



IMPROVED COMMON WEIGHT DEA-BASED DECISION APPROACH FOR ECONOMIC AND FINANCIAL PERFORMANCE ASSESSMENT

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Abstract. Economic and financial performance assessment possesses an important role for efficient usage of available resources. In this study, a novel common weight multiple criteria decision making (MCDM) approach based on data envelopment analysis (DEA) is presented to identify the best performing decision making unit (DMU) accounting for multiple inputs as well as multiple outputs. The robustness of the developed model, which provides a rank-order with enhanced discriminatory characteristics and improved weight dispersion, is illustrated by two case studies that aim to provide economic and financial performance assessment. The first study presents an evaluation of Morgan Stanley Capital International emerging markets, whereas the second case study ranks the Turkish deposit banks using the proposed methodology as well as providing a comparative evaluation with several other approaches addressed in earlier works. The results indicate that the introduced approach guarantees to identify the best performing DMU without including a discriminating parameter requiring an arbitrary step size value in model formulation while also achieving an improved weight dispersion for inputs and outputs.

Keywords: common weight DEA-based models, discriminating power, decision analysis, performance evaluation, MSCI emerging markets, Turkish banking sector.

JEL Classification: C61, C44, C65.

Introduction

It is worth noting that market conditions, competitive environment and resources are considered in combination for developing financial services and products (Grigoroudis et al., 2013). Financial performance evaluation enables countries and their finance sector to concentrate on the efficiency of services provided by them and effective allocation of their available resources. Lately, scholars have contributed to the literature of financial performance evaluation in an effort to segregate the ones that operate better from poor performers. Such classification may be accomplished by data envelopment analysis (DEA), which is a very widely used non-

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parametric technique (Sathye, 2003; Moradi-Motlagh & Babacan, 2015). DEA is a decision making tool that is based on mathematical programming and has been commonly utilized to deal with decision problems which require considering multiple inputs and outputs for assessing the relative efficiency of homogeneous decision making units (DMUs) without a priori knowledge concerning the importance assigned to inputs as well as outputs (Karsak & Ahiska, 2007).

Although conventional DEA models provide performance assessment by classifying the DMUs as efficient or inefficient, they also have several limitations. First of all, n formulations need to be solved to obtain the efficiency scores for all DMUs, where n denotes the total number of DMUs. In addition, traditional DEA models do not usually have an adequate discriminatory characteristic since they consider all DMUs with an efficiency score equal to 1 as efficient. To overcome this issue, super efficiency, which excludes the constraint that the optimal weights for the evaluated DMU do not imply an efficiency exceeding unity, is presented (Andersen & Petersen, 1993). Super efficiency measure has an improved discriminating power through enabling super-efficient units with efficiency scores greater than one; however, it also possesses several limitations such as enabling each DMU to be evaluated according to different weights, assigning specialized DMUs excessively high efficiency scores, and possibility of yielding an infeasible solution, where full ranking of DMUs cannot be provided (Adler et al., 2002). In general, the flexibility of DMUs in determining the weights tends to result in more than one efficient DMU. Techniques including cross-efficiency analysis and weight restrictions were proposed to elaborate the discriminating power of DEA. Moreover, conventional DEA models do not use common set of weights for evaluating DMUs and enable the DMUs to determine their own weights for maximizing their efficiency scores. Hence, ranking DMUs with the efficiency scores acquired from different sets of weights may lead to impossibility for common assessment for some cases (Sun et al., 2013).

In order to overcome the issues mentioned above, common weight DEA-based models have been used by a number of scholars. These models do not require subjective evaluation to determine weights for inputs and outputs while providing a common assessment for all DMUs. Thus, determining input as well as output weights in favor of DMUs is limited that increases the discriminating power of the analysis (Karsak & Ahiska, 2005). Hence, using common weights appears to be practical for computing the efficiency scores and obtaining a rank-order for all DMUs.

Common weight DEA-based models provide common assessment for DMUs in case of weight restriction, dealing with multi-period framework (Hajiagha et al., 2018). These models can also solve resource allocation problem providing common evaluation of DMUs. Furthermore, common weight DEA-based models necessitate a smaller number of mathematical programs to be solved to determine efficiency scores compared with traditional DEA models. Thereby, common weight models also provide computational savings.

This paper contributes to common weight DEA-based modeling literature incorporating multiple inputs as well as multiple outputs by proposing a novel mathematical programming approach that improves the common weight MCDM model developed by Karsak and Ahiska (2007). The contributions of the proposed approach to the DEA literature can be summarized as follows. First, the use of the proposed framework guarantees to identify the best performing DMU via solving one mixed integer linear programming model in addition to

a single linear program for determining the set of minimax efficient DMUs. Second, it does not necessitate an arbitrarily determined parameter, k , value for improving the discriminating power. Third, it provides enhanced weight dispersion for inputs and outputs. Fourth, two real world applications concerning economic and financial performance evaluation are given to show the robustness of the proposed decision framework. Finally, a comparative with those of several other approaches developed in earlier research papers evaluation is provided to set forth the merits of the proposed methodology.

The remaining parts of the manuscript are organized as follows. Section 1 provides a concise review of the literature with regard to common weight DEA models published over the last decade. Section 2 delineates the conventional DEA model, several common weight models also used for comparison purposes, and the proposed improved model. In Section 3, the implementation and comparative evaluation of the proposed decision approach with earlier studies are provided through two case studies which focus on identifying the best performing MSCI emerging market and performance assessment in Turkish banking sector, respectively. Last Section concludes with final remarks and directions for future research.

