A MODEL-DRIVEN DECISION SUPPORT SYSTEM FOR REALLOCATION OF SUPPLY TO ORDERS UNDER UNCERTAINTY IN CERAMIC COMPANIES

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Abstract. In ceramic companies, uncertainty in the tone and gage obtained in first quality units of the same finished good (FG) entails frequent discrepancies between planned homogeneous quantities and real ones. This fact can lead to a shortage situation in which certain previously committed customer orders cannot be served because there are not enough homogeneous units of a specific FG (i.e., with the same tone and gage). In this paper, a Model-Driven Decision Support System (DSS) is proposed to reassign the actual homogeneous stock and the planned homogeneous sublots to already committed orders under uncertainty by means of a mathematical programming model (SP-Model). The DSS functionalities enable ceramic decision makers to generate different solutions by changing model options. Uncertainty in the planned homogeneous quantities, and any other type of uncertainty, is managed via scenarios. The robustness of each solution is tested in planned and real situations with another DSS functionality based on another mathematical programming model (ASP-Model). With these DSS features, the ceramic decision maker can choose in a friendly fashion the orders to be served with the current homogeneous stock and the future uncertainty homogeneous supply to better achieve a balance between the maximisation of multiple objectives and robustness.

Keywords: decision support system, mathematical programming models, lack of homogeneity in the product, shortage planning, uncertainty, ceramic companies.

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Introduction

Improving customer satisfaction efficiently is a key aspect of the positioning of competitive companies. Customer satisfaction should lead to customer loyalty, which is one of the factors required to guarantee the sustainability of any business (Okongwu *et al.* 2012). Customer satisfaction can comprise several dimensions, such as rapid response to customer needs, reliability of commitments, high customer service level, among others.

The most important factor in a company's profitability is demand for its goods (services) (Knyvienė *et al.* 2010). The order fulfilment process plays a critical role by efficiently matching companies' supply and demand in the short term (Pibernik, Yadav 2009). Okongwu *et al.* (2012) considers that order fulfilment can be broken down into two phases: the first comprises the order promising activity and the second comprises the execution activity; here the order is actually served as promised. In order promising, the fundamental role of the Available-To-Promise (ATP) function is to provide a response to customer order requests based on resource availability (Ball *et al.* 2004). As a result, decisions about the rejection/ acceptation of customer orders, definition of the promised due date and allocation of resources to customer orders are made. Depending on the manufacturing strategy adopted, the availability level checked can differ. Meyr (2009) reports different situations of demand fulfilment in relation to the three customer order decoupling points into make-to-order, assemble-to-order and make-to-stock. For the purpose of supporting the order promising and the required availability check, the so-called Advanced Available-To-Promise (AATP) systems have been developed (Pibernik 2005; Thammakoranonta *et al.* 2008). AATP systems are defined as a variety of methods and tools that enhance order promising responsiveness and order fulfilment reliability (Pibernik 2005).

In an ideal situation, customer orders are delivered as promised during execution activity. However due to unforeseen events, such as late delivery of materials, absenteeism from work and machine breakdowns, the initial resource allocation can become suboptimal, unfeasible, and can even lead to a shortage situation (Framiñán, Leisten 2010). If anything is made, these disruptions can lead to some initially promised orders not being served on time, which negatively affects customer satisfaction. Before this shortage situation, it is necessary to find alternative solutions to minimise the negative effect on the supply chain (SC) and customers. In order to find a satisfactory solution for both customers and the SC, some shortage planning models have been developed. Indeed, shortage planning deals with the activities to be accomplished should stock not be available. These activities can include decisions on stock reallocation among committed orders (Boza *et al.* 2014), outsourcing (Bhakoo *et al.* 2012), substitutive products (Balakrishnan, Geunes 2000) or negotiation with customers (late supply, partial shipments, etc.).

