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EVALUATION OF GOVERNMENT INVESTMENT USING NESTED PROBABILISTIC LINGUISTIC PREFERENCE RELATIONS BASED ON GRAPH THEORY

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Abstract. Government investment, as a major government function, is closely related to national development and economic growth. It plays a key role to maximize the benefits of this fund, which requires the government to choose the optimal investment plan. Considering the complex and uncertain decision-making environment, we propose the nested probabilistic linguistic preference relation (NPLPR) based on the nested probabilistic linguistic term sets (NPLTSs), to express preference information from the qualitative and quantitative angle. According to graph theory, we define a consistency index and an acceptable consistency of NPLPR to measure the additive consistency. Based on which, we establish a novel algorithm for unacceptable consistent NPLPR to meet the acceptable consistency. Finally, projects in government investment are evaluated by the proposed decision-making method, and some comparative analyses, discussions, and implications are provided from three angles. This study provides a new perspective for scholars to make scientific and rational decisions with the help of technological and economic development in various fields.

Keywords: government investment, nested probabilistic linguistic term sets, nested probabilistic linguistic preference relation, consistency check, graph theory, cognitive decision-making.

JEL Classification: C02, C60, D70, D81, D91.

Introduction

Government investment could make up for market failure, coordinate the proportion of major investment in the whole society, and then promote economic development and structural optimization. Therefore, it is a necessary means of national macro-economic regulation and control, and plays an important macro-guiding role in social investment and resource allocation. With the development of the society, evaluation information is increasingly uncertain and complex in decision-making problems (Dahooie et al., 2020). Decision makers (DMs) are often unable to take crisp numbers to express their evaluation information because of

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lacking professional knowledge and complex decision-making environment. Considering that human's cognition is of inevitable fuzziness in practice, people prefer to use linguistic expressions to describe assessment results (Zadeh, 1975), and thus linguistic variables could express the DMs' preferences effectively. At present, many researchers have paid attention to establishing various flexible linguistic models or complex expressions to better describe uncertain information. (Ban et al., 2020).

Since the research of Zadeh in 1975, the theoretical developments of linguistic representation models have continued until now. The first phase is the single linguistic term to express opinion. Later, linguistic models were improved to add probability to the linguistic term considering the uncertainty, such as virtual linguistic terms (VLT) (Xu, 2004), and 2-tuple fuzzy linguistic representation model (2TFLRM) (Herrera & Martinez, 2000). Wang and Hao (2006) proposed a novel 2-tuple fuzzy linguistic representation model based on the concept of symbolic proportion. Alcala et al. (2007) proposed a genetic learning of compact fuzzy rule. In addition, the OWA-based consensus operator (Dong et al., 2010) were further defined to calculation. Due to that the single linguistic term is unable to describe people's views comprehensively, and they often tend to hesitate between different linguistic terms, like "between good and very good". Hesitant Fuzzy Linguistic Term Set (HFLTS) was proposed to deal with describing complex information (Rodríguez et al., 2012). Probabilistic Linguistic Term Set (PLTS) (Pang, et al., 2016), and distributed linguistic representation (Zhang et al., 2014) were proposed to express comprehensive information combined probabilistic information and linguistic term. In the face of increasingly uncertain linguistic information, one-layer linguistic model cannot be expressed effectively. Later, Double Hierarchy Hesitant Fuzzy Linguistic Term Set (DHHFLTS) (Gou et al., 2017), 2-Dimension Linguistic Term Set (2DLTS) (Zhao et al., 2019) were proposed to split linguistic expression. Up to now, HFLTSs and PLTSs have been popular and applied in various fields. HFLTSs focus on collecting possible linguistic terms where DMs have some hesitations on them (Torra, 2010; Rodríguez et al., 2013). In the hesitant fuzzy linguistic environment, recent studies have been focused on the distance and similarity measures (Liao et al., 2014), consistency and consensus (Wu & Xu, 2016), correlation coefficients (Liao et al., 2015b), and decision-making methods (Liao et al., 2015a). Considering different importance degrees of linguistic terms, PLTSs is more reasonable to represent people's preference. Under such an environment, Bai et al. (2017) established a comparison method and developed a rational way to apply in the decision-making field.

Under the complex decision-making environment, either one-dimension information or evaluation without degree cannot meet the basic needs to expression for complex decision-making problem, such as "very good (60% degree) in the first-class hospital (20% degree)" or "a little bit (20% degree) for good (70% degree)". To measure such the multi-dimensional and uncertainty of linguistic evaluations, Wang et al. (2019a) proposed the Nested Probabilistic-Numerical Linguistic Term Set (NPNLTS), and extended it to a general concept, defined as the nested probabilistic linguistic term set (NPLTS) (Wang et al., 2021). A NPLTS consisted of "outer and inner" structure, could express four situations in terms of the linguistic type. (1) Case 1: Ordinal variable and ordinal variable. This is suitable to describe complex performance information. For instance, when a doctor asks for a patient about his blood pressure, he may say "a little bit high". (2) Case 2: Ordinal variable and nominal variable. This is applied to express the linguistic information from the overall performance to the local characteristic. For example, when a consumer buys a car, he may first consider the "good" company and then consider the "after-sale service". (3) Case 3: Nominal variable and ordinal variable. This is suitable to describe the linguistic information from the local characteristic to its performance. For example, when a user selects a carrier, he may consider "high bandwidth". (4) Case 4: Nominal variable and nominal variable. This is applied to describe double linguistic information. For instance, when a consumer chooses a mobile phone, he may consider "iPhone and then its price". In this way, multi-dimensional and uncertain information could be expressed with clear structure accurately and comprehensively. To compare the differences of NPLTS and other linguistic models, Table 1 lists the characteristics with some linguistic representation models. The NPLTSs remain the advantages of the PLTSs, and it is necessary and worthful to study NPLTSs in both theory and practice. Up to now, a series of distance and similarity measures of NPLTSs have been established in different cases (Wang et al., 2019b). The nested probabilistic linguistic information has been used to handle the allocation problem of water resources, the maneuvering target tracking, and the consensus-based track association problem (Wang et al., 2020).

DMs prefer to express their judgments through attributes or pairwise comparisons of the alternatives. Preference relation is an efficient and common tool to describe uncertain information over a set of alternatives (Chuang et al., 2020). Currently, there are three preference types to compare two alternatives. The first one is that DM either prefers one to the other. The second one is that DM is indifferent between two alternatives, and the third one is that DM is unable to compare them. Herrera-Viedma et al. (2004) proposed two mathematical models based on the concept of preference relations. These studies have been indicated that it is more accurate for pairwise comparison methods than non-pairwise methods compared with various preference methods. The reason may be that pairwise comparison focuses on the evaluated two alternatives without other alternatives (Meng et al., 2019). Hence, preference relations have become powerful techniques because of describing uncertain information

Table 1. The characteristics with linguistic representation models (source: our research)

Linguistic model	Reference	Element	Importance degree	layer	Variable type
2TFLRM	(Herrera & Martinez, 2000)	Single	√	One	Ordinal
VLT	(Xu, 2004)	Single	√	One	Ordinal
HFLTS	(Rodríguez et al., 2012)	Multiple	×	One	Ordinal
PLTS	(Pang et al., 2016)	Multiple	√	One	Ordinal
DHHFLTS	(Gou et al., 2017)	Multiple	×	Two	Ordinal
2DLTS	(Zhao et al., 2019)	Multiple	×	Two	Ordinal
NPLTS	(Wang et al., 2021)	Multiple	√	Two	Ordinal/ Nominal

Note: The full names of the abbreviations are: 2TFLRM – 2-tuple fuzzy linguistic representation model, VLS – Virtual linguistic terms, HFLTS – hesitant fuzzy linguistic term set, PLTS – probabilistic linguistic term set, DHHFLTS – double hierarchy hesitant fuzzy linguistic term set, 2DLTS – 2-dimension linguistic term set, NPLTS – nested probabilistic linguistic term set.

conformed to people's cognitive (Wu & Xu, 2016). It is worthy and necessary to study linguistic preference relations, and it is an important branch to research the linguistic preference relations (Mi et al., 2020). This study mainly focuses on the nested probabilistic linguistic preference relations (NPLPRs), a new preference relation form, and then apply them to the decision-making problems in government investment.

Due to that an inconsistent preference relation would result in misleading results, consistency is a key property no matter what preference relations, and it is necessary for consistency checking to guarantee DMs to make logical results. It is a hot topic to study the consistency and consistency checking based on linguistic models, and many scholars have studied them (Yang et al., 2021). Since NPLTSs can deal with uncertain and complex information effectively in decision-making, it is worth to define the consistency of NPLPR to make a leading and rational result. Additionally, transitivity is the basic concept for the consistency (Wang, 1997), because the requirement to characterize consistency is to extend the classical requirements of binary preference under an uncertain environment. Many of the properties, such as minimum transitivity, and additive transitivity (Herrera-Viedma et al., 2004), have been proposed for different preference relations to apply transitivity. Considering that it is suitable to use the additive consistency for fuzzy preference relations (Zhang et al., 2018), and graph theory is a very useful method to describe the consistency of preference relation (Boffey, 1982), this study measures the additive consistency of NPLPR with the help of graph theory.

The contributions of this paper lie in the following aspects: (1) From a new perspective of linguistic expression model, i.e., NPLTSs, the concept of the NPLPR is firstly proposed to better present preference information comprehensively in government investment by pairwise comparison of alternatives. (2) To ensure the rationally and scientifically, an additive consistency of the NPLPR is proposed based on graph theory. (3) To improve the unacceptable consistent NPLPR until it is acceptable and help DMs make the reasonable and effective decisions, two novel algorithms are established. One is an automatic improving algorithm, and another is a decision-making algorithm with the consistent NPLPR. (4) An experimental study concerning the evaluation of investment plans for government is given. Some comparative analyses are conducted from three perspectives including the impact for using NPLPR without checking consistency, various preference relations and the changed adjusted parameter.