1. Literature review

In recent years, several studies have contributed to the literature on common weight DEA-based models. Some of these researches have solved the problems with single input and multiple outputs while others have focused on the problems with multiple inputs as well as multiple outputs.

First, common weight DEA-based models including single input and multiple outputs are reviewed. Karsak and Ahiska (2005) developed a minimax efficiency model to calculate the common weight efficiency scores of DMUs with a single formulation, and then incorporated a discriminatory parameter into the model for obtaining a single efficient DMU. They employed this formulation for robot selection with single input and multiple outputs. Karsak and Ahiska (2008) proposed improvements on their previous formulation by integrating a bisection search algorithm to identify discriminating parameter values robustly for single input and multiple outputs cases comprising ordinal as well as cardinal data. Foroughi (2012) used mixed integer programming and developed a minimax efficiency model avoiding discriminating parameter, and employed the model to address robot selection. More recently, Toloo (2013) also proposed a mixed integer linear program by considering single input and multiple outputs. The approach that identified the single most efficient DMU was illustrated through a data set of professional tennis players.

Alternatively, researchers have also focused on common weight DEA-based models that involve multiple inputs as well as multiple outputs. Karsak and Ahiska (2007) developed minimax efficiency models to identify the most efficient DMU for the case incorporating multiple inputs and outputs. Amirteimoori and Emrouznejad (2012) provided the input or output reduction by employing a DEA-based multi objective linear programming model and applied it to the banking sector.

Sun et al. (2013) proposed two mathematical models, where one of them recognizes the virtual ideal unit as the reference object and the other one takes into account the virtual

anti-ideal DMU as the reference object. Omrani (2013) aimed to identify the most efficient provincial gas company in Iran via introducing three DEA-based models.

Recently, Salahi et al. (2016) suggested a robust counterpart model for the envelopment form of CCR, and introduced relations between dual of this model and robust counterpart of multiplier form of CCR. Yang et al. (2016) conducted a case study of Taiwan forests after reorganization. For that reason, they constructed a DEA-based common weight decision aid for calculating the change of efficiency scores of a DMU at different time periods. Carillo and Jorge (2016) proposed a DEA-based common weight modeling framework, which minimizes the total Tchebychev distances of each DMU to an ideal point, and implemented their model on three numerical illustrations that are taken from earlier studies. Tables 1 and 2 provide advantages and limitations of the developed methodologies in a tabular format. Table 1 denotes DEA-based models with single input and multiple outputs, while Table 2 gives DEA-based models including multiple inputs and multiple outputs.

Table 1. Advantages and limitations of DEA-based models including single input and multiple outputs

Author(s)	Year	Advantages	Limitations
Karsak and Ahiska	2005	<ul style="list-style-type: none"> – Improved discriminating power. – Based on both cardinal and ordinal outputs. – Yields the best performing DMU by solving fewer linear programs compared with DEA-based approaches. 	<ul style="list-style-type: none"> – Cannot be employed for problems with multiple inputs and multiple outputs. – In stage II, the model requires an arbitrary step size value for the discriminating parameter, k.
Karsak and Ahiska	2008	<ul style="list-style-type: none"> – Enhances the approach proposed by Karsak and Ahiska (2005) by making use of a bisection search algorithm. – Allows to calculate the values of discriminating parameter, k, in a robust manner rather than requiring an arbitrary step size value. 	<ul style="list-style-type: none"> – Cannot be generalized to multiple inputs and multiple outputs case.
Foroughi	2012	<ul style="list-style-type: none"> – Eliminates the discriminating parameter from the model proposed by Karsak and Ahiska (2005) while maintaining the discriminating power of the approach. – Provides computational efficiency compared to the models that utilize step size value for discriminating parameter, k. 	<ul style="list-style-type: none"> – Requires an additional mathematical programming model that yields the minimax efficient DMUs as well as additional linear programs for each minimax efficient DMU.
Toloo	2013	<ul style="list-style-type: none"> – Improved discriminating power without requiring a discriminating parameter, k. 	<ul style="list-style-type: none"> – Requires a penalty value and an auxiliary binary variable. – Guarantees to identify the single best efficient DMU by transforming the model to a mixed integer linear program. – The model is applicable when the problem has a dummy input and multiple outputs.

Table 2. Advantages and limitations of DEA-based models including multiples input and multiple outputs

Author(s)	Year	Advantages	Limitations
Karsak and Ahiska	2007	<ul style="list-style-type: none"> - Extends the work by Karsak and Ahiska (2005) by integrating multiple inputs into the decision framework. - Provides computational savings compared with conventional DEA models. - Improved discriminating power compared with conventional DEA. 	<ul style="list-style-type: none"> - Requires a decision analyst to determine the value of k subjectively. - May not provide consistent results as for the rankings of minimax efficient DMUs considering two-stage approach provided in the study.
Amirteimoori and Emrouznejad	2012	<ul style="list-style-type: none"> - Enables input or output reduction when some system constraints force to reduce them. 	<ul style="list-style-type: none"> - Requires initially employing CCR model that has to be solved n times, where n is the number of DMUs. - Lacks computational efficiency. - Applicable only when inputs and outputs have to be reduced.
Omrani	2013	<ul style="list-style-type: none"> - Guarantees the obtain the best performing DMU. 	<ul style="list-style-type: none"> - Combines the model that yields common set of weights with another model that allows DMUs to identify the weights in their own favor. - Yields inconsistent models.
Salahi et al.	2016	<ul style="list-style-type: none"> - Applicable for the problems with interval data. 	<ul style="list-style-type: none"> - Solves $n+1$ models to identify the best performing DMU, n is the number of DMUs. - Lacks computational efficiency. - Inconsistent models, where first stage of the developed approach computes different set of weights for each DMU whereas second stage yields common set of weights, are developed.
Yang et al.	2016	<ul style="list-style-type: none"> - Calculates the change of efficiency scores of a DMU at different time periods. 	<ul style="list-style-type: none"> - Lacks computational efficiency.
Carillo and Jorge	2016	<ul style="list-style-type: none"> - Minimizes the total Tchebychev distances of each DMU to an ideal point. 	<ul style="list-style-type: none"> - The computational experiments are performed with randomly generated data sets.