Shortage situations are more likely to appear in the SCs that promise orders according to high uncertainty levels; e.g., ceramic companies that must face the so-called inherent Lack of Homogeneity in Product Uncertainty (Alemany *et al.* 2013b), which is uncertainty in the future homogeneous quantities of the same finished good (FG) in planned production lots. Incorporation of raw materials, which directly originate from nature (e.g., clays) and certain productive factors (e.g., temperature and humidity), means that there are units of the same

FG (subtypes) in the same lot which differ in aspect (quality), tone (colour) and/or gage (thickness). However, the real homogeneous quantities of each subtype in an FG lot are not known until their production is finished (LHP uncertainty).

In ceramic companies, this aspect becomes a problem because customers require the tiles that complete their orders to be homogeneous (e.g., same quality, tone and gage). However, they promise orders to customers based not only on the uncommitted stock, but also on the uncommitted planned lots in the master production schedule (mps) for which homogeneous sublots are not yet known. If there is enough real or planned uncommitted quantities for the requested customer due date, the order is accepted and the corresponding ordered quantity is reserved from stock or planned lots. After manufactured the planned production lots in the master plan, they are classified into the corresponding subtypes. Due to LHP uncertainty, discrepancies between planned and real homogeneous quantities are more likely to occur. For this reason, checks are made to see if there is a sufficient amount of the uniform subtypes to serve previously promised orders. If not, it is quite usual that all the orders with immediate delivery dates are not completely served with the reserved stock as previously planned.

To solve this situation, reallocation of the available stock of each FG subtype to previously committed orders can be considered. This paper also contemplates the reallocation of uncertain future homogeneous quantities of subtypes in the master plan lots. Reallocation of supply to customer orders is very difficult, and LHP makes it even more complex given the larger number of references managed (subtypes) and the number of possibilities when assigning supply (Alemany *et al.* 2013a). Thus in a real situation, finding not only an optimal solution, but also a feasible one to the reallocation problem is very hard given the huge volume of committed orders, each with several order lines, different subtypes and homogeneous sublots. The complexity of the shortage problem under study justifies the use of mathematical programming models. In order to find a satisfactory solution for both customers and the SC, some shortage planning models (Pibernik 2006; Zschorn 2006) have been developed. However, they do not deal with any LHP characteristic. In this research work, two mathematical programming models (SP-Model and ASP-Model) are proposed to support decisions about stock and planned lots reallocation among committed orders when a shortage situation occurs due to LHP uncertainty. Dealing with LHP requires modelling new aspects to ensure that customers are served with homogeneous units of the same FG, which is precisely the contribution of the SP-Model and the ASP-Model.

The problem under study can also be considered a Large Complex System (Filip, Leviskä 2009), where human intervention may be required given the large dimensions of this system, the existing constraints in the information structure, and presence of uncertainties. Thus when human intervention is necessary, DSSs can be a solution. One main advantage of DSSs is that the decision maker (DM) does not need to understand the complexities of mathematical modelling (Gomes da Silva *et al.* 2006). For this reason, the above two models were introduced into a DSS that provides the DM with different functionalities in order to deal with the complexity and uncertainty of the system.

DSSs have been developed for different processes such as order planning (Azevedo, Sousa 2000) and order management (Abid *et al.* 2004; Venkatadri *et al.* 2006; Kalantari *et al.* 2011). In line with the process under study herein, Okongwu *et al.* (2012) propose a DSS

that provides different strategic options when disruption in the SC, which leads to stock-out situations, occurs in general contexts that do not address any LHP characteristic. DSSs have been widely used for different purposes and in distinct sectors with LHP for agricultural sustainable development (Kurlavičius 2009), and for master planning in ceramic companies (Mundi *et al.* 2013). Yet, as far as we know, only one DSS has been developed to solve the stock-out situation in LHP contexts (Boza *et al.* 2014), which is also based on mathematical programming models.