The organization of this study is as follows: Section 1 reviews related contents of NPLTSs. Section 2 establishes the research method. In Section 3, we give a case study to show the proposed method step by step, and conduct comparati,ve analyses and discussions. In the last Section, we end the study with conclusions.

1. Linguistic scale and nested probabilistic linguistic term set

Some concepts are reviewed, including the additive linguistic evaluation scale, NPLTSs and the normalization of NPLTSs. In general, additive linguistic evaluation scale has two types, i.e., the traditional additive linguistic scale (Xu, 2012) and the subscript-symmetric linguistic scale (Xu, 2005). Specifically, the form of traditional additive linguistic scale is $S_1 = \{s_\alpha \mid \alpha = 0,1,\cdots,\tau\}$, where s_α represents a possible value for a linguistic label. And the form of subscript-symmetric linguistic evaluation scale is $S_2 = \{s_\alpha \mid \alpha = -\tau,\cdots,-1,0,1,\cdots,\tau\}$, where s_0 means an indifference, while other labels are distributed around it symmetrically. In this study, subscript-symmetric linguistic evaluation scale is used, and there are some operational laws: (1) if $\alpha > \beta$, then $s_\alpha > s_\beta$; (2) the negation operator is defined as: $neg(s_\alpha) = s_{-\alpha}$, and $neg(s_0) = s_0$.

With the linguistic scale, the NPLTSs was proposed as follows (Wang et al., 2021): A nested linguistic term set (NLTS) consists of outer linguistic term set (OLTS) and inner linguistic term set (ILTS) denoted as $S_O = \{s_\alpha \mid \alpha = -\tau,\cdots,-1,0,1,\cdots,\tau\}$ and $S_I = \{n_\beta \mid \beta = -\varsigma,\cdots,-1,0,1,\cdots,\varsigma\}$, respectively. The NLTS can be written as $S_N = \{s_\alpha\{n_\beta\} \mid \alpha = -\tau,\cdots,-1,0,1,\cdots,\tau; \beta = -\varsigma,\cdots,-1,0,1,\cdots,\varsigma\}$ by merging the expressions, where $s_\alpha\{n_\beta\}$ names as the nested linguistic term. NPLTS is a mapping function from a finite set X to a subset of NLTS S_N, denoted as:

$$P_{S_N} = \left\{\left\langle x_i, p_{S_N}(x_i)\right\rangle \middle| x_i \in X\right\}, \tag{1}$$

where $p_{S_N}(x_i)$ is the element in S_N:

$$p_{S_N}(x_i) = \left\{ \begin{array}{c} s_{a(k)}\left(p_{s(k)}\right)\left\{n_{\beta(l)}\left(p_{n(l)}\right)\right\}(x_i) \Big| s_{a(k)}\left\{n_{\beta(l)}\right\} \in S_N, p_{s(k)} > 0, p_{n(l)} > 0, \\ k = 1,2,\cdots,\# s_\alpha(x_i), l = 1,2,\cdots,\# n_\beta(x_i), \\ \alpha = -\tau,\cdots,-1,0,1,\cdots,\tau, \beta = -\varsigma,\cdots,-1,0,1,\cdots,\varsigma \end{array} \right\}, \tag{2}$$

where $\# s_\alpha(x_i)$ is the length of elements in $s_\alpha(x_i)$; $\# n_\beta(x_i)$ is the length of the elements in $n_\beta(x_i)$; $\sum_{k=1}^{\# s_{\alpha(k)}(x_i)} p_{s(k)} \le 1$, $\sum_{l=1}^{\# n_{\beta(l)}(x_i)} p_{n(l)} \le 1$.

The normalization method is developed as follows (Wang et al., 2021): Let $\overline{P_{S_N}}$ be a normalized NPLTS (N-NPLTS) with a normalized OPLTS $s_\alpha^N(p_s)$ and a normalized IPLTS $n_\beta^N(p_n)$:

$$s_\alpha^N(p_s) = \left\{ s_{\alpha(k)}^N\left(p_{s(k)}^N\right) \mid s_{\alpha(k)}^N \in S_O, p_{s(k)}^N \ge 0, k = 1,2,\cdots,\tau+1, \sum_{k=1}^{\tau+1} p_{s(k)}^N = 1 \right\}; \tag{3}$$

$$n_\beta^N(p_n) = \left\{ n_{\beta(l)}^N\left(p_{n(l)}^N\right) \mid n_{\beta(l)}^N \in S_I, 1 \ge p_{n(l)}^N \ge 0, l = 1,2,\cdots,\varsigma+1, \sum_{l=1}^{\varsigma+1} p_{n(l)}^N = 1 \right\}, \tag{4}$$

where $p_{s(k)}^N = \dfrac{p_{s(k)}}{\sum_{k=1}^{\tau+1} p_{s(k)}}$ and $p_{n(l)}^N = \dfrac{p_{n(l)}}{\sum_{l=1}^{\varsigma+1} p_{n(l)}}$.

To compare the comprehensive preference values of alternatives directly, Wang et al. (2019b) defined the comparison rule to rank $P_i\ (i = 1,2,\cdots,m)$:

(1) If $F_w(P_i) > F_w(P_j)$, then $P_i \succ P_j$;

(2) If $F_w(P_i) < F_w(P_j)$, then $P_i \prec P_j$;

(3) If $F_w(P_i) = F_w(P_j)$, then:

(i) If $\sigma(P_i) > \sigma(P_j)$, then $P_i \prec P_j$;

(ii) If $\sigma(P_i) < \sigma(P_j)$, then $P_i \succ P_j$;

(iii) If $\sigma(P_i) = \sigma(P_j)$, then $P_i \sim P_j$.

where $F_w(P)$ is the score function and $\sigma(P)$ is the variance function denoted as:

$$F_w(P)=\frac{1}{\# s_\alpha}\sum_{k=1}^{\# s_\alpha} f\left(s_{\alpha(k)}\left(p_{s(k)}\right)\left\{n_{\beta(l)}\left(p_{n(l)}\right)\right\}\right);$$

$$\sigma(P)=\frac{1}{\# s_\alpha}\sum_{k=1}^{\# s_\alpha}\left(f\left(s_{\alpha(k)}\left(p_{s(k)}\right)\left\{n_{\beta(l)}\left(p_{n(l)}\right)\right\}\right)-F_w\right)^2$$

and

$$f\left(s_{\alpha(k)}\left(p_{s(k)}\right)\left\{n_{\beta(l)}\left(p_{n(l)}\right)\right\}\right)=\sum_{k=1}^{\# s_\alpha} p_{s(k)}\left(\frac{len\left(s_{\alpha(k)}\right)+1}{4}+\frac{\left(\sum_{l=1}^{\# n_\beta}\left(\beta(l)\times p_{n(l)}\right)+\varsigma\right)}{4\varsigma}\right)=\gamma.$$

Wang et al. (2019a) defined the nested probabilistic linguistic weighted averaging (NPLWA) operator to fuse different NPLTSs. Let $P_{S_{N_i}}$ $(i=1,2,\cdots,n)$ be n NPLTSs, the NPLWA operator was represented as:

$$NPLWA\left(P_{S_{N_1}},P_{S_{N_2}},\cdots,P_{S_{N_n}}\right)=\omega_1 P_{S_{N_1}}\oplus\omega_2 P_{S_{N_2}}\oplus\cdots\oplus\omega_n P_{S_{N_n}}, \tag{5}$$

where $\omega=\left(\omega_1,\omega_2,\cdots,\omega_n\right)^T$ is a weight vector of $P_{S_{N_i}}$, $\omega_i\geq 0, i=1,2,\cdots n$, and $\sum_{i=1}^{n}\omega_i=1$. The operational law of "$\oplus$" could calculate as follows:

$$\lambda_1 s_\alpha\left(v_1\right)\oplus\lambda_2 s_\beta\left(v_2\right)=s_{\frac{(\alpha+\beta)}{2}}\left(\frac{\left(\lambda_1 v_1+\lambda_2 v_2\right)}{\left(\lambda_1+\lambda_2\right)}\right), \tag{6}$$

where $s_\alpha\left(v_1\right),s_\beta\left(v_2\right)\in S$ are two linguistic terms, and $\lambda_1,\lambda_2\geq 0$.

2. Research methodology

2.1. Consistency of NPLPR

Given a set of alternatives $X=\left\{x_1,x_2,\cdots,x_m\right\}$, and the DMs express their preference information through comparing each pair of alternatives using NPLTSs. The NPLPR is defined as follows: A NPLPR $\tilde{P}_{S_N}=\left(p_{S_{N_{ij}}}\right)_{m\times m}\subset X\times X$ for $i,j=1,2,\cdots,m$ on the set X. $p_{S_{N_{ij}}}=\left\{s_{\alpha(k)_{ij}}\left(p_{s(k)_{ij}}\right)\left\{n_{\beta(l)_{ij}}\left(p_{n(l)_{ij}}\right)\right\}\mid k=1,\cdots,\# s_{\alpha_{ij}}, l=1,\cdots,\# n_{\beta_{ij}}\right\}$ is a NPLTS, where $p_{s(k)_{ij}}>0, \sum_{k=1}^{\# s_\alpha} p_{s(k)_{ij}}\leq 1, p_{n(l)_{ij}}>0, \sum_{l=1}^{\# n_\beta} p_{n(l)_{ij}}\leq 1$, and $\# s_{\alpha_{ij}}$ is the length of linguistic terms in $s_{\alpha_{ij}}\left(p_{s_{ij}}\right)$, $\# n_{\beta_{ij}}$ is the length of linguistic terms in $n_{\beta_{ij}}\left(p_{n_{ij}}\right)$. Suppose that the elements of OLTS are ordinal variables, $p_{s(k)_{ij}}=p_{s(k)_{ji}}$, $s_{\alpha(k)_{ij}}=neg\left(s_{\alpha(k)_{ji}}\right)$, $s_{\alpha_{ii}}\left(p_s\right)=\left\{s_0(1)\right\}=\left\{s_0\right\}, \# s_{\alpha_{ij}}=\# s_{\alpha_{ji}}$ and $s_{\alpha(k)_{ij}}\left(p_{s(k)_{ij}}\right)\leq s_{\alpha(k+1)_{ij}}\left(p_{s(k+1)_{ij}}\right)$ for $i\leq j$. Otherwise, $p_{s(k)_{ij}}=1-p_{s(k)_{ji}}$, $s_{\alpha(k)_{ij}}=s_{\alpha(k)_{ji}}$, $s_{\alpha(k)_{ii}}\left(p_s\right)=\left\{s_{(k)}\left(\frac{1}{(\tau+1)}\right)\right\}, \# s_{\alpha_{ij}}=\# s_{\alpha_{ji}}$.

