.333	1.000	1.154	1.154	5.859	3
Pegah	1.200	1.000	1.000	1.071	1.071	6.657	2
Damdaran	2.000	1.000	1.077	1.000	1.154	5.769	4
Kalleh	1.000	1.000	1.000	1.000	1.000	7.000	1

4.1.2.2. ARWEN II application

In this process, the defect rate is added into the decision matrix as a cost criterion. The first step of this process is to transform the original matrix into a benefit matrix (Table 9).

Now determine the elected nominee (A_j^+) using Eq. (27), as shown below and then computing Γ_i values to rank the suppliers according to the larger values of Γ_i. The results are shown in Table 10.

$$A_j^+ = \{0.784, 1.5, 8, 7, 7.5, 7.5\}.$$

Table 9. The transformed supplier selection decision matrix

Supplier	C_1	C_2	C_3	C_4	C_5	C_6
Haraz	0.735	1	6	7	6.5	6.5
Pegah	0.777	1.25	8	7	7	7
Damdaran	0.703	0.75	8	6.5	7.5	6.5
Kalleh	0.784	1.5	8	7	7.5	7.5

Table 10. Prioritization of suppliers

Supplier	C_1	C_2	C_3	C_4	C_5	C_6	Γ_j	Rank
Haraz	1.067	1.500	1.333	1.000	1.154	1.154	4.792	3
Pegah	1.009	1.200	1.000	1.000	1.071	1.071	5.648	2
Damdaran	1.116	2.000	1.000	1.077	1.000	1.154	4.654	4
Kalleh	1.000	1.000	1.000	1.000	1.000	1.000	6.000	1

4.1.2.3. ARWEN III

In contrast to the other forms, ARWEN III assumes that DMs have access to criteria information, therefore the criteria weights could be determined. In the following problem, VIMM is employed for computing the criteria weights. Table 11 illustrates the transformed weighted supplier evaluation matrix, where $W_j = \{0.136, 0.205, 0.159, 0.182, 0.114, 0.205\}$.

Table 11. Weighted transformed supplier evaluation matrix

Supplier	C_1	C_2	C_3	C_4	C_5	C_6
W_j	0.136	0.205	0.159	0.182	0.114	0.205
Haraz	0.735	1	6	7	6.5	6.5
Pegah	0.777	1.25	8	7	7	7
Damdaran	0.703	0.75	8	6.5	7.5	6.5
Kalleh	0.784	1.5	8	7	7.5	7.5

Determining the elected nominee is the next step of the supplier selection process by ARWEN III method.

$A_j^+ = \{0.107, 0.308, 1.272, 1.274, 0.855, 1.538\}$.

The final ranks of suppliers are demonstrated in Table 12 with the computation of the weighted proximity coefficient (Γ_i^*) using Eq. (32).

Table 12. Supplier ranking with Γ_j values

Supplier	C_1	C_2	C_3	C_4	C_5	C_6	Γ_j	Rank
Haraz	1.067	1.500	1.333	1.000	1.154	1.154	1.215	3
Pegah	1.009	1.200	1.000	1.000	1.071	1.071	1.066	2
Damdaran	1.115	2.000	1.000	1.077	1.000	1.154	1.267	4
Kalleh	1.000	1.000	1.000	1.000	1.000	1.000	1.001	1

4.1.2.4. ARWEN IV application

In order to apply ARWEN IV algorithm to analyze the supplier evaluation matrix, a new decision matrix requires to be established in which an elected nominee ($Supplier_5^*$) is defined. The new supplier evaluation matrix is shown in Table 13. Similar to the other forms of ARWEN, converting the decision matrix into a benefit decision matrix is the next step of ARWEN IV. The transformed supplier evaluation matrix is exhibited in Table 14. Now deriving the supplier rankings according to the largest value of Γ_i^*, calculated using Eq. (32), is the final step of ARWEN IV, as shown in Table 15. This table indicates that the elected nominee affected the supplier ranking, while not being actually involved in the final ranking.

Table 13. Supplier evaluation decision matrix with elected nominee as ($Supplier_5^*$)

Supplier	–	+	+	+	+	+
	C_1	C_2	C_3	C_4	C_5	C_6
W_j	0.136	0.205	0.159	0.182	0.114	0.205
Haraz	0.0075	1	6	7	6.5	6.5
Pegah	0.0063	1.25	8	7	7	7
Damdaran	0.0084	0.75	8	6.5	7.5	6.5
Kalleh	0.0061	1.5	8	7	7.5	7.5
$Supplier_5^*$	0.0050	2	8	9	8	8

Table 14. The transformed supplier evaluation matrix

	C_1	C_2	C_3	C_4	C_5	C_6
W_j	0.136	0.205	0.159	0.182	0.114	0.205
Haraz	0.105	0.205	0.954	1.274	0.741	1.333
Pegah	0.110	0.256	1.272	1.274	0.798	1.435
Damdaran	0.102	0.154	1.272	1.183	0.855	1.333
Kalleh	0.111	0.308	1.272	1.274	0.855	1.538
$Supplier_5^*$	0.116	0.410	1.272	1.638	0.912	1.640

Table 15. The suppliers' ranks

Supplier	C_1	C_2	C_3	C_4	C_5	C_6	Γ_j	Rank
Haraz	1.097	2.000	1.333	1.286	1.231	1.231	3.823	3
Pegah	1.048	1.600	1.000	1.286	1.143	1.143	4.780	2
Damdaran	1.137	2.667	1.000	1.385	1.067	1.231	3.515	4
Kalleh	1.040	1.333	1.000	1.286	1.067	1.067	5.207	1

4.1.2.5. E-ARWEN IV application

In this section, application of E-ARWEN for the selection of the best dairy supplier is provided. The application follows Eqs (34)–(39) in order to evaluate the suppliers. Similar to the earlier form of ARWEN, the same set of criteria weights is utilized here. The key element of E-ARWEN process is to determine the positive and negative elected nominees using Eqs (34) and (35) and then converting the supplier evaluation matrix into the weighted benefit decision matrix, as shown in Table 16, where (A_j^+) and (A_j^-) denote positive and negative elected nominees respectively. These two elements helps the algorithm to compute positive proximity coefficient (Γ_i^+) and negative proximity coefficient (Γ_i^-) respectively. To rank the suppliers, the percent convergence coefficient (PCC_i) of each supplier needs to be calculated using Eq. (39), as shown in Table 17.

Table 16. Weighted normalized decision matrix with positive and negative elected nominees

Supplier	C_1	C_2	C_3	C_4	C_5	C_6
W_j	0.136	0.205	0.159	0.182	0.114	0.205
Haraz	0.071	0.088	0.063	0.093	0.052	0.097
Pegah	0.060	0.111	0.084	0.093	0.056	0.104
Damdaran	0.080	0.066	0.084	0.086	0.060	0.097
Kalleh	0.058	0.133	0.084	0.093	0.060	0.112
A_j^+	0.111	0.308	1.272	1.274	0.855	1.538
A_j^-	0.102	0.154	0.954	1.183	0.741	1.333

Table 17. Supplier ranks with PCC_i values

Supplier	C_1	C_2	C_3	C_4	C_5	C_6	Γ_i^+	Γ_i^-	PCC_i	Rank
Haraz	0.105	0.205	0.954	1.274	0.741	1.333	4.805	6.446	48.37%	3
Pegah	0.110	0.256	1.272	1.274	0.798	1.435	5.650	7.315	55.41%	2
Damdaran	0.102	0.154	1.272	1.183	0.855	1.333	4.677	6.487	47.31%	4
Kalleh	0.111	0.308	1.272	1.274	0.855	1.538	6.000	7.810	58.33%	1

4.2. The evaluation of Chain store cheese suppliers

The following supplier selection problem is adopted from Zakeri et al. (2022b) work, where a cheese supplier evaluation is presented. The problem is constructed on nine suppliers and ten different criteria, in which the "defect rate" is a non-beneficial criterion. The decision matrix is illustrated in Table 18, in which $W_j = \{0.114, 0.076, 0.101, 0.101, 0.114, 0.089, 0.127, 0.089, 0.114, 0.076\}$ is the set of criteria weights.

In this section, ARWEN III and E-ARWEN have been applied to solve the supplier selection problem. This case is more complex than the previous supplier selection example, which aids in showcasing the steps of the mentioned two forms of ARWEN more efficiently. In the next sub-section the ctiteita, Appropriateness of the product price to the market price, Numbers of Promotion times, Ability to adapt to increase, decrease, and change of order timing, Make-to-order production, Delivery reliability, Variety, Brand equity, Defect Rate, Reliability of quality, and After sales services are shown as $C_j = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9\}$ respectively. In the both application, the first supplier has been selected as the best supplier.

Table 18. The cheese supplier selection decision matrix

	+	+	+	+	+	+	+	–	+	+
W_j	0.114	0.076	0.101	0.101	0.114	0.089	0.127	0.089	0.114	0.076
	Appropriateness of the product price to the market price	Numbers of Promotion times	Ability to adapt to increase, decrease, and change of order timing	Make-to-order production	Delivery reliability	Variety	Brand equity	Defect Rate	Reliability of quality	After sales services
Kalleh	10	12	9	10	10	9	10	0.048	7	9
Mihan	9	14	7	7	7	3	9	0.021	7	7
Pegah	10	12	9	10	7	7	9	0.090	5	5
Haraz	9	12	9	7	9	7	7	0.043	7	7
Damdaran	7	9	5	7	7	5	7	0.054	7	7
Sabbah	9	18	7	9	9	7	5	0.041	7	5
Alima	5	6	10	10	10	9	5	0.063	9	5
Gela	10	12	9	5	7	3	2	0.047	10	5
Domino	7	10	5	5	5	2	7	0.029	5	7

4.2.1. ARWEN III Application

The application of ARWEN III and the key elements of the method, including the elected nominee and the weighted proximity coefficient, have been displayed in the following Tables 19 and 20.