Although providing viable performance assessment through employing common set of weights and enhancing discriminating power of conventional DEA, some of these abovementioned models suffer from drawbacks including impractical weight dispersion and incoherent ranking of DMUs regarding different stages of the modelling framework. Therefore, novel common weight DEA-based approaches overcoming these limitations need to be developed.

2. Proposed methodology

2.1. Background

The DEA model, also known as the CCR model (the abbreviation for Charnes, Cooper and Rhodes), developed by Charnes et al. (1978) computes the relative efficiencies of a homogeneous set of DMUs without requiring a priori information concerning the importance of inputs as well as outputs. As the fractional model is nonlinear and nonconvex, it can be converted into a linear programming model through a transformation. The linear program for computing the efficiency score of DMU_{*j*0} is as follows:

$$\begin{aligned}
 \max E_{j_0} &= \sum_{r=1}^s u_r y_{rj_0} \\
 \text{subject to} & \\
 \sum_{i=1}^m v_i x_{ij_0} &= 1; \\
 \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} &\leq 0, \forall j, u_r, v_i \geq \epsilon, \forall r, i,
 \end{aligned} \tag{1}$$

where E_{j_0} denotes the efficiency score of the DMU under evaluation, u_r is the weight assigned to output r , v_i is the weight assigned to input i , y_{rj} represents the quantity of output r generated and x_{ij} denotes the quantity of input i used by DMU_{*j*}, respectively, and ϵ is a small positive scalar.

Due to aforementioned limitations of conventional DEA models such as poor discriminating power and impractical assessment of alternatives lack of common weights, researchers intended to extend the evaluation/selection process of alternatives using common weight models. Karsak and Ahiska (2007) developed a common weight multi-criteria decision making (MCDM) framework for decision problems in the presence of multiple inputs as well as multiple outputs that is based on minimizing maximum deviation from efficiency as well as equating the sum of criteria weights to 1 as in other typical MCDM models.

The minimax efficiency model introduced by Karsak and Ahiska (2007) for assessing alternatives with multiple inputs and multiple outputs is as

$$\begin{aligned}
 \min \theta & \\
 \text{subject to} & \\
 \theta - d_j &\geq 0, \forall j; \\
 \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} + d_j &= 0, \forall j; \\
 \sum_{r=1}^s u_r + \sum_{i=1}^m v_i &= 1, u_r, v_i, d_j \geq 0, \forall j, i, j,
 \end{aligned} \tag{2}$$

where d_j is the deviation from the efficiency score of DMU j , (i.e. $d_j = 1 - E_j$, E_j is the efficiency score of DMU_{*j*}), and θ denotes the maximum deviation from efficiency.

Model (2) focuses on minimizing the maximum deviation from efficiency, and results in computation savings by calculating efficiency scores of all DMUs with a single model. Furthermore, this model enables the assessment of relative efficiency of all DMUs with common weights unlike traditional DEA models where each DMU is evaluated by different set of weights.

When a single efficient DMU cannot be obtained via model (2), Karsak and Ahiska (2007) proposed the following common weight model to identify the best performer.

$$\begin{aligned}
 & \min \theta - k \sum_{j \in EF} d_j \\
 & \text{subject to} \\
 & \theta - d_j \geq 0, \forall j; \\
 & \theta - \sum_{j \in EF} d_j \geq 0; \\
 & \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} + d_j = 0, \forall j; \\
 & \sum_{r=1}^s u_r + \sum_{i=1}^m v_i = 1, u_r, v_i, d_j \geq 0, \forall j, i, j,
 \end{aligned} \tag{3}$$

where $k \in (0,1]$ is a discriminatory parameter which is specified by the analyst, while EF indicates the set of DMUs that are identified as minimax efficient via model (2).

The methodology presented by Karsak and Ahiska (2007) enables to rank efficient DMUs with superior computational efficiency compared with traditional DEA models. One short-coming of this approach is that an analyst is required to specify the value of discriminatory parameter k . Another limitation of the model is that model (3) may result in lower final efficiency scores for a number of minimax efficient DMUs than several other DMUs that are found to be inefficient with respect to model (2).