The normalized NPLPR (N-NPLPR) $\overline{\tilde{P}_{S_N}}=\left(\overline{p}_{S_{N_{ij}}}\right)_{m\times m}$ meets the requirement that there are all N-NPLTSs in the upper triangular matrix. To analyze the relationship between NPLPR and the NPLTSs in the NPLPR, we first define the following relationship based on the whole score of the NPLTS: Given two NPLTSs $P_{S_{N_1}}$ and $P_{S_{N_2}}$, if $F_w\left(P_{S_{N_1}}\right)=F_w\left(P_{S_{N_2}}\right)$ (Wang et al., 2021), then $P_{S_{N_1}}$ and $P_{S_{N_2}}$ are equivalent, and is denoted as $P_{S_{N_1}} \cong P_{S_{N_2}}$.

Firstly, the consistency of NPLPR consisting of NPLTSs is discussed with one element both in the OLTS and in the ILTS. Specifically, as for any NPLTS, $P_{S_{N_{ij}}}=\left\{s_{\alpha(k)_{ij}}\left(p_{s(k)_{ij}}\right)\left\{n_{\beta(l)_{ij}}\left(p_{n(l)_{ij}}\right)\right\} \mid k=1,2,\cdots,\#s_{\alpha_{ij}}, l=1,2,\cdots,\#n_{\beta_{ij}}\right\}$ in a NPLPR, if $k=l=1$, i.e., there are one outer linguistic term with its corresponding probability and one inner linguistic term with its corresponding value, then we call it a special NPLPR denoted as $\dot{P}_{S_N}=\left(\dot{p}_{S_{N_{ij}}}\right)_{m\times m}=\left\{s_{\alpha_{ij}}\left(p_{s_{ij}}\right)\left\{n_{\beta_{ij}}\left(p_{n_{ij}}\right)\right\}\right\}_{m\times m}$. When $p_{s_{ij}}=p_{n_{ij}}=1$, it holds full evaluation information; when $p_{s_{ij}}<1$ or $p_{n_{ij}}<1$, there are partial assessment information. In such a circumstance, the special preference relation is defined as: Let $X=\{x_1,x_2,\cdots,x_m\}$, and the evaluation information are obtained by $S_O=\{s_\alpha \mid \alpha=-\tau,\cdots,-1,0,1,\cdots,\tau\}$ and $S_I=\{n_\beta \mid \beta=-\varsigma,\cdots,-1,0,1,\cdots,\varsigma\}$. The preference information is expressed as special NPLTSs. Given a special NPLPR $\dot{P}_{S_N}=\left(\dot{p}_{S_{N_{ij}}}\right)_{m\times m}\subset X\times X$, where $\left(\dot{p}_{S_{N_{ij}}}\right)_{m\times m}=\left\{s_{\alpha_{ij}}\left(p_{s_{ij}}\right)\left\{n_{\beta_{ij}}\left(p_{n_{ij}}\right)\right\}\right\}_{m\times m}$, the special N-NPLPR with one element is denoted as $\dot{\bar{P}}_{S_N}=\left(\dot{\bar{p}}_{S_{N_{ij}}}\right)_{m\times m}\subset X\times X$, where $\left(\dot{\bar{p}}_{S_{N_{ij}}}\right)_{m\times m}=\left\{s_{\alpha_{ij}}\left(\bar{p}_{s_{ij}}\right)\left\{n_{\beta_{ij}}\left(\bar{p}_{n_{ij}}\right)\right\}\right\}_{m\times m}$ and $\bar{p}_{s_{ij}}=\bar{p}_{n_{ij}}\ (i,j=1,2,\cdots,m)$.

Next, we define the special NPLPR satisfied additive consistency. Let $\dot{P}_{S_N}=\left(\dot{p}_{S_{N_{ij}}}\right)_{m\times m}$ be a special NPLPR, where $\left(\dot{p}_{S_{N_{ij}}}\right)_{m\times m}=\left\{s_{\alpha_{ij}}\left(p_{s_{ij}}\right)\left\{n_{\beta_{ij}}\left(p_{n_{ij}}\right)\right\}\right\}_{m\times m}$, and $\dot{\bar{P}}_{S_N}=\left(\dot{\bar{p}}_{S_{N_{ij}}}\right)_{m\times m}$ be its special N-NPLPR, where $\left(\dot{\bar{p}}_{S_{N_{ij}}}\right)_{m\times m}=\left\{s_{\alpha_{ij}}\left(\bar{p}_{s_{ij}}\right)\left\{n_{\beta_{ij}}\left(\bar{p}_{n_{ij}}\right)\right\}\right\}_{m\times m}$, then $\dot{P}_{S_N}$ is called an additively consistent special NPLPR if $\dot{\bar{p}}_{S_{N_{ij}}}=\dot{\bar{p}}_{S_{N_{ie}}}\oplus\dot{\bar{p}}_{S_{N_{ej}}}$, for any $i,e,j=1,2,\cdots,m$, i.e.,

$$s_{\alpha_{ij}}\left(\bar{p}_{s_{ij}}\right)=s_{\alpha_{ie}}\left(\bar{p}_{s_{ie}}\right)\oplus s_{\alpha_{ej}}\left(\bar{p}_{s_{ej}}\right), n_{\beta_{ij}}\left(\bar{p}_{n_{ij}}\right)=n_{\beta_{ie}}\left(\bar{p}_{n_{ie}}\right)\oplus n_{\beta_{ej}}\left(\bar{p}_{n_{ej}}\right). \tag{7}$$

Motivated by the preference relation graph (P-graph) and the symmetric preference relation graph (S-P-graph), we define the P-graph and the S-P-graph with respect to special NPLPR are defined as follows: Let $\dot{P}_{S_N}=\left(\dot{p}_{S_{N_{ij}}}\right)_{m\times m}$ be a special NPLPR and $\dot{\bar{P}}_{S_N}=\left(\dot{\bar{p}}_{S_{N_{ij}}}\right)_{m\times m}$ be the corresponding special N-NPLPR, $G_P=(V,A)$ is a weighted P-graph with priority, where $V=\{v_1,v_2,\cdots,v_m\}$ and $A=\left\{\left(v_i,v_j\right) \mid i\neq j, i,j=1,2,\cdots,m\right\}$ are the set of vertices and the set of arcs, respectively. The $\left(v_i,v_j\right)$ from v_i to v_j is a directed line segment. If $s_{\alpha_{ij}}>s_0$, then the arc $\left(v_i,v_j\right)$ links v_i and v_j. $w\left(v_i,v_j\right)$ and $p\left(v_i,v_j\right)$ are the weight and the priority of the arc $\left(v_i,v_j\right)$, respectively, and $w\left(v_i,v_j\right)=\alpha_{ij}\bar{P}_{s_{ij}}, p\left(v_i,v_j\right)=\beta_{ij}\bar{p}_{n_{ij}}, i\neq j, i,j=1,2,\cdots,m$.

Let $\dot{P}_{S_N} = \left(\dot{p}_{S_{N_{ij}}}\right)_{m\times m}$ be a special NPLPR and $\bar{\dot{P}}_{S_N} = \left(\bar{\dot{p}}_{S_{N_{ij}}}\right)_{m\times m}$ be the corresponding special N-NPLPR, the $\dot{P}_{S_N}$ is a weighted S-P-graph with priority $G_{S-P} = (V, A)$, where $V = \{v_1, v_2, \cdots, v_m\}$ is a set of vertices and $A = \{(v_i, v_j) \mid i \neq j, i, j = 1, 2, \cdots, m\}$ is a set of arcs. The (v_i, v_j) from the v_i to vertex v_j is a directed line segment. The weight and the priority of the arc (v_i, v_j) is denoted as $w(v_i, v_j)$ and $p(v_i, v_j)$, and $w(v_i, v_j) = \alpha_{ij}\, \bar{P}_{s_{ij}}, p(v_i, v_j) = \beta_{ij}\, \bar{p}_{n_{ij}}, i \neq j, i, j = 1, 2, \cdots, m$.

To represent the information of a path from the one vertex to another vertex in P-graph or S-P-graph, the length and the importance of the path are defined as follows: Let $G = (V, A)$ be the weighted digraph with priority, and $\left(v_{i_1}, (v_{i_1}, v_{i_2}), v_{i_2}, \cdots, (v_{i_{k-1}}, v_{i_k}), v_{i_k}\right)$ be a path in $G = (V, A)$. The length of path is the sum of weights of its arcs, denoted as $len\left(v_{i_1}, (v_{i_1}, v_{i_2}), v_{i_2}, \cdots, (v_{i_{k-1}}, v_{i_k}), v_{i_k}\right)$, and the importance of the path is the sum of the priorities of its arcs, denoted as $imp\left(v_{i_1}, (v_{i_1}, v_{i_2}), v_{i_2}, \cdots, (v_{i_{k-1}}, v_{i_k}), v_{i_k}\right)$.