Table 19. The normalized decision matrix and the elected nominee (A_i^+)

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
Kalleh	0.132	0.114	0.129	0.143	0.141	0.173	0.164	0.890	0.109	0.158
Mihan	0.118	0.133	0.100	0.100	0.099	0.058	0.148	0.952	0.109	0.123
Pegah	0.132	0.114	0.129	0.143	0.099	0.135	0.148	0.794	0.078	0.088
Haraz	0.118	0.114	0.129	0.100	0.127	0.135	0.115	0.901	0.109	0.123
Damdaran	0.092	0.086	0.071	0.100	0.099	0.096	0.115	0.876	0.109	0.123
Sabbah	0.118	0.171	0.100	0.129	0.127	0.135	0.082	0.906	0.109	0.088
Alima	0.066	0.057	0.143	0.143	0.141	0.173	0.082	0.856	0.141	0.088
Gela	0.132	0.114	0.129	0.071	0.099	0.058	0.033	0.892	0.156	0.088
Domino	0.092	0.095	0.071	0.071	0.070	0.038	0.115	0.933	0.078	0.123
A_i^+	0.132	0.171	0.143	0.143	0.141	0.173	0.164	0.952	0.156	0.158

Table 20. The weighted proximity coefficient and suppliers' ranks

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	Γ_i^*	Rank
Kalleh	0.132	0.114	0.129	0.143	0.141	0.173	0.164	0.890	0.109	0.158	16.895	1
Mihan	0.118	0.133	0.100	0.100	0.099	0.058	0.148	0.952	0.109	0.123	16.567	5
Pegah	0.132	0.114	0.129	0.143	0.099	0.135	0.148	0.794	0.078	0.088	16.669	3
Haraz	0.118	0.114	0.129	0.100	0.127	0.135	0.115	0.901	0.109	0.123	16.726	2
Damdaran	0.092	0.086	0.071	0.100	0.099	0.096	0.115	0.876	0.109	0.123	16.477	7
Sabbah	0.118	0.171	0.100	0.129	0.127	0.135	0.082	0.906	0.109	0.088	16.653	4
Alima	0.066	0.057	0.143	0.143	0.141	0.173	0.082	0.856	0.141	0.088	16.523	6
Gela	0.132	0.114	0.129	0.071	0.099	0.058	0.033	0.892	0.156	0.088	16.047	9
Domino	0.092	0.095	0.071	0.071	0.070	0.038	0.115	0.933	0.078	0.123	16.070	8

4.2.2. E-ARWEN III Application

The extended form of ARWEN has been applied to solve the cheese supplier selection. In contrast to other forms, it includes an additive element called the negative elected nominee which represents an abstract alternative fabricated out of ser of the minim value of the alternatives against the criteria. The E-ARWEN application is displayed in Tables 21–24.

Table 21. The normalized decision matrix and the positive and negative elected nominee (A_i^+, A_i^-)

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
Kalleh	0.132	0.114	0.129	0.143	0.141	0.173	0.164	0.890	0.109	0.158
Mihan	0.118	0.133	0.100	0.100	0.099	0.058	0.148	0.952	0.109	0.123
Pegah	0.132	0.114	0.129	0.143	0.099	0.135	0.148	0.794	0.078	0.088
Haraz	0.118	0.114	0.129	0.100	0.127	0.135	0.115	0.901	0.109	0.123
Damdaran	0.092	0.086	0.071	0.100	0.099	0.096	0.115	0.876	0.109	0.123
Sabbah	0.118	0.171	0.100	0.129	0.127	0.135	0.082	0.906	0.109	0.088
Alima	0.066	0.057	0.143	0.143	0.141	0.173	0.082	0.856	0.141	0.088
Gela	0.132	0.114	0.129	0.071	0.099	0.058	0.033	0.892	0.156	0.088
Domino	0.092	0.095	0.071	0.071	0.070	0.038	0.115	0.933	0.078	0.123
A_i^+	0.132	0.171	0.143	0.143	0.141	0.173	0.164	0.952	0.156	0.158
A_i^-	0.066	0.057	0.071	0.071	0.070	0.038	0.033	0.794	0.078	0.088

Table 22. Supplier selection converted matrix in accordance with the positive elected nominee

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
Kalleh	0.114	0.114	0.112	0.101	0.114	0.089	0.127	0.095	0.163	0.076
Mihan	0.127	0.098	0.144	0.144	0.163	0.267	0.141	0.089	0.163	0.098
Pegah	0.114	0.114	0.112	0.101	0.163	0.114	0.141	0.107	0.228	0.137
Haraz	0.127	0.114	0.112	0.144	0.127	0.114	0.181	0.094	0.163	0.098
Damdaran	0.163	0.152	0.202	0.144	0.163	0.160	0.181	0.097	0.163	0.098
Sabbah	0.127	0.076	0.144	0.112	0.127	0.114	0.254	0.094	0.163	0.137
Alima	0.228	0.228	0.101	0.101	0.114	0.089	0.254	0.099	0.127	0.137
Gela	0.114	0.114	0.112	0.202	0.163	0.267	0.635	0.095	0.114	0.137
Domino	0.163	0.137	0.202	0.202	0.228	0.401	0.181	0.091	0.228	0.098

Table 23. Supplier selection converted matrix in accordance with the negative elected nominee

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
Kalleh	0.057	0.038	0.056	0.051	0.057	0.020	0.025	0.079	0.081	0.042
Mihan	0.063	0.033	0.072	0.072	0.081	0.059	0.028	0.074	0.081	0.054
Pegah	0.057	0.038	0.056	0.051	0.081	0.025	0.028	0.089	0.114	0.076
Haraz	0.063	0.038	0.056	0.072	0.063	0.025	0.036	0.078	0.081	0.054
Damdaran	0.081	0.051	0.101	0.072	0.081	0.036	0.036	0.081	0.081	0.054
Sabbah	0.063	0.025	0.072	0.056	0.063	0.025	0.051	0.078	0.081	0.076
Alima	0.114	0.076	0.051	0.051	0.057	0.020	0.051	0.083	0.063	0.076
Gela	0.057	0.038	0.056	0.101	0.081	0.059	0.127	0.079	0.057	0.076
Domino	0.081	0.046	0.101	0.101	0.114	0.089	0.036	0.076	0.114	0.054

Table 24. The positive and negative proximity coefficients, the percent convergence coefficient, and the suppliers' ranks

	Γ_i^+	Γ_i^-	PCC_i	Rank
Kalleh	16.895	0.507	51.004	1
Mihan	16.567	0.619	40.892	4
Pegah	16.669	0.616	40.878	5
Haraz	16.726	0.569	45.359	2
Damdaran	16.477	0.675	35.692	7
Sabbah	16.653	0.592	43.297	3
Alima	16.523	0.640	38.931	6
Gela	16.047	0.732	31.761	8
Domino	16.070	0.812	23.818	9

5. Discussion and comparison

The level of information between ARWEN III and ARWEN IV is different. While DMs in ARWEN III are aware of the problem, they are also capable of assuming an optimal alternative with optimum criteria. In ARWEN IV, an optimal alternative influences the ranking process when there is perfect data available for the problem. The optimal alternative will not be included in the final ranking because it is only an abstract alternative without any independent existence. When compared to other traditional scales, ARWEN algorithms use VIMM scale and provide DMs with a greater variety of options (Figures 3 and 4). These scales consist of three turning points very low, moderate, and very high. These three points aid DMs to construct a bounded framework to translate their choice into the closest numbers based on their perceptions and expectations.

In the following section, results obtained from the method are compared with some very popular MCDM methods in order to render the advantages, limitations, and capabilities of the novel algorithm.

5.1. The first example

5.1.1. Comparisons

The application of a new MCDM method. Called ARWEN, is demonstrated in this paper for a chain store dairy goods supplier selection problem. Six criteria are defined for the evaluation process including defect rates, appropriateness of product price to market price, promotions, ability to adapt to increase, decrease, and change of order timing, after-sales services, and delivery reliability. Except "defect rate", all criteria are benefit in nature in which suppliers aim to maximize their scores. While as a cost criterion, minimum defect rate is always suitable. In the paper, the considered supplier selection problem dealt with each ARWEN family member which delivered the same results. The policy of ARWEN algorithms is the selection of the alternative that has the lowest changing rates compared to the elected nominee. As an instance, the following figure illustrates the fluctuation of each supplier than the elected nominee in ARWEN IV process. Results of ARWEN, based on Table 8, are displayed in Figure 5.

From Figure 5, it is observed that Damdaran has the most fluctuation than the elected nominee, as marked with the red line in the figure. Kalleh has the lowest fluctuation in contrast to Damdaran, as indicated by the violet line. All the four forms of ARWEN method indicate Kalleh as the best ranked alternative. In fact, Kalleh is at the optimum point in each decision-making matrix, whereas Damdaran stands at the last rank position in every method. Furthermore, "Pegah" and "Haraz" have the second and third ranks respectively in all ARWEN forms. Since ARWEN I only evaluates the decision matrices with benefit criteria, the obtained results are ignored in this section to provide a comprehensive view of the other forms of ARWEN. Table 25 shows the obtained results from the different forms of ARWEN members.