More recently, Sun et al. (2013) employed two mathematical programming models to rank the common weight efficiency scores. First, they introduced the model given below.

$$\begin{aligned}
 & \min \sum_{j=1}^n d_j \\
 & \text{subject to} \\
 & \sum_{i=1}^m v_i x_{ij} - d_j = \sum_{r=1}^s u_r y_{rj}, \forall j; \\
 & \sum_{i=1}^m v_i x_{\min} = 1; \\
 & \sum_{r=1}^s u_r y_{\max} = 1, \\
 & u_r, v_i \geq \varepsilon, \forall r, i, d_j \geq 0, \forall j,
 \end{aligned} \tag{4}$$

where $x_{\min} = \min\{x_{ij} | j = 1, \dots, n\}$, ($i = 1, \dots, m$) and $y_{\max} = \max\{y_{rj} | j = 1, \dots, n\}$, ($r = 1, \dots, s$) Sun et al. (2013) demonstrated that the optimal weights computed by model (4) might not be unique and diverse software can result in disparate optimal weights. Hence, they developed the nonlinear programming model given below in order to improve the convenience of this approach.

$$\begin{aligned} & \max \sum_{i=1}^m v_i^2 + \sum_{r=1}^s u_r^2 \\ & \text{subject to} \\ & \sum_{j=1}^n \left(\sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s u_r y_{rj} \right) = D^*; \\ & \sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s u_r y_{rj} \geq 0, \forall j; \\ & \sum_{i=1}^m v_i x_{\min} = 1; \\ & \sum_{r=1}^s u_r y_{\max} = 1, u_r, v_i \geq \varepsilon, \forall r, i, \end{aligned} \tag{5}$$

where D^* represents the optimal value for objective function of model (4). As an alternative to their model given above, Sun et al. (2013) suggested the following model.

$$\begin{aligned} & \min \sum_{j=1}^n \left(\sum_{i=1}^m v_i x_{\max} - \sum_{i=1}^m v_i x_{ij} \right) + \sum_{j=1}^n \left(\sum_{r=1}^s u_r y_{rj} - \sum_{r=1}^s u_r y_{\min} \right) \\ & \text{subject to} \\ & \sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s u_r y_{rj} \leq 0, \forall j; \\ & \sum_{i=1}^m v_i x_{\max} = 1; \\ & \sum_{r=1}^s u_r y_{\min} = 1, u_r, v_i \geq \varepsilon, \forall r, i. \end{aligned} \tag{6}$$

Although these models enable full ranking of DMUs and thus improve the discriminatory characteristics of DEA, the weight dispersion for inputs and outputs is relatively poor.

2.2. Proposed common weight decision making approach

The common weight approach developed in this study aims to improve the modeling framework in Karsak and Ahiska (2007). The proposed approach incorporates a non-Archimedean infinitesimal ϵ to model (2), and the resulting common weight mathematical programming model is given below.

$$\begin{aligned}
 & \min \theta \\
 & \text{subject to} \\
 & \theta - d_j \geq 0, \forall j; \\
 & \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} + d_j = 0, \forall j; \\
 & \sum_{r=1}^s u_r + \sum_{i=1}^m v_i = 1, \quad u_r, v_i \geq \epsilon, \forall r, i, \quad d_j \geq 0, \forall j,
 \end{aligned} \tag{7}$$

where $\theta = \max d_j$ and ϵ is a small positive scalar. Next, in lieu of model (3), an improved model is proposed by eliminating discriminating parameter k . The developed model for multiple inputs as well as multiple outputs is as

$$\begin{aligned}
 & \min \theta \\
 & \text{subject to} \\
 & \theta - d_j \geq 0, \forall j; \\
 & \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} + d_j = 0, \forall j; \\
 & \sum_{r=1}^s u_r + \sum_{i=1}^m v_i = 1; \\
 & d_j + Mz_j \geq \epsilon, \quad j \in EF; \\
 & \sum_{j \in EF} z_j = 1, \quad z_j \in \{0,1\}, \quad j \in EF, \quad u_r, v_i \geq \epsilon, \forall r, i, \quad d_j \geq 0, \forall j,
 \end{aligned} \tag{8}$$

where M denotes a very large positive number, and z_j is a 0–1 decision variable. The proposed model assures to have a single efficient DMU as demonstrated below, and yields improved weight dispersion for inputs and outputs. Moreover, the obtained rankings are consistent with the differentiation between inefficient and efficient DMUs in model (7).

Theorem. Model (8) yields a single efficient DMU.

Proof. Let DMU_k and DMU_l be decision making units found to be efficient ($k, l \in EF$) by the common set of optimal weights u_r^* and v_i^* . This will lead to $u_r^* y_{rk}^* - v_i^* x_{ik}^* = 0$ and $u_r^* y_{rl}^* - v_i^* x_{il}^* = 0$ which result in $d_k^* = d_l^* = 0$, and thus, $z_k^* = z_l^* = 1$. However, this is not possible according to constraint $\sum_{j \in EF} z_j = 1$. Consequently, DMU_k and DMU_l cannot be efficient simultaneously. \square

3. Economic and financial performance assessment using the proposed approach

This paper provides performance evaluation of MSCI emerging markets and banks operating in Turkish banking sector by taking into consideration economic and financial indicators. Some of these indicators are to be minimized whereas others should be maximized to increase the efficiency of DMUs. For that reason, the indicators that are to be minimized and

maximized are considered as inputs and outputs, respectively. The logic of the categorization of indicators is in conformance with the principles of DEA, and thus, it is appropriate to develop a DEA-based decision framework for economic and financial assessment of MSCI emerging markets and banks operating in Turkish banking sector. DEA was previously used as a decision aid in economic and financial performance assessment. In this section, several studies that employ DEA to provide economic and financial performance assessment of countries as well as banks employing DEA are reviewed.