Example 1. Given an OLTS $S_O = \{s_\alpha \mid \alpha = -4, \cdots, -1, 0, 1, \cdots, 4\}$ and an ILTS $S_I = \{n_\beta \mid \beta = -4, \cdots, -1, 0, 1, \cdots, 4\}$, respectively. A special NPLPR is:

$$\dot{P}_{S_N} = \begin{bmatrix} s_0\{n_0\} & \{s_1(0.8)\{n_0(0.4)\}\} & \{s_2(0.4)\{n_1(0.2)\}\} & \{s_3(0.7)\{n_1(0.2)\}\} \\ \{s_{-1}(0.8)\{n_0(0.4)\}\} & s_0\{n_0\} & \{s_1(0.7)\{n_1(0.2)\}\} & \{s_2(0.5)\{n_1(0.4)\}\} \\ \{s_{-2}(0.4)\{n_{-1}(0.2)\}\} & \{s_{-1}(0.7)\{n_{-1}(0.2)\}\} & s_0\{n_0\} & \{s_1(0.2)\{n_0(0.1)\}\} \\ \{s_{-3}(0.7)\{n_{-1}(0.2)\}\} & \{s_{-2}(0.5)\{n_{-1}(0.4)\}\} & \{s_{-1}(0.2)\{n_0(0.1)\}\} & s_0\{n_0\} \end{bmatrix}.$$

The corresponding special N-NPLPR is:

$$\bar{\dot{P}}_{S_N} = \begin{bmatrix} s_0\{n_0\} & \{s_1(1)\{n_0(1)\}\} & \{s_2(1)\{n_1(1)\}\} & \{s_3(1)\{n_1(1)\}\} \\ \{s_{-1}(1)\{n_0(1)\}\} & s_0\{n_0\} & \{s_1(1)\{n_1(1)\}\} & \{s_2(1)\{n_1(1)\}\} \\ \{s_{-2}(1)\{n_{-1}(1)\}\} & \{s_{-1}(1)\{n_{-1}(1)\}\} & s_0\{n_0\} & \{s_1(1)\{n_0(1)\}\} \\ \{s_{-3}(1)\{n_{-1}(1)\}\} & \{s_{-2}(1)\{n_{-1}(1)\}\} & \{s_{-1}(1)\{n_0(1)\}\} & s_0\{n_0\} \end{bmatrix}.$$

The P-graph and S-P-graph of B are presented in Figure 1.

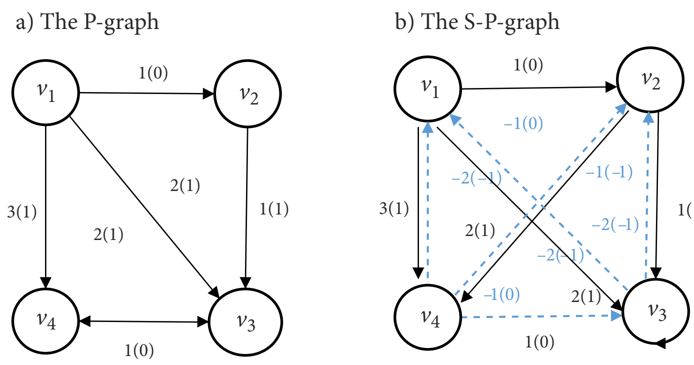

Figure 1. The P-graph and S-P-graph of the special NPLPR B (sources: authors' own research)

Next, we study the situation where at least one NPLTS has multiple elements in a NPLPR. The additively consistent NPLPR is defined as follows:

Let $\tilde{P}_{S_N} = \left(p_{S_{N_{ij}}}\right)_{m\times m}$ $(i,j=1,2,\cdots,m)$ be a NPLPR, where $p_{S_{N_{ij}}} = \left\{ s_{\alpha(k)_{ij}}\left(p_{s(k)_{ij}}\right) \left\{ n_{\beta(l)_{ij}}\left(p_{n(l)_{ij}}\right)\right\} \middle| k=1,2,\cdots,\#s_{\alpha_{ij}}, l=1,2,\cdots,\#n_{\beta_{ij}}\right\}$, and $\bar{\tilde{P}}_{S_N} = \left(\bar{p}_{S_{N_{ij}}}\right)_{m\times m}$ be its N-NPLPR, where $\bar{p}_{S_{N_{ij}}} = \left\{ s_{\alpha(k)_{ij}}\left(\bar{p}_{s(k)_{ij}}\right)\left\{ n_{\beta(l)_{ij}}\left(\bar{p}_{n(l)_{ij}}\right)\right\}\right\}$, then $\tilde{P}_{S_N}$ is called an additively consistent NPLPR if $\bar{p}_{S_{N_{ij}}} = \bar{p}_{S_{N_{ie}}} \oplus \bar{p}_{S_{N_{ej}}}$, i.e., for any$i,e,j=1,2,\cdots,n$, $k=1,2,\cdots,\#s_{\alpha_{ij}}$ and $l=1,2,\cdots,\#n_{\beta_{ij}}$,

$$s_{\alpha(k)_{ij}}\left(\bar{p}_{s(k)_{ij}}\right) = s_{\alpha(k)_{ie}}\left(\bar{p}_{s(k)_{ie}}\right) \oplus s_{\alpha(k)_{ej}}\left(\bar{p}_{s(k)_{ej}}\right);$$

$$n_{\beta(l)_{ij}}\left(\bar{p}_{n(l)_{ij}}\right) = n_{\beta(l)_{ie}}\left(\bar{p}_{n(l)_{ie}}\right) \oplus n_{\beta(l)_{ej}}\left(\bar{p}_{n(l)_{ej}}\right). \quad (8)$$

Let $\tilde{P}_{S_N} = \left(p_{S_{N_{ij}}}\right)_{m\times m}$ be a NPLPR and $\bar{\tilde{P}}_{S_N} = \left(\bar{p}_{S_{N_{ij}}}\right)_{m\times m}$ be the corresponding N-NPLPR, $G_P = (V,A)$ is a weighted P-graph with priority, where $V = \{v_1, v_2, \cdots, v_m\}$ and $A = \left\{\left(v_i, v_j\right) \mid i \neq j, i,j = 1,2,\cdots,m\right\}$ are a set of vertices and a set of arcs, respectively. $\left(v_i, v_j\right)$ from v_i to v_j is a directed line segment. if $s_{\alpha(k)_{ij}} > s_0$, the arc $\left(v_i, v_j\right)$ links v_i and v_j. The $w(k)\left(v_i, v_j\right)$ and $p(k)\left(v_i, v_j\right)$ are represented the k-th weight and priority of the arc $\left(v_i, v_j\right)$, and $w(k)\left(v_i, v_j\right) = C^1_{\#s_{\alpha_{ij}}} \alpha(k)_{ij}\, \bar{p}_{s(k)_{ij}}, p(k)\left(v_i, v_j\right) = \sum_{l=1}^{\#n_{\beta_{ij}}} \beta(l)_{ij}\, \bar{p}_{n(l)_{ij}}, i \neq j, i,j = 1,2,\cdots,m$.

Let $\tilde{P}_{S_N} = \left(p_{S_{N_{ij}}}\right)_{m\times m}$ be a NPLPR and $\bar{\tilde{P}}_{S_N} = \left(\bar{p}_{S_{N_{ij}}}\right)_{m\times m}$ be the corresponding N-NPLPR, a weighted S-P-graph with priority is denoted as $G_{S-P} = (V,A)$, where $V = \{v_1, v_2, \cdots, v_m\}$ and $A = \left\{\left(v_i, v_j\right) \mid i \neq j, i,j = 1,2,\cdots,m\right\}$ are a set of vertices and a set of arcs. The $\left(v_i, v_j\right)$ from v_i to v_j is a directed line segment. $w(k)\left(v_i, v_j\right)$ and $p(k)\left(v_i, v_j\right)$ are the k-th weight and priority of the $\left(v_i, v_j\right)$, and $w(k)\left(v_i, v_j\right) = C^1_{\#s_{\alpha_{ij}}} \alpha(k)_{ij}\, \bar{p}_{s(k)_{ij}}$, $p(k)\left(v_i, v_j\right) = \sum_{l=1}^{\#n_{\beta_{ij}}} \beta(l)_{ij}\, \bar{p}_{n(l)_{ij}}$, $i \neq j, i,j = 1,2,\cdots,m$.

Let $\tilde{P}_{S_N} = \left(p_{S_{N_{ij}}}\right)_{m\times m}$ be a NPLPR and $\bar{\tilde{P}}_{S_N} = \left(\bar{p}_{S_{N_{ij}}}\right)_{m\times m}$ be the corresponding N-NPLPR, then $\tilde{P}_{S_N} = \left(p_{S_{N_{ij}}}\right)_{m\times m}$ is an additively consistent NPLPR if $\bar{p}_{S_{N_{ij}}} \cong \bar{p}_{S_{N_{ie}}} \oplus \bar{p}_{S_{N_{ej}}}$, for any $i,e,j = 1,2,\cdots,m$, i.e.,

$$s_{\alpha(k)_{ij}}\left(\bar{p}_{s(k)_{ij}}\right) \cong s_{\alpha(k)_{ie}}\left(\bar{p}_{s(k)_{ie}}\right) \oplus s_{\alpha(k)_{ej}}\left(\bar{p}_{s(k)_{ej}}\right);$$

$$n_{\beta(l)_{ij}}\left(\bar{p}_{n(l)_{ij}}\right) \cong n_{\beta(l)_{ie}}\left(\bar{p}_{n(l)_{ie}}\right) \oplus n_{\beta(l)_{ej}}\left(\bar{p}_{n(l)_{ej}}\right). \quad (9)$$

In general, we denote $\tilde{P}_{S_N} = \left(p_{S_{N_{ij}}}\right)_{m\times m}$ as a NPLPR and $\hat{\tilde{P}}_{S_N} = \left(\hat{p}_{S_{N_{ij}}}\right)_{m\times m}$ as the corresponding consistent NPLPR, $\bar{\tilde{P}}_{S_N} = \left(\bar{p}_{S_{N_{ij}}}\right)_{m\times m}$ and $\bar{\hat{P}}_{S_N} = \left(\bar{\hat{p}}_{S_{N_{ij}}}\right)_{m\times m}$ as the corresponding N-NPLPR and the consistent NPLPR respectively.