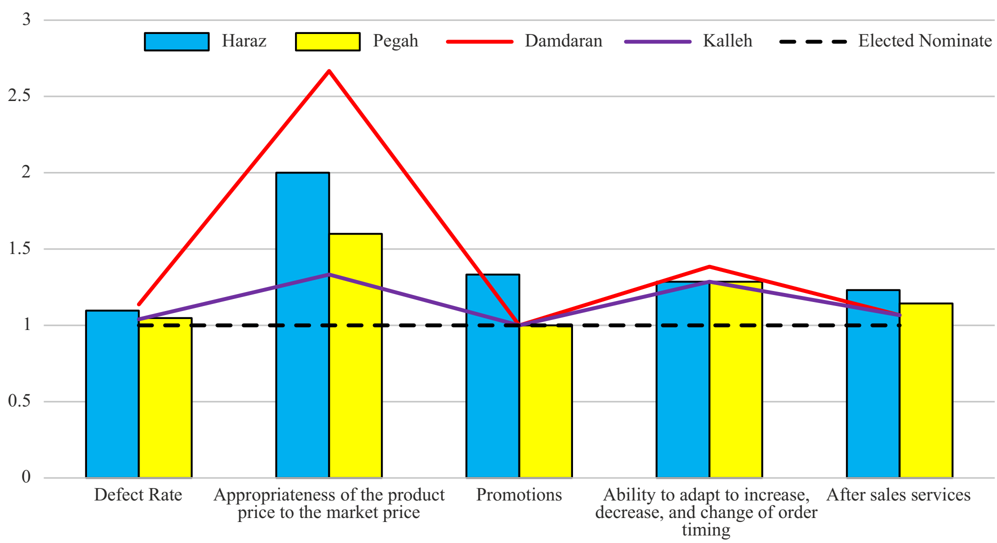

Figure 5. Fluctuation of suppliers rather than the elected nominee in ARWEN IV process

Table 25. Results of ARWEN methods

	Ranking		
Supplier	ARWEN II	ARWEN III	ARWEN IV
Haraz	3	3	3
Pegah	2	2	2
Damdarn	4	4	4
Kalleh	1	1	1

The methods produced the same findings, as shown in the Table 15, as a result of Kalleh's supremacy in practically every stated area of a dairy product supplier. Additionally, the unequal distribution of the cost and benefit criteria and, as a result, the lower weight assigned to the cost criterion, could be cited as contributing factors to the similar outputs.

5.1.2. Comparison

In this section, results of the considered supplier selection problem are compared with the results of SAW (MacCrimmon, 1968), TOPSIS, and VIKOR methods in order to validate the outcomes of ARWEN. Among the variations, ARWEN III uses the traditional concept of MCDM problems which includes alternatives, criteria, and a specified set of criteria weights and hence, the results of ARWEN III are compared with TOPSIS and SAW methods, as shown in Table 26.

The information in Table 14 suggests that the four MCDM methods rank suppliers identically. Due to the fact that both TOPSIS and ARWEN algorithms are devised using distance-based logic, they are comparable. In the decision matrices, both methods seek to identify the ideal solutions. They are referred to as elected nominees in ARWEN and positive/negative ideal solutions in TOPSIS, respectively. In general, ARWEN ranks alternatives according to which one has the least impact on the elected nominee. The alternatives that TOPSIS selects are those that are situated closest to the ideal solution and farthest from the worst solution. The origin of the solutions is another distinction between TOPSIS and ARWEN. TOPSIS draws the ideal solutions from the decision matrix, which implies that the decision matrix generates the ideal solutions; in contrast, ARWEN has been developed to consider the elected nominee even outside of the decision matrix (see ARWEN IV).

As shown in Table 26, even though ARWEN and SAW produced the same results when solving the supplier selection problem, a comparison shows that ARWEN is a little more complex than SAW method.

Table 26. The comparative analysis of results: ARWEN III, TOPSIS, SAW and VIKOR

	Ranking Result			
Supplier	ARWEN III	SAW	TOPSIS	VIKOR
Haraz	3	3	3	3
Pegah	2	2	2	2
Damdaran	4	4	4	4
Kalleh	1	1	1	1

VIKOR method is more complicated than ARWEN, yet it produced the same results (see Table 26). In addition, ARWEN features a straightforward normalizing procedure that is only applicable to decision matrices with cost criteria. ARWEN is hence far uncomplicated than other MCDM methods, which employ many normalizing procedures. It can be concluded that ARWEN is reliable to use for MCDM problems with a less complicated algorithm and a more flexible approach if simplicity is considered as a factor for reliability of an MCDM method.

The results revealed that ARWEN is just as effective as the other methods. The ARWEN family members offer simpler algorithms for MCDM problems than TOPSIS and VIKOR, which is a benefit. The new method offers a flexible algorithm with four members to fit itself with the context of the decision-making problems and solve it in various situations with respect to the level of access to information.

E-ARWEN is proposed as an expanded version of ARWEN to analyze alternatives with two elected nominees in order to address decision-making challenges. Results of E-ARWEN are consistent with those of other established methods. Results from the comparison of EARWEN, ARWEN III, SAW, TOPSIS, and VIKOR are presented in Table 27.

Table 27. The comparative analysis of results: ARWEN III, EARWEN, SAW, TOPSIS and VIKOR

Supplier	ARWEN III	E-ARWEN	SAW	TOPSIS	VIKOR
Haraz	3	3	3	3	3
Pegah	2	2	2	2	2
Damdaran	4	4	4	4	4
Kalleh	1	1	1	1	1

Unlike TOPSIS method, E-ARWEN selects the best alternative based on its changes rates against the elected nominees, both positive and negative. According to its distance calculation from the two borders of the best and worst ideal solutions, TOPSIS selects the most favorable alternative. The changes rates are shown in Figure 6 using EARWEN's application to the supplier selection issue for chain stores.

According to Figure 6, the dairy brand Damdaran exhibits the highest fluctuation compared to the positive elected nominee, while Kalleh, in contrast to Damdaran, exhibits the highest fluctuation compared to the negative elected nominee and the least fluctuation compared to the positive elected nominee. In light of Table 27 and as shown in Figure 6, Kalleh is ranked first, and Damdaran is last among the other suppliers.

5.2. RPI application

To demonstrate comparability of the derived results, ARWEN III has been compared with other MCDM methods in the previous section. All the proposed five methods including E-ARWEN produced the same results. Because of the relative dominance of the alternatives or distribution of criteria weights, rankings produced by different MCDM methods may be the same. Therefore, validating the results of MCDM methods through another process becomes necessary. In this paper, RPI has been proposed to validate the result. RPI is used

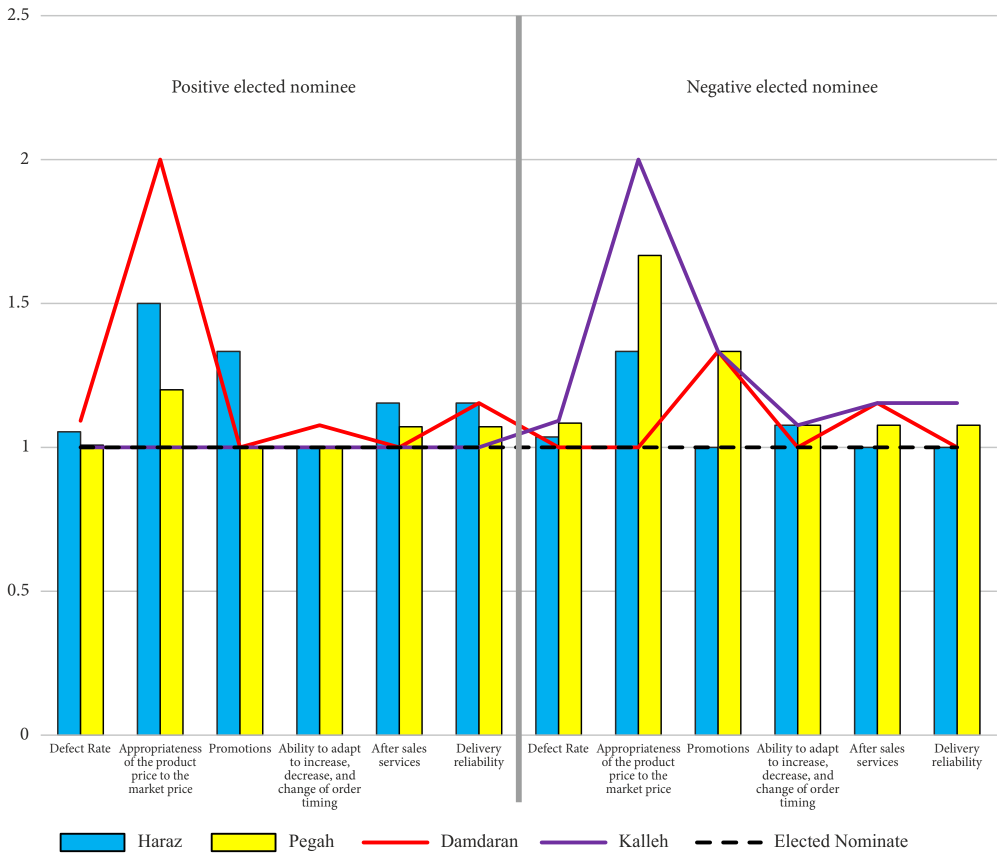

Figure 6. The fluctuation of Kalleh and Damdaran compated to A_j^+ and A_j^- in E-ARWEN process

to assess performances of the considered methods in evaluating the suppliers in order to validate the obtained results. The relevant criteria must first be ranked according to their weights from maximum to minimum to start this validation process, as shown in Table 28. Form this table, it can be observed that the weights are equally distributed between delivery reliability and appropriateness of product price to market price criteria. The results of the mentioned Eqs (2) and (3) for determining the criteria priority (C_2 and C_6) are shown in Table 29. The new order of criteria could be found in Table 30.