First, research works that implement DEA models for analyzing performance of countries considering economic and financial indicators are reviewed. Lozano-Vivas and Pastor (2006) employed DEA to evaluate financial performance of 15 OECD countries over time period by demonstrating the correlation between banking productivity and macroeconomic efficiency. Giambona and Vassallo (2014) proposed a DEA-based approach to evaluate 27 European Union (EU) countries in terms of their social inclusion, which denotes a key component of EU sustainable development strategy. Hsu and Lee (2014) employed DEA to evaluate the performance of public spending and observe how productivity has changed over time for 18 OECD countries from 1995 to 2002. Degl'Innocenti et al. (2017) used a two-stage DEA model for examining the bank productivity for 28 EU countries during global financial crisis. Lately, Feng et al. (2017) employed DEA for evaluating 41 regions by considering green development performance index that provides ecologic and economic improvement. Chodakowska and Nazarko (2017) developed a DEA-based decision approach for evaluating European countries by taking into account financial and environmental factors.

A number of research papers focus on employing DEA for evaluating economic and financial performance of banks. Sathye (2003) used the conventional DEA model to compute the efficiency scores of Indian banks by classifying them under three subclasses. Sherman and Zhu (2006) determined the efficiency of bank branches by developing an integrated approach that incorporates quality criterion into DEA efficiency. Grigoroudis et al. (2013) proposed a multi-stage DEA network model to compute the efficiency scores of bank branches, and utilized a set of performance factors combining business success indicators, customer satisfaction and personnel assessment. Titko et al. (2014) applied input-oriented DEA model under VRS assumption to compute efficiency of Latvian banks, and solved the introduced model with fourteen different input and output combinations. Puri and Yadav (2016) developed a DEA approach with uncertain data and undesirable outputs to evaluate cost efficiency and revenue efficiency of Indian banks as well as bank groups for the periods 2011–2012 and 2012–2013. Ray (2016) aimed to identify the optimum number of bank branches in Indian banking sector for efficient allocation of the available resources. For that reason, a DEA-based mixed integer linear programming model, which aims at minimizing cost subject to the number of bank branches of the market region, is developed.

This section shows the implementation of the proposed decision making approach via case studies conducted in MSCI emerging markets and Turkish banking sector, and provides a comparative evaluation with the models developed by Karsak and Ahiska (2007) and Sun et al. (2013). GAMS software with CPLEX solver is used for performing the computations.

3.1. Performance evaluation of emerging markets

Morgan Stanley Capital International (MSCI) indices were originally designed as capital international indices in 1968 to measure the performance of global capital for non-US markets. In 1986, international capital indices were taken over by Morgan Stanley and became popularly known as MSCI indices. The use of MSCI indices in the 1980s have become widespread and they have been among primary indices for non-US markets (GCM Forex, n.d.).

The MSCI emerging markets indices were first introduced via market capitalization of 21 countries. Nowadays, MSCI indices are composed of 23 countries representing 10% of the world markets. (GCM Forex, n.d.). MSCI was established to provide investment decision aid tools worldwide. MSCI offers Market Cap Indexes that are divided into four different indexes as “MSCI World Index”, “MSCI Emerging Markets Index”, “MSCI Frontier Markets Index”, and “MSCI Standalone Market Indexes” (Morgan Stanley Capital International, n.d.).

This subsection illustrates the application of the proposed decision aid using MSCI Emerging Markets data, and compares the results with those of former models introduced by Karsak and Ahiska (2007) and Sun et al. (2013). To identify the best performer among the countries in MSCI Emerging Markets Index, 23 countries are evaluated according to two inputs, namely “inflation” and “government debt to gross domestic product (GDP)”, and four outputs, namely “GDP growth rate”, “gross savings to GDP”, “current account balance (% of GDP)”, and “market capitalization of listed domestic companies (% of GDP)”. Raw data for related inputs and outputs for 2015 are provided in Table 3. Since the raw data

includes negative as well as positive values, input data are normalized as
$$\left[\frac{1 - \frac{x_i^* - x_{ij}}{\sum_{i=1}^m (x_i^* - x_{ij})}} \right],$$

where $x_i^* = \max_j x_{ij}$ for $\forall i$, while output data are normalized using
$$\left[\frac{1 - \frac{y_r^* - y_{rj}}{\sum_{r=1}^s (y_r^* - y_{rj})}} \right],$$
 where $y_r^* = \max_j y_{rj}$ for $\forall r$ (Jahan et al. 2016).

Overall ranking results according to efficiency scores are shown in Table 4. To provide a comparative evaluation of models, ϵ is taken to be 0.000001 as in Sun et al. (2013). The DEA-CCR model yields five efficient countries that are DMU₃, DMU₁₂, DMU₁₈, DMU₂₀ and DMU₂₁, whereas other DMUs are inefficient. On the other hand, DMU₁₈, DMU₂₀ and DMU₂₁ are considered as minimax efficient as regards model (2). Subsequently, three additional linear programs are solved with a step size of 0.1 until k equals to 0.3 in model (3), and DMU₂₀ is identified as the best performing country. Although DMU₁₈ and DMU₂₁ obtain efficiency score of 1 according to model (2), with respect to model (3) they are rank-ordered after DMU₁₂ which is a minimax inefficient alternative. Alternatively, DMU₃ and DMU₂₀ are the most efficient countries with regard to model (5) and model (6), respectively, developed by Sun et al. (2013). Instead, model (8) of the developed methodology identifies DMU₁₈ as the best performing country. DMU₂₀ and DMU₂₁, which are the minimax efficient countries as well regarding model (7), follow DMU₁₈ in the efficiency rankings of model (8).