Example 2. Given an OLTS $S_O = \{s_\alpha \mid \alpha = -4,\cdots,-1,0,1,\cdots,4\}$ and an ILTSS$_I = \{n_\beta \mid \beta = -4,\cdots,-1,0,1,\cdots,4\}$, respectively. Let $P_{S_N} = \left(p_{S_{N_{ij}}}\right)_{m\times m}$ be a NPLPR as follows:

$$S_N = \begin{bmatrix} s_0\{n_0\} & \left\{\begin{array}{c} s_2(0.2)\{n_1(0.3)\}, \\ s_1(0.8)\{n_0(0.4), n_1(0.6)\}\end{array}\right\} & \{s_1(0.4)\{n_1(0.2)\}\} & \{s_3(0.7)\{n_0(0.2)\}\} \\ \left\{\begin{array}{c} s_{-2}(0.2)\{n_{-1}(0.3)\}, \\ s_{-1}(0.8)\{n_0(0.4), n_{-1}(0.6)\}\end{array}\right\} & s_0\{n_0\} & \{s_2(0.7)\{n_1(0.2)\}\} & \{s_1(0.5)\{n_0(0.6), n_1(0.4)\}\} \\ \{s_{-1}(0.4)\{n_{-1}(0.2)\}\} & \{s_{-2}(0.7)\{n_{-1}(0.2)\}\} & s_0\{n_0\} & \{s_2(0.2)\{n_0(0.1)\}\} \\ \{s_{-1}(0.4)\{n_{-1}(0.2)\}\} & \{s_{-1}(0.5)\{n_0(0.6), n_{-1}(0.4)\}\} & \{s_{-2}(0.2)\{n_0(0.1)\}\} & s_0\{n_0\} \end{bmatrix}.$$

The corresponding N-NPLPR is:

$$\bar{P}_{S_N} = \begin{bmatrix} s_0\{n_0\} & \left\{\begin{array}{c} s_2(0.2)\{n_1(1)\}, \\ s_1(0.8)\{n_0(0.4), n_1(0.6)\}\end{array}\right\} & \{s_1(1)\{n_1(1)\}\} & \{s_3(1)\{n_0(1)\}\} \\ \left\{\begin{array}{c} s_{-2}(0.2)\{n_{-1}(1)\}, \\ s_{-1}(0.8)\{n_0(0.4), n_{-1}(0.6)\}\end{array}\right\} & s_0\{n_0\} & \{s_2(1)\{n_1(1)\}\} & \{s_1(1)\{n_0(0.6), n_1(0.4)\}\} \\ \{s_{-1}(1)\{n_{-1}(1)\}\} & \{s_{-2}(1)\{n_{-1}(1)\}\} & s_0\{n_0\} & \{s_2(1)\{n_0(1)\}\} \\ \{s_{-3}(1)\{n_0(1)\}\} & \{s_{-1}(1)\{n_0(0.6), n_{-1}(0.4)\}\} & \{s_{-2}(1)\{n_0(1)\}\} & s_0\{n_0\} \end{bmatrix}.$$

The additively consistent N-NPLPRs of $\bar{P}_{S_N}$ is obtained by Eq. (8) as:

$$\bar{\tilde{P}}_{S_N} = \begin{bmatrix} s_0\{n_0\} & \left\{\begin{array}{c} s_{1.25}(0.65)\{n_{0.5}(1), n_{0.25}(0)\}, \\ s_{0.75}(0.35)\{n_0(0.5), n_{0.25}(0.5)\}\end{array}\right\} & \left\{\begin{array}{c} s_{1.75}(0.78)\{n_1(1)\}, \\ s_{1.5}(0.22)\{n_{0.75}(0.75), n_1(0.25)\}\end{array}\right\} & \left\{\begin{array}{c} s_3(0.87)\{n_{0.5}(0.5), n_{0.75}(0.5)\}, \\ s_{2.75}(0.13)\{n_{0.25}(0.25), n_{0.75}(0.75)\}\end{array}\right\} \\ \left\{\begin{array}{c} s_{-1.25}(0.65)\{n_{-0.5}(1), n_{-0.25}(0)\}, \\ s_{-0.75}(0.35)\{n_0(0.5), n_{-0.25}(0.5)\}\end{array}\right\} & s_0\{n_0\} & \left\{\begin{array}{c} s_{0.5}(0.70)\{n_{0.5}(1), n_{0.75}(0)\}, \\ s_{0.75}(0.30)\{n_{0.75}(1)\}\end{array}\right\} & \left\{\begin{array}{c} s_{1.75}(0.78)\{n_0(0.5), n_{0.5}(0.5)\}, \\ s_2(0.22)\{n_{0.25}(0.25), n_{0.5}(0.75)\}\end{array}\right\} \\ \left\{\begin{array}{c} s_{-1.75}(0.78)\{n_{-1}(1)\}, \\ s_{-1.5}(0.22)\{n_{-0.75}(0.75), n_{-1}(0.25)\}\end{array}\right\} & \left\{\begin{array}{c} s_{-0.5}(0.70)\{n_{-0.5}(1), n_{-0.75}(0)\}, \\ s_{-0.75}(0.30)\{n_{-0.75}(1)\}\end{array}\right\} & s_0\{n_0\} & \{s_{1.25}(1)\{n_{-0.5}(0.5), n_{-0.25}(0.5)\}\} \\ \left\{\begin{array}{c} s_{-3}(0.87)\{n_{-0.5}(0.5), n_{-0.75}(0.5)\}, \\ s_{-2.75}(0.13)\{n_{-0.25}(0.25), n_{-0.75}(0.75)\}\end{array}\right\} & \left\{\begin{array}{c} s_{-1.75}(0.78)\{n_0(0.5), n_{-0.5}(0.5)\}, \\ s_{-2}(0.22)\{n_{-0.25}(0.25), n_{-0.5}(0.75)\}\end{array}\right\} & \{s_{-1.25}(1)\{n_{0.5}(0.5), n_{0.25}(0.5)\}\} & s_0\{n_0\} \end{bmatrix}.$$

Figure 2 shows the P-graph and the P-graph with additively consistent of P_{S_N}.

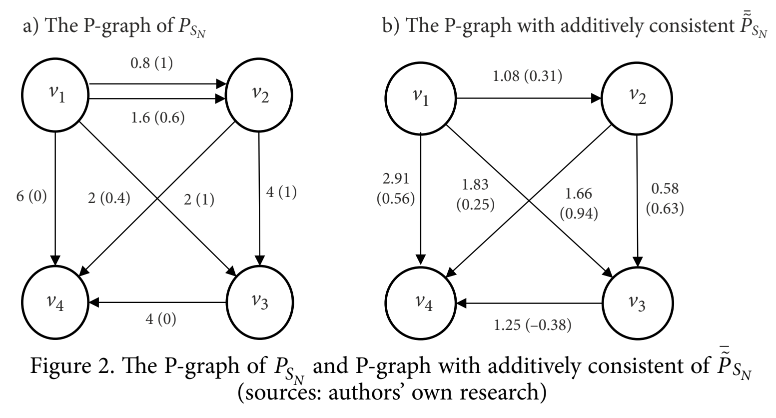

Figure 2. The P-graph of P_{S_N} and P-graph with additively consistent of $\bar{\tilde{P}}_{S_N}$ (sources: authors' own research)

2.2. Consistency improving for NPLPR

Consistency is an important issue for any preference relation before decision-making. In this section, a consistency checking and improving algorithm are established, and a NPLPR-based decision-making method is further developed.

2.2.1. Consistency index of NPLPR

A distance measure between two NPLTSs is defined as: Let $P_{S_{N_1}}=\left(p_{S_{N_{1_{ij}}}}\right)_{m\times m}$ and $P_{S_{N_2}}=\left(p_{S_{N_{2_{ij}}}}\right)_{m\times m}$ be two NPLTSs, $\bar{P}_{S_{N_1}}=\left(\bar{p}_{S_{N_{1_{ij}}}}\right)_{m\times m}$ and $\bar{P}_{S_{N_2}}=\left(\bar{p}_{S_{N_{2_{ij}}}}\right)_{m\times m}$ be the corresponding N-NPLTSs, respectively, which satisfy $\#s_{\alpha_{1_{ij}}}=\#s_{\alpha_{2_{ij}}}=\#s_{\alpha_{ij}}$ and $\#n_{\beta_{1_{ij}}}=\#n_{\beta_{2_{ij}}}=\#n_{\beta_{ij}}$. Then

$$d\left(P_{S_{N_1}},P_{S_{N_2}}\right)=\frac{1}{\#s_{\alpha_{ij}}\times\#n_{\beta_{ij}}}\sum_{k=1}^{\#s_{\alpha_{ij}}}\sum_{l=1}^{\#n_{\beta_{ij}}}\frac{\left(\frac{\left|s_{\alpha_{1_{ij}}}-s_{\alpha_{2_{ij}}}\right|}{2\tau}+\left|p_1^{(k)}-p_2^{(k)}\right|+\frac{\left|n_{\beta_{1_{ij}}}-n_{\beta_{2_{ij}}}\right|}{2\varsigma}+\left|v_1^{(l)}-v_2^{(l)}\right|\right)}{4}. \quad (10)$$

It satisfies the fundamental properties of distance, i.e., non-negativity, symmetry, and boundedness. Therefore, it is a normalized distance. In the following, we define the distance between any two NPLPRs. Let $\tilde{P}_{S_{N_1}}=\left(p_{S_{N_{1_{ij}}}}\right)_{m\times m}$ and $\tilde{P}_{S_{N_2}}=\left(p_{S_{N_{2_{ij}}}}\right)_{m\times m}$ be two NPLPRs, $\bar{\tilde{P}}_{S_{N_1}}=\left(p_{S_{N_{1_{ij}}}}\right)_{m\times m}$ and $\bar{\tilde{P}}_{S_{N_2}}=\left(p_{S_{N_{2_{ij}}}}\right)_{m\times m}$ be the corresponding N-NPLPRs, respectively. Then

$$d\left(\tilde{P}_{S_{N_1}},\tilde{P}_{S_{N_1}}\right)=\sqrt{\frac{2}{n(n-1)}\sum_{j=i+1}^{n}\sum_{i=1}^{n}\left(d\left(P_{S_{N_1}},P_{S_{N_2}}\right)\right)^2}. \quad (11)$$

Furthermore, the consistency index of NPLPR $\tilde{P}_{S_N}$ is denoted as:

$$CI\left(\tilde{P}_{S_N}\right)=d\left(\bar{\tilde{P}}_{S_N},\bar{\tilde{P}}_{S_N}\right). \quad (12)$$

The smaller the value of *CI*, the higher the consistency level of $\tilde{P}_{S_N}$. The values of $\overline{CI}\left(\tilde{P}_{S_N}\right)$ were provided for different order n when $\alpha = 0.1$ and $\sigma = 2$, shown in Table 2 (Dong et al., 2008).