The next step begins with the deletion of the criterion with the lowest weight, which in present example is the after-sales service (C_5), and the step is completed by evaluating the alternatives against the remaining criteria. Table 31 exhibits the CPI values calculated using Eqs (8) and (9). This index is crucial in determining how well the MCDM method performs.

Table 28. Arrangement of criteria weights in descending order

Criteria	C_2	C_6	C_4	C_3	C_1	C_5
Weight	0.205	0.205	0.182	0.159	0.136	0.114

Table 29. The computation of the priority between C_2 and C_6

W_j	0.136	0.205	0.159	0.182	0.114	0.205
Supplier	C_1	C_2	C_3	C_4	C_5	C_6
Haraz	0.2451	0.2222	0.2000	0.2545	0.2281	0.2364
Pegah	0.2591	0.2778	0.2667	0.2545	0.2456	0.2545
Damdaran	0.2344	0.1667	0.2667	0.2364	0.2632	0.2364
Kalleh	0.2614	0.3333	0.2667	0.2545	0.2632	0.2727
$\left(\sum_{j=l}^{n} r_{ij}\right)^{w_{C_j}}$		2.9947				3.0096

Table 30. Order of the new criteria weights

Criteria	C_6	C_2	C_4	C_3	C_1	C_5
Weight	0.205	0.205	0.182	0.159	0.136	0.114

Table 31. CPI values for the criteria

CPI	C_6	C_2	C_4	C_3	C_1	C_5
P_{C_j}	0.2582	0.2074	0.1655	0.1350	0.1026	0.1313

Table 32 and Figure 7 exhibit the analytical comparison between the two variables (P_{C_j} and w_j) to illustrate the difference between performances of each criterion and associated relative weight.

Table 32. Criteria performance index, the criteria weights, and their difference

Criteria	C_6	C_2	C_4	C_3	C_1	C_5
P_{C_j}	0.1026	0.2074	0.135	0.1655	0.1313	0.2582
w_j	0.136	0.205	0.159	0.182	0.114	0.205
Difference	0.0334	0.0024	0.024	0.0165	0.0173	0.0532

Comparison between the weighted proximity coefficient obtained from the MCDM method and the generated weighted proximity coefficient with the impact of *CPI* as the new set of criteria weight is the next significant step in the computation of *RPI* values of an MCDM method. Table 33 displays the *RPI*, the weighted proximity coefficients produced by two different weights, and its differences, as shown in Figure 8.

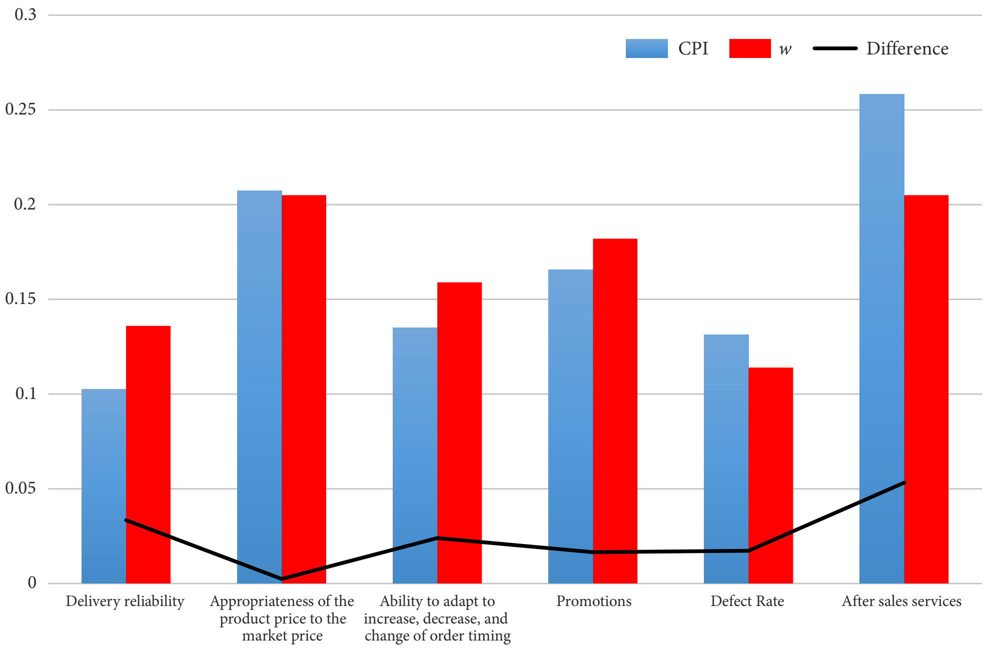

Figure 7. Analytical comparison between the criteria performance and their weights

Table 33. Ranking performance index (*RPI*)

<table>
<tr><th>Supplier</th><th>Γ_i^*</th><th>$\Gamma_i^{*'}$</th><th colspan="2"></th></tr>
<tr><td>Haraz</td><td>6.785</td><td>6.785</td><td colspan="2">0.001</td></tr>
<tr><td>Pegah</td><td>6.934</td><td>6.930</td><td colspan="2">0.004</td></tr>
<tr><td>Damdaran</td><td>6.733</td><td>6.728</td><td colspan="2">0.004</td></tr>
<tr><td>Kalleh</td><td>6.999</td><td>7.000</td><td colspan="2">0.001</td></tr>
<tr><td colspan="3"></td><td>RPI</td><td>0.011</td></tr>
</table>

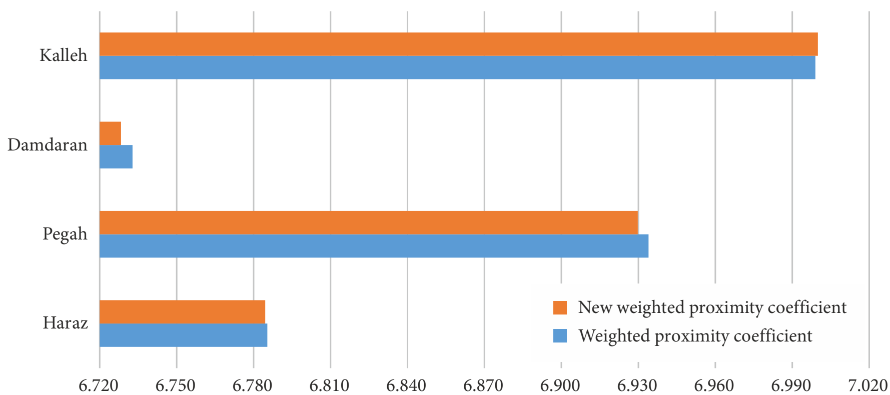

Figure 8. Difference between Γ_i^* and $\Gamma_i^{*'}$ in which performance of an MCDM method could be interpreted

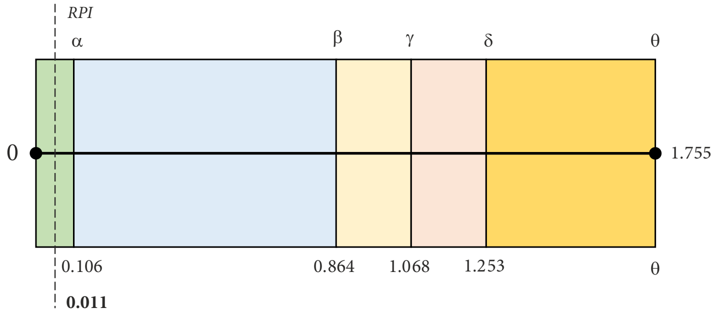

Figure 9. Performance of ARWEN III in terms of *RPI*

The interpretation of the RPI calculation is the most crucial step. In order to be regarded as a reliable MCDM method, RPI must be oriented as close to 0 value as possible, as shown in Figure 2. To display different performance spectra, five variables are defined here. These variables can be computed based on the number of criteria using Eqs (15)–(24). For the considered case study of supplier evaluation, this paper considers six criteria, thus, Eqs (15)–(19) are applicable here to estimate *RPI* values. Figure 9 shows that the calculated *RPI* values of ARWEN III method is located on the best performance spectrum ($0 < RPI \leq \alpha$) which establishes its reliability and acceptability. The other spectrum boundaries are as follows: $\theta, \delta, \gamma, \beta, \alpha$ $[\alpha, \beta] = [0.106, 0.864]$; $[\beta, \gamma] = [0.864, 1.068]$; $[\gamma, \delta] = [1.068, 1.253]$; $[\delta, \theta] = [1.253, 1.755]$.