Table 3. Input and output data for MSCI Markets (source: Trading Economics, n.d.; World Bank, n.d.)

DMU (j)	Country	Input1	Input2	Output1	Output2	Output3	Output4
1	Brazil	9	65.45	-3.8	14	-3.3	27.2
2	Chile	4.3	17.4	2.3	21	-2	79.1
3	China	1.4	42.6	6.9	48	3	74
4	Colombia	5	50.7	3.1	18	-6.5	29.4
5	Czech Republic	0.3	40.3	4.5	27	0.9	17.39
6	Egypt	10.4	85	4.2	11	-5.1	16.7
7	Greece	-1.7	177.4	-0.2	10	0.1	21.6
8	Hungary	-0.1	74.7	3.1	25	3.2	14.5
9	India	4.9	69.6	7.9	32	-1.1	72.6
10	Indonesia	6.4	26.9	4.8	32	-2	41
11	Korea	0.7	37.8	2.6	36	7.7	89.4
12	Malaysia	2.1	54.5	5	28	3	129.3
13	Mexico	2.7	43.2	2.5	22	-2.9	35.2
14	Pakistan	2.5	63.2	4.7	23	-0.6	15.25
15	Peru	3.6	23	3.3	19	-4.9	29.9
16	Philippines	1.4	45.05	5.9	44	2.5	81.7
17	Poland	-1	51.1	3.9	20	-0.6	28.9
18	Qatar	1.9	34.9	3.6	47	8.4	86.6
19	Russia	15.5	15.9	-2.8	27	5.1	28.8
20	South Africa	4.6	49.3	1.3	16	-4.3	234
21	Thailand	-0.9	43.9	2.8	30	8.1	88.3
22	Turkey	7.7	27.5	4	25	-4.5	26.3
23	United Arab Emirates	4.1	18.1	3.8	29	5.3	52.9

The developed approach that enables to identify the best performing country obviously enhances the discriminatory characteristics of DEA-CCR model yielding five efficient countries. In comparison to models in Karsak and Ahiska (2007), the ranking inconsistency encountered in the outcomes of model (3) among the DMUs found to be efficient according to model (2) is avoided in the proposed approach. Moreover, in contrast with model (3), a discriminating parameter is not required in the proposed model.

Although presenting a new common weight modelling perspective, the models addressed in Sun et al. (2013) are inapt to provide robust weight distribution for assessing MSCI emerging markets as given in Table 5. Model (6) developed by Sun et al. (2013) considers only one input ($v_2 > \epsilon$) and one output ($u_4 > \epsilon$), while model (5) considers single output ($u_1 > \epsilon$) and two inputs ($v_1, v_2 > \epsilon$). One shall also note that v_1 equals to 0.000008 that is quite close to ϵ . All the other weights are determined to be equal to ϵ . Alternatively, model (8) given in this paper yields improved weight dispersion with all inputs and outputs possessing weights significantly greater than ϵ as depicted in Table 5.

Table 4. Rankings with respect to efficiency scores of countries

DMU (<i>i</i>)	DEA-CCR	Model (2) developed by Karsak and Ahiska (2007)	Model (3) developed by Karsak and Ahiska (2007)	Model (5) developed by Sun et al. (2013)	Model (6) developed by Sun et al. (2013)	Proposed model (7)	Efficiency scores obtained from the proposed model (8)	Proposed model (8)
1	23	21	20	22	19	21	0.975589	21
2	10	9	8	13	2	9	0.992814	9
3	1	6	4	1	9	6	0.997661	6
4	21	19	17	14	17	19	0.983308	19
5	14	10	12	6	16	10	0.990833	10
6	22	21	20	17	22	21	0.975589	21
7	18	21	20	23	23	21	0.975589	21
8	17	14	16	19	21	14	0.987695	14
9	7	10	10	2	14	10	0.990833	10
10	11	13	14	4	11	13	0.989178	13
11	6	4	4	15	5	4	0.999447	4
12	1	5	2	8	3	5	0.99893	5
13	20	14	14	18	15	14	0.987695	14
14	19	17	17	10	20	17	0.986594	17
15	16	14	12	9	12	14	0.987695	14
16	9	6	4	3	8	6	0.997661	6
17	12	10	10	12	18	10	0.990833	10
18	1	1	4	11	4	1	1	1
19	13	20	20	21	10	20	0.980523	20
20	1	1	1	20	1	1	0.999999	2
21	1	1	3	16	7	1	0.999999	2
22	15	18	17	7	13	18	0.985039	18
23	8	8	9	5	6	8	0.995224	8

Table 5. Comparative evaluation of input and output weights

Weight	Model (3) developed by Karsak and Ahiska (2007)	Model (5) developed by Sun et al. (2013)	Model (6) developed by Sun et al. (2013)	Proposed model (8)
v_1	0.233098	0.000008	0.000001	0.21042
v_2	0.27573	1.058481	0.999999	0.294784
u_1	0.056867	0.999997	0.000001	0.065238
u_2	0	0.000001	0.000001	0.023153
u_3	0.00454	0.000001	0.000001	0.079532
u_4	0.429765	0.000001	1.057122	0.326874

3.2. Performance evaluation of banks in Turkish banking sector

Banking is an important component of the financial framework that mediates the transfer of resources in the economy. The banking sector is a major field that affects the whole economy in addition to being a crucial element of the Turkish financial framework. The relative share of the banking sector in the financial system may vary in each country depending on the level of economic and social development. The relative share of the banking sector in the financial system generally decreases in parallel to social and economic development.