Table 2. The values of $\overline{CI}\left(\tilde{P}_{S_N}\right)$ when $\alpha = 0.1$ and $\sigma = 2$ (source: Dong et al., 2008)

$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 7$	$n = 8$
0.2207	0.3030	0.3488	0.3774	0.3970	0.4112
0.1103	0.1515	0.1744	0.1887	0.1985	0.2056
0.0552	0.0758	0.0872	0.0944	0.0993	0.1028

2.2.2. Automatic improving algorithm

If the consistency of a NPLPR $\tilde{P}_{S_N} = \left(p_{S_{N_{ij}}} \right)_{m\times m}$ is unacceptable, i.e., $CI\left(\tilde{P}_{S_N}\right) > \overline{CI}\left(\tilde{P}_{S_N}\right)$, then the consistency of $\tilde{P}_{S_N}$ is supposed to be improved to meet consistent NPLPR $\hat{\tilde{P}}_{S_N} = \left(\hat{p}_{S_{N_{ij}}} \right)_{m\times m}$. Based on the discussion above, an automatic improving algorithm for the NPLPR with unacceptable consistency is established, presented as follows:

Algorithm 1

Input: A NPLPR $\tilde{P}_{S_N} = \left(p_{S_{N_{ij}}} \right)_{m\times m}$, the number of iterations $z = 0$ and an adjusted parameter $\theta \left(0 \le \theta \le 1\right)$.

Output: The modified and consistent NPLPR $\tilde{P}_{S_N} = \left(p_{S_{N_{ij}}} \right)_{m\times m}$.

Step 1. Calculate the N-NPLPR $\bar{\tilde{P}}_{S_N} = \left(\bar{p}_{S_{N_{ij}}} \right)_{m\times m}$ by Eqs (3) and (4). Go to Step 2.

Step 2. Calculate the consistent NPLPR $\hat{\tilde{P}}_{S_N} = \left(\hat{p}_{S_{N_{ij}}} \right)_{m\times m}$ by Eq. (10). Go to Step 3.

Step 3. Determine $\overline{CI}\left(\tilde{P}_{S_N}\right)$ according to Table 1. Go to Step 4.

Step 4. Calculate $CI\left(\tilde{P}_{S_N}\right)$ by Eq. (12). If $CI\left(\tilde{P}_{S_N}\right) > \overline{CI}\left(\tilde{P}_{S_N}\right)$, then go to Step 5; otherwise, go to Step 6.

Step 5. Let $\left(p_{S_{N_{ij}}} \right)_{m\times m}^{(z+1)} = (1-\theta)\left(p_{S_{N_{ij}}} \right)_{m\times m}^{(z)} \oplus \theta \left(p_{S_{N_{ij}}} \right)_{m\times m}$ and $z = z + 1$. Go back to Step 3.

Step 6. Let $\left(\tilde{P}_{S_N}\right)^{(z)} = \left(p_{S_{N_{ij}}} \right)_{m\times m}^{(z)}$, and output the modified NPLPR $\tilde{P}_{S_N} = \left(p_{S_{N_{ij}}} \right)_{m\times m}$. Go to Step 7.

Step 7. End.

When $\theta = 0$, the modified NPLPR is the original NPLPR, and when $\theta = 1$, the modified NPLPR satisfies additive consistency. The larger the adjusted parameter θ, the faster the speed to satisfy consistency. Therefore, Algorithm 1 is convergent. Moreover, the adjusted parameter θ is provided by the DMs and depends on specific practical problems. Furthermore, the adjusted parameter θ can be regarded as a preference coefficient of the DMs.

2.2.3. A NPLPR-based decision-making process

Let $X = \{x_1, x_2, \cdots, x_n\}$ be a set of alternatives. The DMs compare the alternatives with a weighting vector $\omega = \{\omega_1, \omega_2, \cdots, \omega_n\}^T$, where $0 \le \omega_j \le 1$ and $\sum_{j=1}^{n} \omega_j = 1 \left(j = 1, 2, \cdots, n\right)$, and then a preference relation with NPLTS information can be obtained as follows:

$$\tilde{P}_{S_N} = \left(p_{S_{N_{ij}}} \right)_{m\times m} = \begin{bmatrix} p_{S_{N_{11}}} & p_{S_{N_{12}}} & \cdots & p_{S_{N_{1m}}} \\ p_{S_{N_{21}}} & p_{S_{N_{22}}} & \cdots & p_{S_{N_{2m}}} \\ \vdots & \vdots & \ddots & \vdots \\ p_{S_{N_{n1}}} & p_{S_{N_{n2}}} & \cdots & p_{S_{N_{mm}}} \end{bmatrix},$$

where $p_{S_{N_{ij}}}\left(1,2,3,\cdots,m\right)$ is a NPLTS, denoting the preference information that the alternative x_i compares to the alternative x_j.

Algorithm 2

Input: A NPLPR $\tilde{P}_{S_N}=\left(p_{S_{N_{ij}}}\right)_{m\times m}$, the weight vector $\omega=\left(\omega_1,\omega_2,\cdots,\omega_m\right)^T$, the iterations $z = 0$ and the parameter $\theta\left(0\le\theta\le 1\right)$.

Output: The best alternative x_i^*.

Step 1. Calculate the N-NPLPR $\bar{\tilde{P}}_{S_N}=\left(\bar{p}_{S_{N_{ij}}}\right)_{m\times m}$ based on the given NPLPR $\tilde{P}_{S_N}=\left(p_{S_{N_{ij}}}\right)_{m\times m}$. Go to Step 2.

Step 2. Calculate the consistent NPLPR $\widehat{\tilde{P}}_{S_N}=\left(\widehat{p}_{S_{N_{ij}}}\right)_{m\times m}$ by Eq. (10). Go to Step 3.

Step 3. Determine $\overline{CI}\left(\tilde{P}_{S_N}\right)$ according to Table 1 and calculate $CI\left(\tilde{P}_{S_N}\right)$ by Eq. (12). Go to Step 4.

Step 4. If $CI\left(\tilde{P}_{S_N}\right)>\overline{CI}\left(\tilde{P}_{S_N}\right)$, then go to Step 5; otherwise, go to Step 6.

Step 5. Improve the NPLPR $\tilde{P}_{S_N}=\left(p_{S_{N_{ij}}}\right)_{m\times m}$ by Algorithm 1, and obtain the modified NPLPR $\tilde{P}_{S_N}=\left(p_{S_{N_{ij}}}\right)_{m\times m}$. Go to Step 6.

Step 6. Integrate the preference information in $\tilde{P}_{S_N}=\left(p_{S_{N_{ij}}}\right)_{m\times m}$ with NPLWA by Eq. (5). Go to Step 7.

Step 7. Rank the alternatives $P_i\left(i=1,2,\cdots,m\right)$ according to comparison rule. Go to Step 8.

Step 8. Obtain the optimal alternative x_i^*. Go to Step 9.

Step 9. End.

3. A case study

Each project has its advantages and disadvantages, and they are evaluated by experts from an overall perspective. NPLPR, as a new preference relation, is suitable for practical problems of multiple dimension and nested structure. In this section, we consider "new energy development", an important scientific and technological problem to be breakthrough in the world, as an example to explain how the government chooses the overall optimal investment project after expert evaluation and make some comparative analyses to illustrate the reliance of the proposed NPLPR-based method.

3.1. Problem description

New energy, such as geothermal energy, ocean energy, biomass energy and nuclear fusion energy, is different from traditional energy, which has just started to be developed and utilized or is being actively studied and to be promoted (Geng et al., 2021). In people's life, the main energy sources used are fossil fuels. With the improvement of social life, it is expected that the world energy consumption will grow at a rate of 2.7% per year in the future. According to the current international common energy forecast, oil resources will be exhausted in 40 years, natural gas resources will be used up in 60 years, coal resources can only be used for 220 years. According to the prediction of the international authority unit, by the 2060s, that is, 2060, the proportion of the global new energy will develop to account for more than 50% of the world's energy composition, become the cornerstone of the future energy of human society, the leading role in the world energy stage. Burning of fossil fuels, is the main factor causing air pollution, and how to use the energy in the development has become a major global problem, and as a result, limit and reduce emissions of carbon dioxide from the burning of various fuels and gases, has become the international community to slow global climate change is an important part of (Wang & Xin, 2020). New energy is a clean energy to protect the ecological environment. Using new energy to gradually reduce and replace the use of fossil energy is a major measure to protect the ecological environment and move towards the road of sustainable economic and social development (Yao et al., 2021). Therefore, it is very important for the government to choose and promote excellent new energy development projects for the development of the country.