5.3. The second example

This paper's second example represents a cheese supplier selection, where ARWEN III and E-ARWEN have been applied to the case. In order to explore the behavior of the extensions above of the ARWEN algorithm and draw the comparisons, seven MCDM methods, including ARPASS, TOPSIS, VIKOR, SAW, COPRAS, WPM, and EDAS, have been applied to extract rankings from the cheese supplier selection's decision matrix. Each MCDM method's ranking is shown in Figure 10 and Table 34, where Kalleh is dominant and has been selected as the best supplier by each MCDM method. As demonstrated in Figure 10, the rankings are almost similar and consistent; however, some differences exist. In the following sections, the ARWEN's two extensions' similarities of rankings with other MCDM methods have been comprehensively evaluated via various statistical measures.

5.3.1. Spearman's rank correlation

One of the popular statistical measures for evaluating two data sets is Spearman's rank correlation. It is applied in many studies to measure the similarities of rankings generated by different MCDM methods. However, like other measures that could be employed for evaluating similarities of rankings, Spearman's rank correlation suffers from a fundamental lack, they are not designed to evaluate the similarities between two different rankings.

The rankings are the data sets in which each member has a different value. At the same time, statistical measures such as Spearman's rank correlation assume the same value for all data sets' members. Therefore, another measure is needed to determine the similarities between

Table 34. The comparison of the MCDM methods' outputs for the cheese supplier selection problem

	ARWEN III	E-ARWEN	ARPASS	TOPSIS	VIKOR	SAW	COPRAS	WPM	EDAS
Kalleh	1	1	1	1	1	1	1	1	1
Mihan	5	4	3	2	4	3	2	2	4
Pegah	3	5	4	7	6	5	5	5	6
Haraz	2	2	2	3	2	2	3	3	2
Damdaran	7	7	8	9	7	8	7	7	7
Sabbah	4	3	6	4	3	4	4	4	3
Alima	6	6	5	5	5	6	6	6	5
Gela	9	8	7	6	8	7	8	8	8
Domino	8	9	9	8	9	9	9	9	9

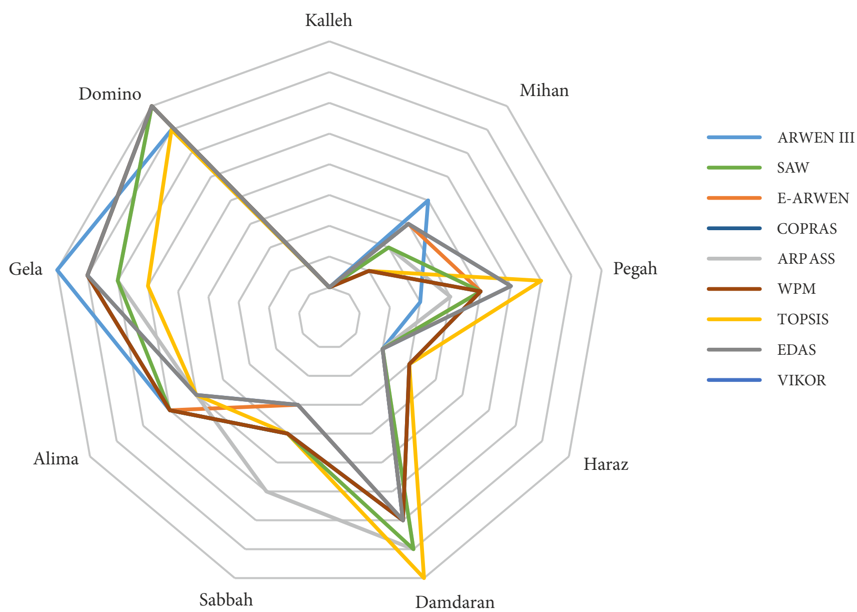

Figure 10. The analytical comparison of the rankings generated by different MCDM methods

rankings. In the next section, the different similarity measures have been applied to study the similarities between the ARWEN III and E-ARWEN with the other seven MCDM methods. The Zakeri-Konstantas performance correlation coefficient is applied to show the similarities between MCDM methods by focusing on their performances in generating rankings.

The similarities determined by Spearman's rank correlation coefficient in solving cheese supplier selection have been portrayed in Figure 11, 12. Besides E-ARWEN, Figure 11 reveals that ARWEN III has the slightest similarity with TOPSIS, while EDAS, SAW, and VIKOR exhibit the most similarity. On the other hand, E-ARWEN demonstrates the highest similarity to VIKOR and EDAS, with two differences in the rankings (see Figure 12). The overall analytical comparisons are shown in Figure 13, and the Spearman's rank correlation coefficient's results are provided in (Table 35).

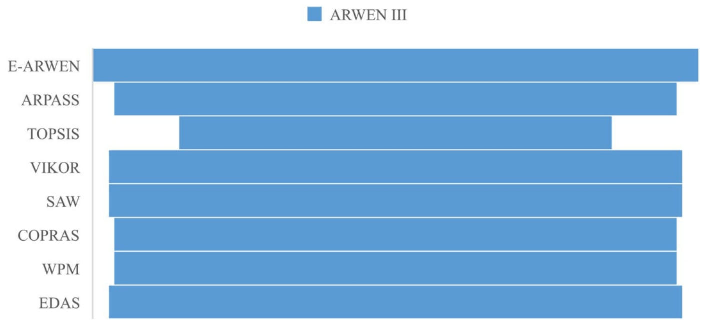

Figure 11. The similarity of ARWEN III and other MCDM methods

Figure 12. The similarity of E-ARWEN and other MCDM methods

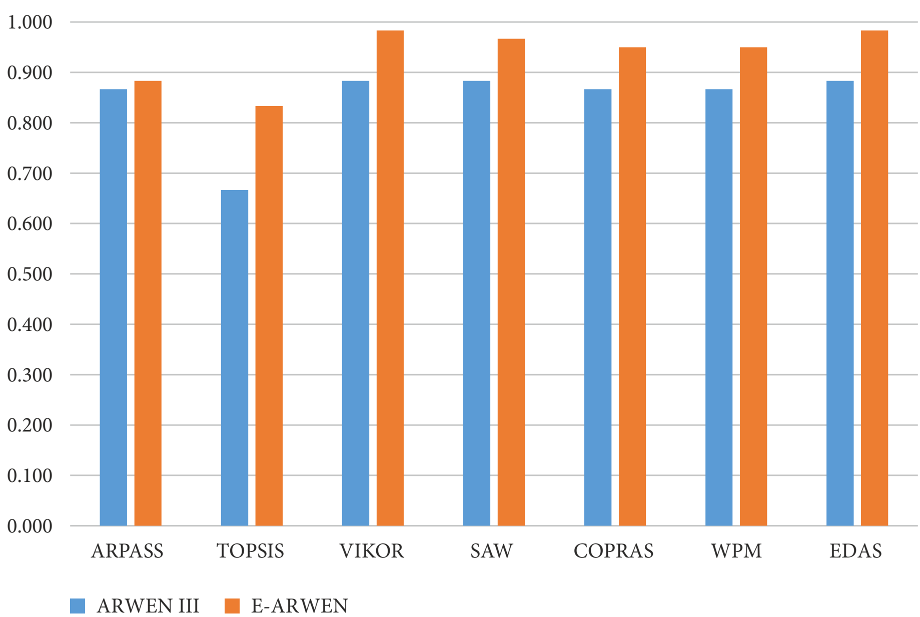

Figure 13. The comparative analysis of the similarities between E-ARWEN and ARWEN II with other MCDM methods

Table 35. The similarities between ARWEN III and E-ARWEN with other MCDM methods using Spearman's rank correlation

	ARWEN III	E-ARWEN	ARPASS	TOPSIS	VIKOR	SAW	COPRAS	WPM	EDAS
ARWEN III		0.933	0.867	0.667	0.883	0.883	0.867	0.867	0.883
E-ARWEN	0.933		0.883	0.833	0.983	0.967	0.950	0.950	0.983

5.3.2. Manhattan distance

The results of Manhattan distance are shown in Figure 14 and 15, where TOPSIS has the least similarity to ARWEN III, and VIKOR and EDAS show the highest similarity to E-ARWEN. The results almost deliver the same range of similarity compared to Spearman's rank correlation coefficient results.

5.3.3. Zakeri-Konstantas distance product correlation coefficient

The Zakeri-Konstantas distance product correlation coefficient $\left(\mathbb{Z}_{(l:h)}\right)$ is a symmetric statistical measure of linear correlation between two sets of rankings. In contrast to other distance-based similarity and dissimilarity measures, similar to the Zakeri-Konstantas performance correlation coefficient, it considers unique values for each member of data sets of rankings.