After the financial crisis in 2001, the Turkish banking sector recovery continued until 2003. The number of branches decreased due to the bankruptcies in the industry. After 2004, number of branches started to increase rapidly although the number of banks continued to decrease between 2004 and 2010. Due to the upsurge in total number of branches, the number of personnel has also increased. Moreover, the banks have managed to increase regularly the number of ATM and POS machines along with the number of credit cards (Türkiye Bankalar Birliği, n.d.).

The increasing level of competition in the banking industry has led the banks to focus on provided services and efficient allocation of available resources (Grigoroudis et al., 2013). Turkish financial services developed dramatically during the period 2002–2012 when Turkish banks obtained return on equity (ROE) ratios above 20% every year. Although the global economic crisis contracted the gross domestic product (GDP) by 4.8% in 2009, a drastic rebound of 9.2% growth in GDP followed in 2010. By the end of 2012, Turkish banking system was no longer an underdeveloped sector, while also being a client-focused market. The average GDP growth rate for Turkey decreased to 3.1% during 2013–2016 period, whereas average ROEs of banks decreased to about 12% (Elhadef et al., 2016). In Turkish banking sector, loans to total assets ratio is high compared with a number of other countries while non-deposit funds are substantially based on liabilities (Türkiye Bankalar Birliği, n.d.). As of 2016, there are 34 deposit banks, 13 investment banks and 6 participation banks in Turkey.

This subsection illustrates the implementation of the proposed approach through a case study conducted in Turkish banking sector based on real data, and provides a comparative evaluation with the models developed by Karsak and Ahiska (2007) and Sun et al. (2013). The case study, which aims to identify the most efficient bank, evaluates 18 deposit banks selected considering the size of total assets according to three inputs as “total non-deposit funds”, “deposits” and “number of branches”, and three outputs namely “total loans”, “total income from interest” and “total financial assets”, respectively. Raw data for related inputs and outputs for the year 2015 are depicted in Table 6. Normalization for data is performed via linear normalization scheme. Normalization for input data using x_{ij}/x_i^* , where $x_i^* = \max_j x_{ij}$ for $\forall i$, whereas output data are normalized as y_{rj}/y_r^* , where $y_r^* = \max_j y_{rj}$ for $\forall r$ (Karsak & Ahiska, 2007).

Overall ranking outcomes with regard to efficiency scores are reported in Table 7. In order to enable a comparison of the results, ϵ is taken to be 0.000001 as in Sun et al. (2013). The DEA-CCR model results in nine efficient banks that are DMU₃, DMU₄, DMU₅, DMU₈, DMU₁₁, DMU₁₂, DMU₁₅, DMU₁₆ and DMU₁₈ while the other DMUs are considered as inefficient. Besides, DMU₄, DMU₈ and DMU₁₁ are minimax efficient DMUs according to model (2).

Table 6. Input and output data for deposit banks (source: Türkiye Bankalar Birliği, n.d.)

DMU (j)	Bank	Input1 (millions of TRY)	Input2 (millions of TRY)	Input3	Output1 (millions of TRY)	Output2 (millions of TRY)	Output3 (millions of TRY)
1	Türkiye Cumhuriyeti Ziraat Bankasi	73847.159	186469.435	1812	186813	22050.495	64871.349
2	Türkiye Is Bankasi	68258.775	153802.426	1377	177934	19200.361	46044.289
3	Türkiye Garanti Bankasi	62704.727	140899.332	980	159140	17420.007	44805.077
4	Akbank	57808.513	138942.497	902	141763	15247.388	55524.719
5	Yapi ve Kredi Bankasi	45478.233	126908.893	1000	148779	15292.461	31862.498
6	Türkiye Halk Bankasi	39540.87	122145.965	949	126745	13656.908	28155.131
7	Türkiye Vakiflar Bankasi	41852.025	109922.534	920	123781	13630.05	25337.165
8	Finans Bank	14615.651	48565.837	642	57226	7597.377	14721.072
9	Denizbank	19142.426	46587.577	692	51349	6804.782	12882.837
10	Türk Ekonomi Bankasi	14250.187	44395.86	532	53213	6219.447	5226.274
11	ING Bank	16688.666	23648.977	298	35205	3726.152	5078.701
12	Odea Bank	3445.322	25333.496	55	21807	2352.473	1587.352
13	HSBC Bank	6142.302	19056.359	284	20491	2402.378	2318.259
14	Sekerbank	5422.869	14867.633	301	16726	2283.308	3151.962
15	Alternatifbank	4588.029	6288.12	59	9345	1071.06	843.604
16	Fibabanka	2033.009	7460.485	67	8615	891.475	728.302
17	Anadolubank	1789.426	7322.809	106	6815	959.21	1513.841
18	Burgan Bank	2122.614	6695.608	56	8186	846.777	774.839

Afterwards, seven additional linear programs are solved with a step size of 0.1 until k equals to 0.7 in model (3), and DMU_8 is found to be the best performing bank. However, DMU_4 and DMU_{11} with efficiency scores of 1 according to model (2) rank below a number of minimax inefficient alternatives with respect to model (3) that results in an inconsistency between the outcomes of models (2) and (3). On the other hand, DMU_{11} and DMU_{15} are the best alternatives as regards model (5) and model (6), respectively, addressed in Sun et al. (2013). Alternatively, DMU_8 is the best performing bank according to model (8) of the proposed methodology. DMU_4 and DMU_{11} , which are also the minimax efficient banks with respect to model (7), follow DMU_8 in the efficiency rankings of model (8).