The present research work proposes a mathematical programming model (SP-Model), which extends the previous model of Boza *et al.* (2014) by incorporating the following main novel aspects: 1) reallocation of planned production lots in addition to stock among customer orders; 2) consideration of multiple objectives; 3) possibility of late delivery; and 4) LHP modelling by splitting the master plan production lots into homogeneous sublots of different FG subtypes. This SP-Model is incorporated into a DSS to enable ceramic DMs the optimal reallocation of real and planned FG availability to already committed orders if a shortage situation occurs. Different DSS functionalities help the DM select the customer orders to be served among already committed ones based on the reallocation of existing subtypes in stock and uncertain planned homogeneous sublots. The robustness of each solution is tested in planned and real situations with a DSS functionality based on another mathematical programming model (ASP-Model).

The rest of the paper is structured as follows: in Section 1, the assumptions of the problem under study and a general DSS overview are presented. The mathematical programming models inserted into the DSS are described in Section 2. The DSS architecture and its main user functionalities are detailed in Section 3. Finally, the research conclusions are stated in the last Section.

1. Assumptions and the DSS overview

The main assumptions made in this section when formulating the mathematical models are described. Then before detailing each DSS component, and in order to provide a better understanding, a general overview of the interactions among the main actors involved in this problem is presented.

In the reallocation process, several assumptions, limitations and objectives are taken into account:

- The existence of several production plants with parallel production lines (resources) capable of processing all the FGs and work according to a Make-To-Stock strategy is assumed.
- FG stock quantities are expressed in terms of subtypes. A subtype is defined as the first quality units of the same FG with the same tone and gage.
- The subtype quantities in stock are known with certainty because they have been already manufactured and classified.
- Master plan production lots are divided into homogeneous sublots of different subtypes of uncertain size. The subtypes in the master plan are modelled by considering that

each planned lot is divided into homogeneous sublots expressed by fractions of the lot through beta coefficients. For instance, $\beta1 = 0.6$, $\beta2 = 0.3$ and $\beta3 = 0.1$ mean that a lot in the master plan of 1,000 m² will be divided into three homogeneous sublots whose sizes are 600 m², 300 m² and 100 m², respectively.

– The input customer orders for the reallocation process are all the already committed orders.

– Each customer order proposal has an associated due date that indicates the time bucket when the customer wishes to receive the FGs in the order. It is possible to define a certain delay in the initial promised due date to less important customer orders.

– Each customer order is integrated by several order lines with quantities of different FGs. A customer order is served if, and only if, all its order lines are served.

– Each order line should be served through homogeneous units of the requested FG for the due date of its customer order. To guarantee homogeneity, each order line should be completed from either a unique subtype in the stock or a unique homogeneous sublot.

– Two customer order classes are distinguished:
  – Orders with a shorter due date than the delivery horizon. The delivery horizon is the period length immediately after the current point of time required to prepare any orders to be immediately delivered. This concept helps identify the orders that should be prepared immediately for delivery in order to meet promised customer due dates.
  – Priority orders: in order to represent the variety of customer classes and contractual relationships, the model allows the definition of the priority orders that should be served before any others, regardless of their profit or due date.

– In the reallocation process, three main objectives are pursued: 1) maximisation of the gross margin deriving from the served orders, 2) maximisation of the early orders served; 3) maximisation of the order lines served. If it is not possible to serve all the priority orders or all the orders in the delivery horizon, two other objectives can be defined: 4) maximisation of the priority orders to be served; and 5) maximisation of the orders in the delivery horizon to be served. The DM should assign a weight to each objective so that the sum of them all comes to one.

In a real situation, finding an optimal and feasible solution for the reallocation problem is very hard and time-consuming given the vast volume of committed orders, each with several order lines, and the existence of different subtypes and homogeneous sublots. Therefore, using mathematical programming models with a DSS is an appropriate measure.