In general, the system aims to choose the best investment plan from four alternative plans:

- Sustainable capacity. If the invested project can achieve sustained phased results and continue to move forward, it shows that the project can create more new blood for the new energy industry. Of course, the government hopes that the funds invested in the project can play a long-term role and create long-term value for the relevant industries.
- Project size. The size of the project scale reflects the importance of the project initiated by the project unit from the side. Larger projects tend to have less risk and contain greater development potential.
- Economic benefits. Governments, of course, would like the same amount of money spent to produce a greater degree of financial output.
- Environmental friendliness. With the continuous advancement of modernization and the progress of science and technology, environmental protection has brought great challenges. As a national regulator, the government needs to pay attention to environmental protection. It is very important to choose environmentally friendly projects with a low degree of environmental damage.
- Talent and technology reserves. The research of new energy is a high-tech research and development project, and the number of cutting-edge talents and cutting-edge technology in the project team has an impact on the actual value of the project results.

3.2. Solve the problem

Suppose that four new type projects plans $\{x_1, x_2, x_3, x_4\}$ are put forward above which can be invested by government. The system compares each pair of plans using the OLTS and the ILTS as follows:

$$S_O = \left\{ \begin{matrix} s_{-2} = \text{sustainable capacity}, s_{-1} = \text{project size}, s_0 = \text{economic benefits}, \\ s_1 = \text{environmental friendliness}, s_2 = \text{talent and technology reserves} \end{matrix} \right\};$$

$$S_I = \left\{ n_{-1} = \text{poor}, n_0 = \text{common}, n_1 = \text{excellent} \right\}.$$

Table 3. The preference information with NPLTSs between project plans (source: our research)

	x_1	x_2	x_3	x_4
x_1	$s_0\{n_0\}$	$\left\{\begin{matrix} s_2(0.3)\{n_1(0.3)\}, \\ s_1(0.7)\{n_0(0.2),n_1(0.8)\} \end{matrix}\right\}$	$\{s_1(0.4)\{n_1(0.2)\}\}$	$\{s_2(0.8)\{n_0(0.1)\}\}$
x_2	$\left\{\begin{matrix} s_{-2}(0.3)\{n_{-1}(0.3)\}, \\ s_{-1}(0.7)\{n_0(0.2),n_{-1}(0.8)\} \end{matrix}\right\}$	$s_0\{n_0\}$	$\{s_2(0.7)\{n_1(0.2)\}\}$	$\{s_1(0.5)\{n_0(0.6),n_1(0.4)\}\}$
x_3	$\{s_{-1}(0.4)\{n_{-1}(0.2)\}\}$	$\{s_{-2}(0.7)\{n_{-1}(0.2)\}\}$	$s_0\{n_0\}$	$\{s_2(0.2)\{n_0(0.1)\}\}$
x_4	$\{s_{-2}(0.8)\{n_0(0.1)\}\}$	$\{s_{-1}(0.5)\{n_0(0.6),n_{-1}(0.4)\}\}$	$\{s_{-2}(0.2)\{n_0(0.1)\}\}$	$s_0\{n_0\}$

Table 3 gives the preference information with NPLTSs. In terms of the preference degree of x_1 over x_3, the DMs are 40% sure that the outer preference is well and are 20% sure that the inner preference is lower cost performance.

The preference information in Table 2 can be denoted as a preference matrix $\tilde{P}_{S_N}$ as follows:

$$\tilde{P}_{S_N}=\begin{bmatrix} s_0\{n_0\} & \left\{\begin{matrix} s_2(0.3)\{n_1(0.3)\}, \\ s_1(0.7)\{n_0(0.2),n_1(0.8)\} \end{matrix}\right\} & \{s_1(0.4)\{n_1(0.2)\}\} & \{s_2(0.8)\{n_0(0.1)\}\} \\ \left\{\begin{matrix} s_{-2}(0.3)\{n_{-1}(0.3)\}, \\ s_{-1}(0.7)\{n_0(0.2),n_{-1}(0.8)\} \end{matrix}\right\} & s_0\{n_0\} & \{s_2(0.7)\{n_1(0.2)\}\} & \{s_1(0.5)\{n_0(0.6),n_1(0.4)\}\} \\ \{s_{-1}(0.4)\{n_{-1}(0.2)\}\} & \{s_{-2}(0.7)\{n_{-1}(0.2)\}\} & s_0\{n_0\} & \{s_2(0.2)\{n_0(0.1)\}\} \\ \{s_{-2}(0.8)\{n_0(0.1)\}\} & \{s_{-1}(0.5)\{n_0(0.6),n_{-1}(0.4)\}\} & \{s_{-2}(0.2)\{n_0(0.1)\}\} & s_0\{n_0\} \end{bmatrix}.$$

Next, we calculate the consistency index of $\tilde{P}_{S_N}$:

$$CI\left(\tilde{P}_{S_N}\right)=d\left(\bar{\tilde{P}}_{S_N},\hat{\bar{p}}_{S_N}\right)=0.3928>\overline{CI}\left(\tilde{P}_{S_N}\right)=0.2930.$$

It means that the NPLPR $\tilde{P}_{S_N}$ is of unacceptable consistency. According to Algorithm 1, when $\theta=0.05$ and $z=3$, the modified NPLPR $\left(\tilde{P}_{S_N}\right)^{(3)}$ is of acceptable consistency, and $CI\left(\left(\tilde{P}_{S_N}\right)^{(3)}\right)=0.2923<0.2930$, shown as:

$$\left(\tilde{P}_{S_N}\right)^{(3)}=\begin{bmatrix} s_0\{n_0\} & \left\{\begin{matrix} s_1(0.72)\{n_{0.5}(0.2),n_0(0.8)\}, \\ s_{0.5}(0.28)\{n_{0.25}(1)\} \end{matrix}\right\} & \left\{\begin{matrix} s_{1.5}(0.86)\{n_1(1)\}, \\ s_{1.25}(0.14)\{n_1(1)\} \end{matrix}\right\} & \left\{\begin{matrix} s_{1.5}(0.86)\{n_0(1)\}, \\ s_{2.25}(0.14)\{n_0(1)\} \end{matrix}\right\} \\ \left\{\begin{matrix} s_{-1}(0.72)\{n_{-0.5}(0.2),n_0(0.8)\}, \\ s_{-0.5}(0.28)\{n_{-0.25}(1)\} \end{matrix}\right\} & s_0\{n_0\} & \left\{\begin{matrix} s_{0.5}(0.86)\{n_1(1)\}, \\ s_{0.75}(0.14)\{n_1(1)\} \end{matrix}\right\} & \left\{\begin{matrix} s_{1.5}(0.86)\{n_0(1)\}, \\ s_{1.75}(0.14)\{n_1(1)\} \end{matrix}\right\} \\ \left\{\begin{matrix} s_{-1.5}(0.86)\{n_{-1}(1)\}, \\ s_{-1.25}(0.14)\{n_{-1}(1)\} \end{matrix}\right\} & \left\{\begin{matrix} s_{-0.5}(0.86)\{n_{-1}(1)\}, \\ s_{-0.75}(0.14)\{n_{-1}(1)\} \end{matrix}\right\} & s_0\{n_0\} & \{s_1(1)\{n_0(1)\}\} \\ \left\{\begin{matrix} s_{-1.5}(0.86)\{n_0(1)\}, \\ s_{-2.25}(0.14)\{n_0(1)\} \end{matrix}\right\} & \left\{\begin{matrix} s_{-1.5}(0.86)\{n_0(1)\}, \\ s_{-1.75}(0.14)\{n_{-1}(1)\} \end{matrix}\right\} & \{s_{-1}(1)\{n_0(1)\}\} & s_0\{n_0\} \end{bmatrix}.$$

Suppose that there is no preference in system for these four alternatives, i.e., $\omega = \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)^T$. The NPLWA operator is used to integrate the preference information of x_i over $x_j\ (i, j = 1,2,3,4)$ in $\left(\tilde{P}_{S_N}\right)^{(3)}$, and the results of the alternatives $x_i\ (i = 1,2,3,4)$ are calculated:

$P_1 = \left\{s_0\{n_0\}, s_{0.86}\{n_{0.175}\}, s_{1.465}\{n_1\}, s_{1.605}\{n_0\}\right\};$

$P_2 = \left\{s_{-0.86}\{n_{-0.175}\}, s_0\{n_0\}, s_{0.535}\{n_1\}, s_{1.535}\{n_{0.5}\}\right\};$

$P_3 = \left\{s_{-1.465}\{n_{-1}\}, s_{-0.535}\{n_{-1}\}, s_0\{n_0\}, s_1\{n_0\}\right\};$

$P_4 = \left\{s_{-1.605}\{n_0\}, s_{-1.535}\{n_{-0.5}\}, s_{-1}\{n_0\}, s_0\{n_0\}\right\}.$

Next, the scores of the alternatives $x_i\ (i = 1,2,3,4)$ are obtained:

$F(P_1) = 0.6381, F(P_2) = 0.5319, F(P_3) = -0.375, F(P_4) = -0.58.$

Therefore, the ranking of alternative projects is $x_1 \succ x_2 \succ x_3 \succ x_4$, and the best project is x_1.

3.3. Comparative analyses and discussions

Some comparative analyses are conducted to show the advantages of the proposed NPLPR-based method by simulation experiments from three perspectives:

(1) The impact for NPLPR without checking consistency

In this part, we use the NPLPR without checking consistency in the proposed method for the applied case above, and compare the result with the proposed method. Table 4 shows the rankings of the projects in the above case study.

Table 4. Rankings based on two methods (source: our research)

Methods	Alternative ranking
The proposed method with consistency checking and improving	$x_1 > x_2 > x_3 > x_4$
The proposed method without consistency checking and improving	$x_1 > x_2 > x_4 > x_3$

In Table 3, the ranking results are a little different by using two methods. x_1 is still the best project. However, the priorities of x_3 and x_4 are reversed. There are two different rankings by using two methods. If the decision-making method based on preference relations without consistency checking and improving, the results may be unreliable. Therefore, it is scientific to obtain the consistent NPLPR to get the optimal alternative through consistency checking and improving process.