The proposed approach provides enhanced discriminating power by enabling to determine the best performing bank while DEA-CCR model results in nine efficient banks.

Table 7. Rankings with respect to efficiency scores of banks ($\epsilon = 0.000001$)

DMU (<i>j</i>)	DEA-CCR	Model (2) developed by Karsak and Ahiska (2007)	Model (3) developed by Karsak and Ahiska (2007)	Model (5) developed by Sun et al. (2013)	Model (6) developed by Sun et al. (2013)	Proposed model(7)	Efficiency scores obtained from proposed model (8)	Proposed model (8)
1	15	17	16	16	15	17	0.96117	17
2	11	11	16	7	10	11	0.98837	11
3	1	7	14	9	12	7	0.99481	7
4	1	1	11	15	17	1	0.999999	2
5	1	9	13	6	13	9	0.991674	9
6	16	17	16	14	16	17	0.96117	17
7	14	16	15	10	11	16	0.975237	16
8	1	1	1	5	3	1	1	1
9	17	13	9	12	5	13	0.984778	13
10	10	14	12	4	6	14	0.982564	14
11	1	1	8	1	2	1	0.999999	2
12	1	15	9	18	18	15	0.981258	15
13	18	12	7	13	9	12	0.987331	12
14	13	9	6	11	4	9	0.991674	9
15	1	4	5	2	1	4	0.999089	4
16	1	6	4	8	14	6	0.996946	6
17	12	7	2	17	7	7	0.99481	7
18	1	5	2	3	8	5	0.998176	5

Contrary to the models developed in Karsak and Ahiska (2007), model (8) provides consistent rankings regarding the sets of efficient and inefficient banks obtained from model (7). Furthermore, the discriminating parameter, which is required in Karsak and Ahiska (2007) despite its previously discussed drawbacks, is not used in the proposed framework.

The formulations employed in Sun et al. (2013) do not provide sound weight distribution compared to model (8) for evaluating the banks as depicted in Table 8. Model (5) proposed by Sun et al. (2013) considers a single input ($v_2 > \epsilon$) and an output ($u_1 > \epsilon$), whereas model (6) takes into account one input ($v_2 > \epsilon$) and one output ($u_2 > \epsilon$) to rank the alternatives. All the other weights are determined to be equal to ϵ . Contrarily, model (8) proposed in this paper results in enhanced dispersion of weights assigned to inputs and outputs as shown in Table 8. These diversity in weights result in different efficiency scores for assessing alternatives, and thus, obtained rankings are obviously not the same for three approaches. The unrealistic weighting schemes, which result in a single input and a single output to be considered while ignoring other inputs as well as outputs in efficiency computations, distort the robustness of the related models.

Table 8. Comparative evaluation of input and output weights

Weight	Model (3) developed by Karsak and Ahiska (2007)	Model (5) developed by Sun et al. (2013)	Model (6) developed by Sun et al. (2013)	Proposed model (8)
v_1	0.179158	0.000001	0.000001	0.063572
v_2	0.350691	29.65423	0.999998	0.416027
v_3	0	0.000001	0.000001	0.039816
u_1	0.253216	0.999998	0.000001	0.327263
u_2	0	0.000001	26.071539	0.000001
u_3	0.216935	0.000001	0.000001	0.153321

Conclusions

This paper introduces an enhanced common weight MCDM framework, which may be employed with success to identify the best performer when multiple inputs and outputs exist. An earlier two-stage efficiency model recommended by Karsak and Ahiska (2007) with an objective given in the second stage as $\min M - k \sum_{j \in EF} d_j$ for determining the most efficient unit has several limitations. First of all, their approach requires a step size value for k that is chosen arbitrarily, and further, it may not provide consistent results as for the rankings of minimax efficient DMUs considering two minimax efficiency models provided in their study. The model is solved by increasing k value by an arbitrary step size until a single efficient DMU is obtained. The proposed formulation that provides identifying best performing DMU via solving one mixed integer linear programming model in addition to a linear program for yielding the set of minimax efficient DMUs does not demand setting an arbitrary value for k . Furthermore, minimax efficient DMUs obtained in model (7) rank above minimax inefficient DMUs with respect to model (8) as well, and thus, coherence in the outcomes of two stages is maintained.

Two case studies, aiming to identify the best DMU in terms of economic and financial performance, are conducted to show the implementation of the developed decision making procedure. Ranking outcomes are compared with those of the models provided in Karsak and Ahiska (2007) and Sun et al. (2013). Two minimax efficiency models employed in Karsak and Ahiska (2007) do not provide consistent results in the rankings of minimax efficient DMUs while the proposed methodology reveals consistent rankings. The models developed by Sun et al. (2013) fall short of providing practical weight dispersion for inputs and outputs, whereas the proposed methodology yields improved weight distribution compared with both of their models.

Last but not least, a limitation of the developed model is that it is applicable only when exact input and output data are available. Enhancing the proposed approach in a way to incorporate qualitative data will be the focus of future research. This will enable management capability, which is an essential factor in financial efficiency assessment, to be taken into consideration.

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