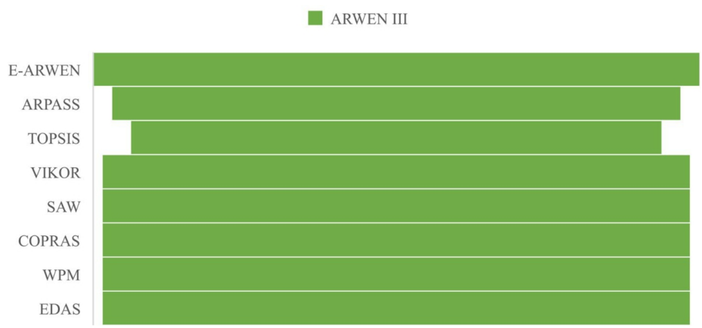

Figure 14. The similarity analysis of ARWEN III and other MCDM methods using the Manhattan distance

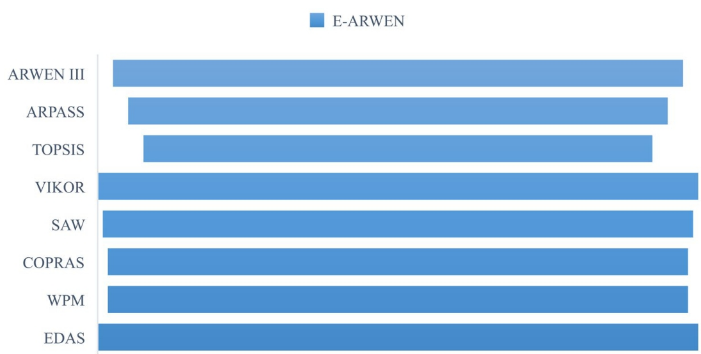

Figure 15. The similarity analysis of E-ARWEN and other MCDM methods using the Manhattan distance

For instance, the first rank's value is different from the second rank, and so on, while other measures, including Spearman's rank correlation, assume the exact value of each member of data sets. Utilizing the value, it measures the correlation between the members and the total similarity of the two rankings. The Zakeri-Konstantas distance product correlation coefficient offers a simpler process compared to the Zakeri-Konstantas performance correlation coefficient. Nevertheless, it keeps the similarity between 0 and 1, while the Zakeri-Konstantas performance correlation coefficient expresses the similarity in percentage in its classic form. The Zakeri-Konstantas distance product correlation coefficient equation is reflected in Eqs (40), where the aim is to measure the similarity of lth MCDM with hth MCDM. The equation is employed when the aim of comparison is to measure the similarity of merely two rankings.

$$\mathbb{Z}_{(l,h)} = \left\langle 1 - \sum_{i=1}^{m} \left(\frac{\left\langle \left(\Delta_i^{R_{(l,h)}} \right)^2 \right\rangle^{0.5}}{\max\limits_{1 \le i \le m} \left\langle \left(\Delta_i^{R_{(l,h)}} \right)^2 \right\rangle^{0.5}} w_i \right) \right\rangle \left\langle \frac{m}{10} \right\rangle^{-1}, \; i = \{1,..,m\},\; 0 < \mathbb{Z}_{(l,h)} < 1. \quad (40)$$

When the comparison comprises more than two rankings, the following equation ought to be employed to measure the similarity between l MCDM method, as the generator of lth ranking, and other MCDM methods. The Zakeri-Konstantas distance product correlation coefficient for more than two rankings is shown in Eqs (41).

$$\mathbb{Z}_{(l,J)} = 1 - \left\langle \sum_{J=1}^{G} \left\langle \frac{\left\langle \left(\Delta_i^{R_{(l,J)}} \right)^2 \right\rangle^{0.5}}{\max\limits_{1 \le i \le m} \max\limits_{1 \le J \le G} \left\langle \left(\Delta_i^{R_{(l,J)}} \right)^2 \right\rangle^{0.5}} w_i \right\rangle \left(\sum_{i=1}^{m} \sum_{J=1}^{G} \left\langle \frac{\left\langle \left(\Delta_i^{R_{(l,J)}} \right)^2 \right\rangle^{0.5}}{\max\limits_{1 \le i \le m} \max\limits_{1 \le J \le G} \left\langle \left(\Delta_i^{R_{(l,J)}} \right)^2 \right\rangle^{0.5}} w_i \right\rangle \right)^{-1} \right\rangle,$$

$$J = \{1, ..., G\}. \quad (41)$$

In the equations, w_i stands for the weights of each rank in lth MCDM method's ranking. The weights are delivered in Table 36.

The example of measuring similarity between two MCDM methods using the Zakeri-Konstantas distance product correlation coefficient, ARWEN III and ARPASS, is displayed in Table 37. The main contrast between the mentioned measures and the Zakeri-Konstantas distance product correlation coefficient is that the latter considers a connection between the data sets generated by MCDM methods in measuring similarity, while Spearman's rank correlation and the Manhattan distance execute it separately. We measured the similarities of the other MCDM methods with the ARWEN III and E-ARWEN separately (see Table 38 and Figures 16, 17) to compare the similarities computed by Spearman's rank correlation and the Manhattan distance with the outputs of the Zakeri-Konstantas distance product correlation coefficient, where the comparison is displayed in Figures 18–21.

Table 36. The constant values of w_i with respect to the number of ranking's variables

Rank \ m	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0.500	0.400	0.333	0.286	0.250	0.222	0.200	0.182	0.167	0.154	0.143	0.133	0.125
2	0.333	0.300	0.267	0.238	0.214	0.194	0.178	0.164	0.152	0.141	0.132	0.124	0.117
3	0.167	0.200	0.200	0.190	0.179	0.167	0.156	0.145	0.136	0.128	0.121	0.114	0.108
4		0.100	0.133	0.143	0.143	0.139	0.133	0.127	0.121	0.115	0.110	0.105	0.100
5			0.067	0.095	0.107	0.111	0.111	0.109	0.106	0.103	0.099	0.095	0.092
6				0.048	0.071	0.083	0.089	0.091	0.091	0.090	0.088	0.086	0.083
7					0.036	0.056	0.067	0.073	0.076	0.077	0.077	0.076	0.075
8						0.028	0.044	0.055	0.061	0.064	0.066	0.067	0.067
9							0.022	0.036	0.045	0.051	0.055	0.057	0.058
10								0.018	0.030	0.038	0.044	0.048	0.050
11									0.015	0.026	0.033	0.038	0.042
12										0.013	0.022	0.029	0.033
13											0.011	0.019	0.025
14												0.010	0.017
15													0.008

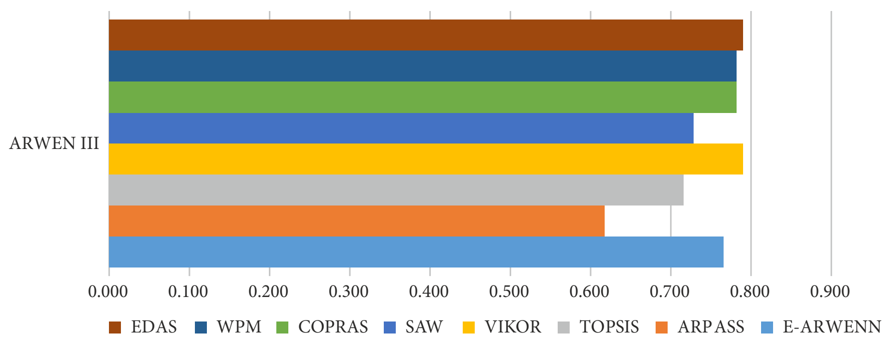

Figure 16. The difference between similarities of MCDM methods with ARWEN III

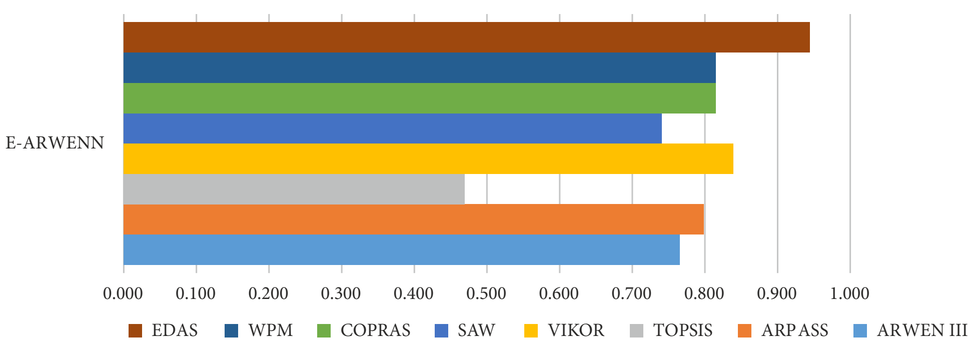

Figure 17. The difference between similarities of MCDM methods with E-ARWEN

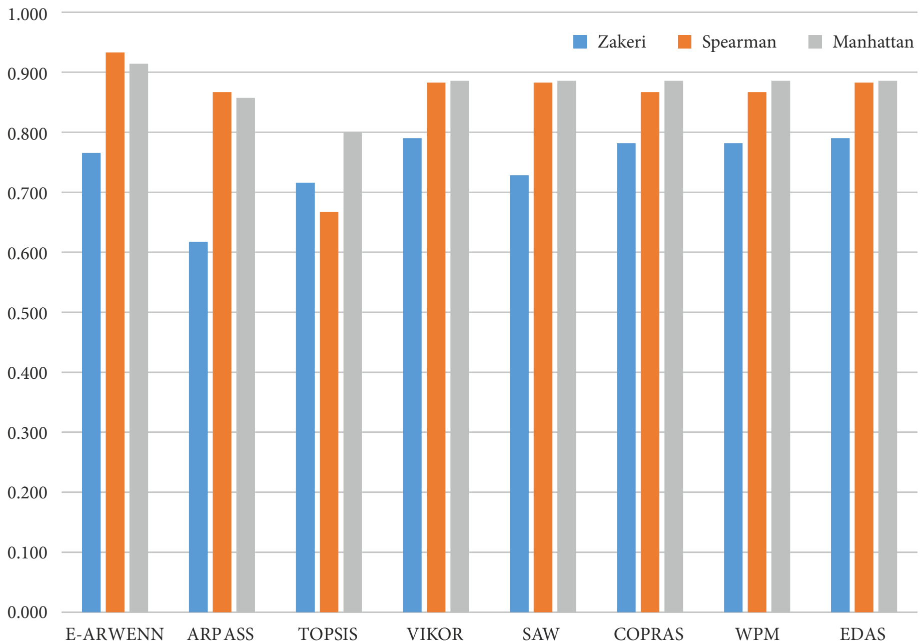

Figure 18. The difference between similarities of MCDM methods and ARWEN III computed by three different measures