(2) The impact for various preference relations

To compare and analyze the results clearly, we use the hesitant fuzzy preference relation (HFPR), the hesitant fuzzy linguistic preference relation (HFLPR) and the probabilistic linguistic preference relation (PLPR), respectively, to deal with the same case. As an extending of PLTS, the HFPR B_1, the HFLPR B_2, and the PLPR B_3 based on original information are:

$$\tilde{P}_{S_{N_1}} = \begin{bmatrix} \{0.5\} & \{0.6,0.7\} & \{0.6\} & \{0.7\} \\ \{0.4,0.3\} & \{0.5\} & \{0.7\} & \{0.6\} \\ \{0.4\} & \{0.3\} & \{0.5\} & \{0.7\} \\ \{0.3\} & \{0.4\} & \{0.3\} & \{0.5\} \end{bmatrix};$$

$$\tilde{P}_{S_{N_2}} = \begin{bmatrix} \{s_0\} & \{s_2,s_1\} & \{s_1\} & \{s_2\} \\ \{s_{-2},s_{-1}\} & \{s_0\} & \{s_2\} & \{s_1\} \\ \{s_{-1}\} & \{s_{-2}\} & \{s_0\} & \{s_2\} \\ \{s_{-2}\} & \{s_{-1}\} & \{s_{-2}\} & \{s_0\} \end{bmatrix};$$

$$\tilde{P}_{S_{N_3}} = \begin{bmatrix} \{s_0(1)\} & \{s_2(0.3),s_1(0.7)\} & \{s_1(0.4)\} & \{s_2(0.8)\} \\ \{s_{-2}(0.3),s_{-1}(0.7)\} & \{s_0(1)\} & \{s_2(0.7)\} & \{s_1(0.5)\} \\ \{s_{-1}(0.4)\} & \{s_{-2}(0.7)\} & \{s_0(1)\} & \{s_2(0.2)\} \\ \{s_{-2}(0.8)\} & \{s_{-1}(0.5)\} & \{s_{-2}(0.2)\} & \{s_0(1)\} \end{bmatrix}.$$

After a set of calculations, Table lists the rankings with different preference relations.

Table 5. Rankings based on four preference relations (source: our research)

Preference relations	Alternative rankings
The NPLPR	$x_1 > x_2 > x_3 > x_4$
The HFPR	$x_1 > x_3 > x_2 > x_4$
The HFLPR	$x_1 > x_2 > x_4 > x_3$
The PLPR	$x_1 > x_2 > x_3 > x_4$

As we can see, all the best project is x_1, and the rankings are the same by using NPLPR and PLPR. However, the results are different when using HFPR and HFLPR, the reason may be that they are not considered the preference degrees. Although results are the same based on PLPR and NPLPR, the PLPR cannot reflect the complete preference information than the NPLPR. Therefore, compared with other three preference relation, the ranking results with NPLPR are reasonable and reliable.

(3) The impact for the adjusted parameter θ

In the following, we mainly study the impact for the adjusted parameter θ in the process of improving consistency. Let m be the number of alternatives, t be the average operation time (AOT). Then, the situation of AOTs with the certain value of θ is shown in Figure 3.

AOTs are monotonically increasing with the increase of the m and the decrease of θ. Besides, it tends to increase exponentially. Therefore, the adjusted parameter θ is an improving preference coefficient from this point of view. In order to see the relationships clearly among the parameters θ, m and t, suppose that θ is from 0 to 1, m is taken from 3 to 14, and the calculation step size is 0.001. Some simulation experiments are provided after 1000 simulation times, shown in Figure 4. The larger the adjusted parameter θ is, the faster the consistency of NPLPR is achieved and the less the AOT is. Similarly, the more the number of the alternative *m* is, the more the AOT is. As we can see, it is consistent with the actual decision-making process.

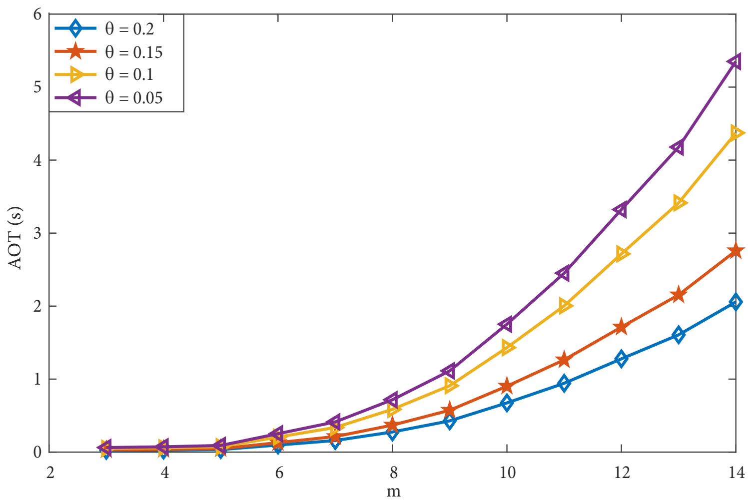

Figure 3. The AOT with various values of θ when *m* is taken from 3 to 14
(sources: authors' own research)

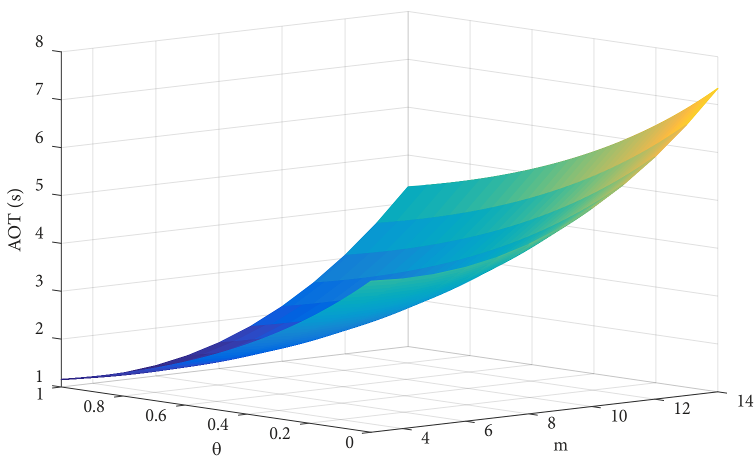

Figure 4. The three-dimensional diagram about AOT, θ and *m*
(sources: authors' own research)

3.4. Implications

There are various complex decision-making problems in government investment, and the consideration of factors such as national defense, by the government through the fiscal investment or local financial bonds, the use of foreign government grants and the domestic and international financial organizations loan of national finance guarantee way owned or joint venture to build assets investment project (Leeper et al., 2010), to promote the national economy and the development of regional economy, to meet the needs of the social culture and life. The multi-dimensional and uncertain decision-making problem has become increasingly difficult for people to make a scientific and rational decision. The purpose of the following discussions and implications is to provide the meaning of the NPLPR to deal with multi-dimensional and complex practical applications effectively. (1) To describe uncertain evaluation information with nested structure with respect to attributes, and it is suitable to apply for the complex decision-making problems; (2) To express multi-dimensional evaluation information whether the preference degree or feature of alternative, which is realized by ordinal variables and nominal variables; (3) To be applicable to various research directions in terms of the type of elements in the nested linguistic term set, and it could be applied to decision-making, optimization, and discrimination problem. According to discussions and comparative analyses, the proposed NPLPR-based method not only considers the consistency, but also describes quantitative and qualitative information of the linguistic terms, and it is more flexible than other popular preference relations and could better reflect the actual situation in real life.

Results in this study provide a novel decision-making approach with NPLPR, and it benefits researchers, policymakers, and practitioners in several ways. (1) In terms of consistency of NPLPR, scholars could further study other forms, like multiplicative consistency, and then propose advanced decision-making approaches. On the other hand, consistency index is a key factor in preference relations, and how to get the rational and scientific index with various number of experts is also an interesting direction for researchers to make contributions. (2) NPLPR is a useful tool for policymakers to evaluate projects or resources, because it considers both qualitative and quantitative information, and nested structure, which conforms to decision-making problems in real life. (3) From the perspective of practice, policymakers could not only adopt qualitative or quantitative decision, but also heterogeneous, interactive, and multi-dimensional decision. From the perspective of theory, there are research gap related to different consistency, automatic improved algorithm of inconsistency preference relation, and decision-making approaches, especially under the complex and uncertain environment. These points have important implications for academics, companies, and policymakers.

Conclusions

Government investment is related to the national economy and the life of the public. Due to the limitation of the amount, it is necessary to use appropriate methods to choose the optimal investment projects. Nested probabilistic linguistic term set (NPLTS) is a novel and practical way in information expression from the perspective of its structure, and it is more consistent with human's cognition. In this study, the nested probabilistic linguistic preference relation (NPLPR) is established, and the additive consistency of NPLPR based on graph theory is defined. Then, we have developed the additive consistency index of NPLPR with various orders to check whether the NPLPR is with acceptable consistency or not. When NPLPR is with unacceptable consistency, then the NPLPR needs to be improved, and an automatic improving algorithm has been established. Moreover, we have proposed a NPLPR-based decision-making method containing the consistency checking and the consistency improving to help the DMs make the scientific decisions. To show the efficiency and the applicability of the proposed NPLPR-based method, a simulative case study related to government investment has been conducted, and a comprehensive comparative analyses have been taken from three perspectives of checking consistency, various preference relations and the changed adjusted parameter θ. The results show that proposed NPLPR-based method gives the DMs choice to improve preferences by adjusted parameter θ, but also is effective and practical. This study has important implications for academics, companies, and policymakers. Some interesting topics related to NPLPR would be studied for further research. For instance, the multiplicative consistency of NPLPR will be investigated, and the consistency index of NPLPR could be explored based on mathematical statistics theory.

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Author contributions

Xinxin Wang conceived the study, was responsible for the design, data analysis, and wrote the first draft of the article. Zeshui Xu was responsible for the development of the data analysis. Honghui Li was responsible for data collection and data interpretation.

Disclosure statement

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