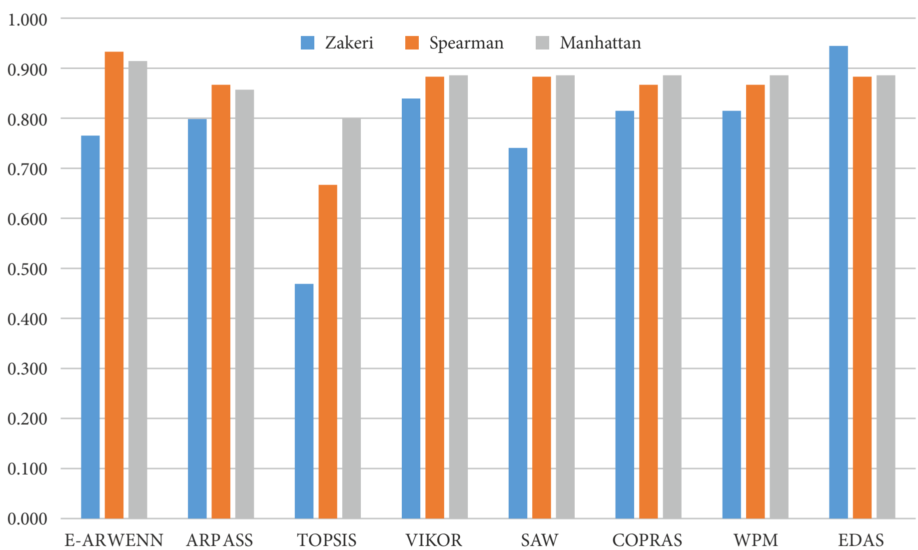

Figure 19. The difference between similarities of MCDM methods and ARWEN III computed by three different measures

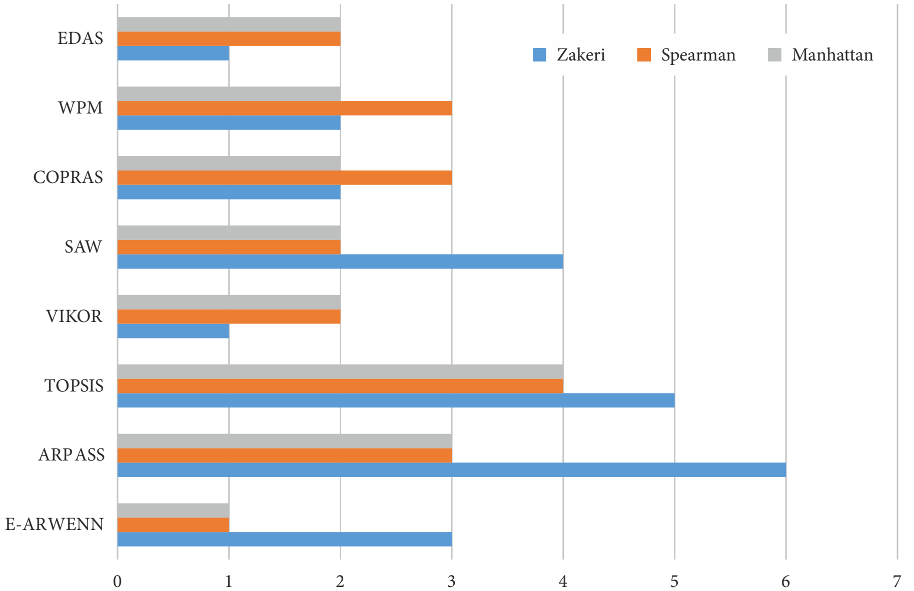

Figure 20. The comparative analysis of rankings of similarity of MCDM methods and ARWEN III

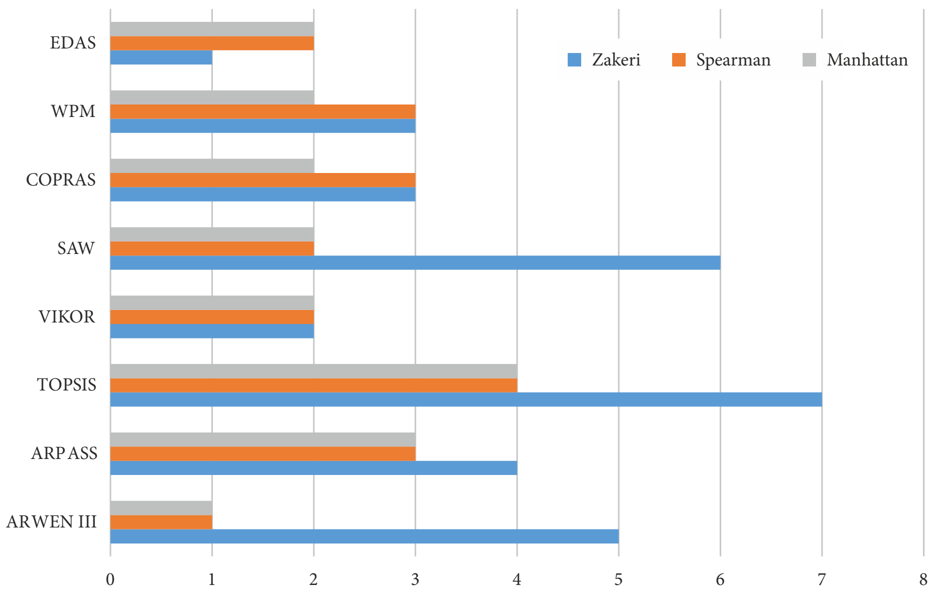

Figure 21. The comparative analysis of rankings of similarity of MCDM methods and E-ARWEN

The three methods' outputs are obviously different since they use different procedures to extract the similarities, however in the ranks of similarities (see Figures 20, 21), Spearman's rank correlation and the Manhattan distance delivered almost the same results, while the Zakeri-Konstantas distance product correlation coefficient revealed the fundamental difference. The primary reason is the impact of rank values, w_i, which is reflected in the different outputs. In most MCDM problems, the aim is to have/select the best alternative instead of a precise ranking of alternatives. Hence, in the validation of MCDM methods through comparison of their outputs, the key element is the best alternative which gives a perspective of an accord in the first rank. Spearman's rank correlation and other similarity measures ignore this fact, and the mentioned compromise on the best alternative could not be interpreted from the results they provide. On the other hand, another shortage of those two methods in providing similarities is the disability to reverberate the difference between the rankings MCDM methods generated. As mentioned, in the data sets MCDM methods generate, each member, i.e., each rank, possesses a unique value that makes it distinct from other members of the set. Ignoring this value leads the incorrect similarities between rankings and makes using Spearman's rank correlation and other distance-based similarity/dissimilarity measures inoperative. Using the weights "the weights of each rank" and considering a connection between the MCDM methods employed for the comparison process makes the Zakeri-Konstantas distance product correlation coefficient the most reliable distance-based measure for computing similarities between rankings.

Table 37. The Zakeri-Konstantas distance product correlation coefficient for E-ARWEN and ARPASS

	w_i	E-ARWEN	ARPASS	Distance Product
Kalleh	0.200	0	0	0
Mihan	0.111	1	2	0.111111
Pegah	0.156	2	1	0.077778
Haraz	0.178	0	0	0
Damdaran	0.067	0	1	0.033333
Sabbah	0.133	1	2	0.133333
Alima	0.089	0	1	0.044444
Gela	0.022	1	2	0.022222
Domino	0.044	1	1	0.022222
			SUM	0.444444
			$\mathbb{Z}_{(ARWENNII,\ ARPASS)}$	0.765

Table 38. similarity between ARWEN III with other MCDM methods

	ARWEN III	E-ARWENN	ARPASS	TOPSIS	VIKOR	SAW	COPRAS	WPM	EDAS
ARWEN III		0.765	0.617	0.716	0.790	0.728	0.782	0.782	0.790
E-ARWENN	0.765		0.798	0.469	0.840	0.741	0.815	0.815	0.944

Measuring the similarities of the MCDM methods with ARWEN III and E-ARWEN, not separately this time, conducts by Eqs (41). The results are exhibited in Table 39 and Figures 22–25 E-ARWEN parades the highest similarity to EDAS and VIKOR in solving the cheese supplier selection. However, similar to another extension of the ARWEN algorithm, ARWEN III, it does not compromise with TOPSIS. SAW, VIKOR, and EDAS are three MCDM algorithms that generate the most similar outputs to the ARWEN III and E-ARWEN. Using a similar philosophy with VIKOR explains the similarity of E-ARWEN and VIKOR results.

Table 39. similarity between ARWEN III with other MCDM methods

	ARWEN III	E-ARWENN	ARPASS	TOPSIS	VIKOR	SAW	COPRAS	WPM	EDAS
ARWEN III		0.913	0.875	0.801	0.879	0.903	0.875	0.875	0.879
E-ARWENN	0.862		0.813	0.744	0.946	0.926	0.882	0.882	0.946

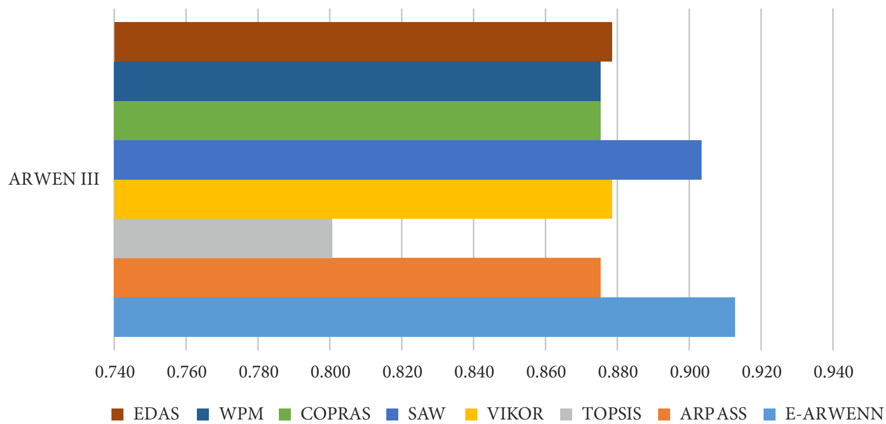

Figure 22. The difference between similarities of MCDM methods with ARWEN III

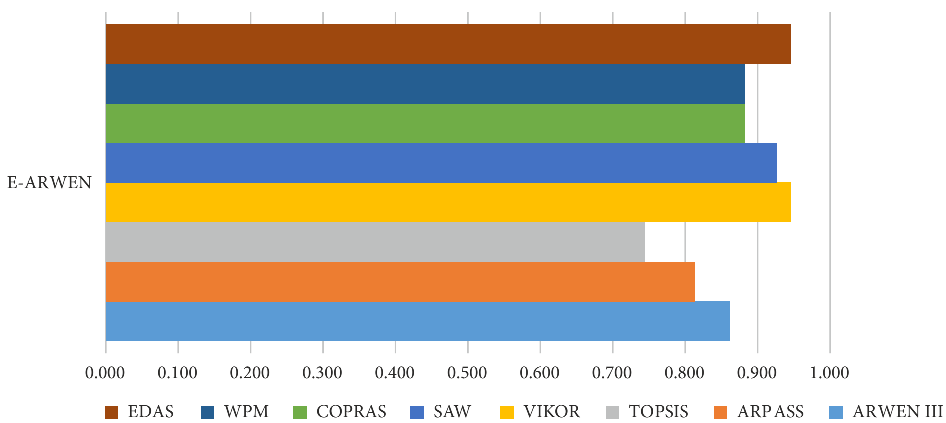

Figure 23. The difference between similarities of MCDM methods with E-ARWEN

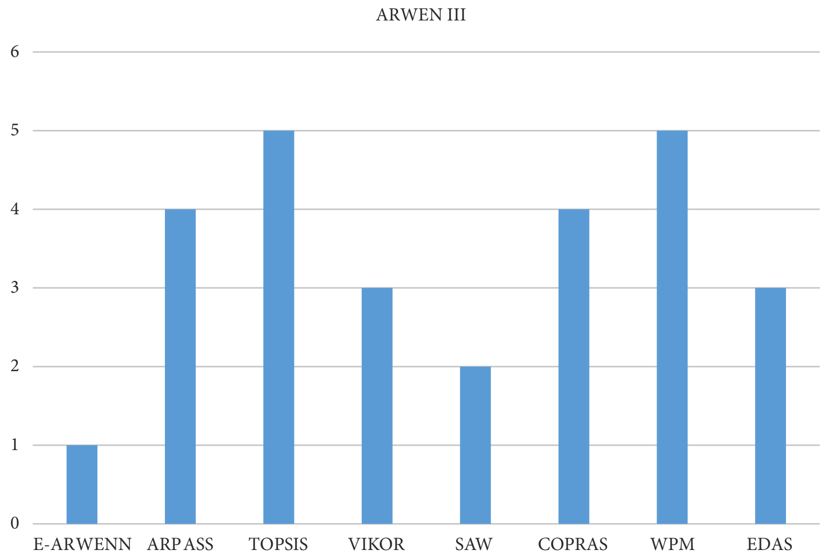

Figure 24. The ranks of similarity between ARWEN III and other MCDM methods

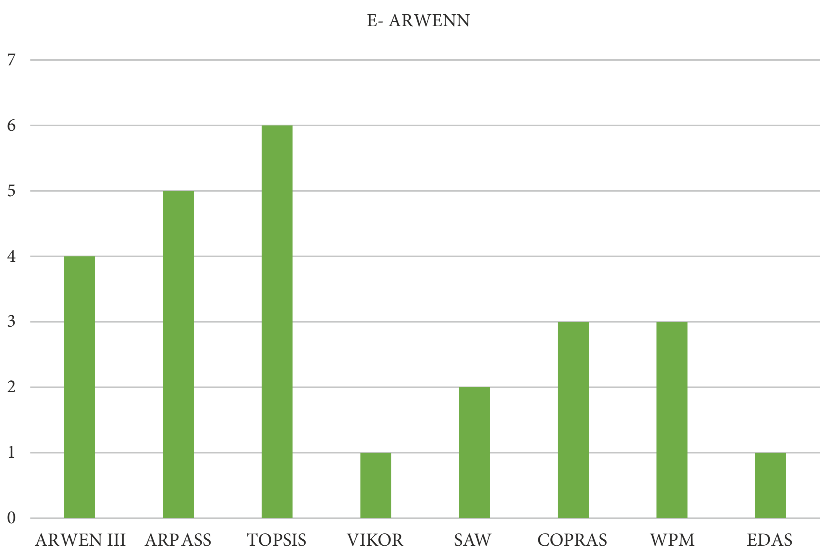

Figure 25. The ranks of similarity between E-ARWEN and other MCDM methods

Conclusions and future works

One of the most prevalent MCDM problems that many researchers have addressed in their studies is supplier selection. The most widely used MCDM method segments for resolving supplier selection issues are distance-based methods. This section focuses on two key components of handling decision-making problems: 1. resolving supplier selection issues through the application of MCDM methods; and 2. validating the outcomes. The family of distance-based MCDM methods assesses alternatives using distance measurement functions. The computation of the best alternatives, which may be taken directly from the decision matrix or taken into account outside of it, is a feature shared by all of these methods. As a distance based MCDM method, the alternatives ranking with elected nominee (ARWEN) is introduced in this paper as a new MCDM method. ARWEN computes change rates between the ideal alternatives – referred to as elected nominees and other alternatives instead of computing the distance between them.

Based on DMs' access to available information regarding the criteria and alternatives, ARWEN is advocated in four different forms. The first group consists of ARWEN I and II. ARWEN I is the most basic version and can be applied to the problems with only benefit criteria when the DMs do not have access to all available information on the problem. DMs in the second group, which consists of ARWEN III and ARWEN IV, have full access to information on the criteria and alternatives. The elected nominee in each of the four variations of ARWEN is comparable to the positive ideal alternative in TOPSIS method. E-ARWEN takes into account the negative elected nominee in the decision matrix for analysis.

The negative elected nominee and the negative ideal alternative in TOPSIS and VIKOR share a similar concept. In E-ARWEN, the alternative is required to have the highest change rates relative to the negative elected nominee. A new statistical measure (RPI) is also introduced in this research to assess performance of MCDM methods.

The criteria impact index (CII), which depicts the influence of each criterion in the final ranking produced by an MCDM method, and the criteria performance index (CPI), which depicts the effectiveness of the criteria in ranking alternatives in diverse situations, are the two statistical measures on which the RPI is built. There are five spectrums that can be used to understand the performance of MCDM methods based on the RPI values. The spectrums are bounded between zero and θ. In addition, α, β, γ, and δ are other variables that separate the spectrums. These variables and the interval they produce are used to interpret the performance of MCDM methods. The higher performance indicates greater reliability of the method in evaluating the specific problem, which afterwards reveals the accuracy of the results produced by the method. The most significant advantage that RPI has over well-known statistical tools like Spearman's rank correlation and sensitivity analysis for validating the results of any MCDM method is that performance evaluation of an MCDM method is not constrained by the number of cases it applies to and the number of other MCDM methods adopted. To show the benefits and potential drawbacks of the new statistical measure, it would be interesting to compare the findings obtained by RPI with comparisons with other existing methods.

Zakeri-Konstantas distance product correlation coefficient ($\mathbb{Z}$) is another statistical measure introduced in this paper. $\mathbb{Z}$ is a distance-based similarity measure which is developed to measure the similarities of data sets, alternatives' rankings, generated by MCDM algorithms. The main advantage of the new measure is that it considers unique values for each member of data sets in order to compute the similarity, while other measures consider a same value or ignore fundamentally the values of each member of rankings. This makes the new measure is reliable tool compared to other measures such as Spearman's rank correlation other distance-based measures.

The ARWEN is designed in accordance with DM's access to the source of information, hence at least two of its extensions have fundamental limitations compared with other MCDM methods since they have proposed to solve the fundamental limitations of sources of information which have the major role in extracting the optimal decision from the decision matrix. Therefore, Further extensions of ARWEN algorithms to solve uncertain decision-making problems is another interesting research area. To a certain extent, algorithms of ARWENs follow human behavior pattern in decision-making. Applications of the five extensions of ARWEN to other MCDM problems can be considered as a recommendation for future research. In order to determine the elected nominee, integration of ARWEN IV with other operational research modeling techniques can also be considered as a future scope. With the E-ARWEN concept, there is a lot of room for this method to be expanded upon and integrated with other MCDM methods for group decision-making environments.

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