A general overview of the interaction among the DSS (its functionalities and the mathematical programming models), the DM and the information flow is shown in Figure 1. The numbers in parentheses on the arrows indicate the usual sequence of the information flow. Through the first DSS functionality, the "SP-Model Configuration", the DM specifies different SP-Model options (1), like weights of each model's objectives, set of priority orders, delay allowed for each order, etc. The SP-Model is solved for each defined configuration (2) and the solutions obtained are reported to the DM along with the SP-Model objective values (3). The DM can compare them and save those solutions considered satisfactory in the candidate solution set (CSS) (4). The second DSS functionality, "Generating Solutions
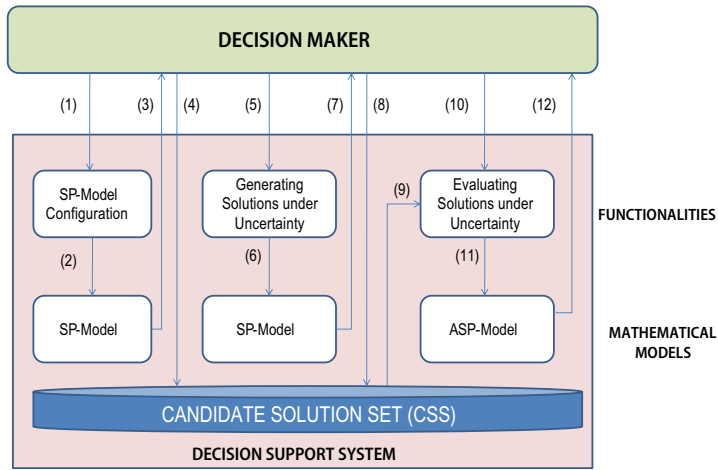
Fig. 1. Interaction among the main actors involved in the LHP reallocation problem

under Uncertainty", allows LHP uncertainty management via definitions of scenarios. The DM can define different scenarios by modifying environmental input data (5) like homogeneity distribution in lots (β coefficients), solve them by the SP-Model (6), and compare different solutions based on the SP-Model objectives (7) in order to save additional ones in the CSS (8). At this point, different "satisfactory" solutions will have been generated and saved in the CSS. However, the DM should choose only one solution to be implemented, but he/she does not know which scenario will finally occur and, therefore, what the real values of the uncertainty data will be. So the DM might be interested in knowing what will happen if the optimal solution of "scenario i" is implemented and if "scenario j" finally occurs. Through the last functionality, "Evaluating Solutions under Uncertainty", the DM chooses (9, 10) which CSS solutions are evaluated in which scenarios. The DSS passes the value of some of the SP-Model decision variables (e.g., served orders) selected by the DM to the ASP-Model (11), which determines the exact orders to be really served when different scenarios or projections of reality occur by deriving the corresponding values achieved for the five objectives in each situation (12). If the changes in the new objective values are minor, the solution will be robust, otherwise it will very sensitive to changes in input data. Finally, the DM should choose one CSS solution to be implemented by taking into account not only the performance (quality) of the different objectives obtained by the SP-Model, but also their robustness (dispersion in their value), as assessed by the ASP-Model. In the following section, the two models integrated into the DSS are described.

2. DSS mathematical programming models

Two MILP deterministic models integrate the DSS: the Shortage Planning Model (SP-Model) and the Auxiliary SP-Model. The following subsections describe them both.

2.1. Shortage Planning Model (SP-Model)

The SP-Model should be at least executed every time customer orders are prepared for delivery, and when discrepancies between planned and real lots appear in homogeneity and/or in total input quantities. The SP-Model solution provides the DM with already committed orders, which can be served after the reallocation procedure, and their real delivery date. For served orders, the delivery date equals the promised due date if no delay is allowed. For each served order line, the solution indicates if it is served from either the LHP stock or from the planned homogeneous sublots. If it is reserved from stock, the specific subtype of the product from the current LHP stock used is also reported. The SP-Model computes not only the remaining current inventory of each subtype, but also the uncommitted quantities for each FG in the planned sublots that have not been reserved for any order after reallocation; i.e., homogeneous real and planned available to promise quantities (ATP-LHP), respectively, that will be used later as input for the order promising process. The formulation of the mixed integer linear programming model for shortage planning in LHP manufacturing contexts is described below.

2.1.1. Nomenclature

Table 1 shows the general model data: indices, sets and parameters. Table 2 provides the set of decision variables.

Table 1. Model data

Indices	
o	Committed customer orders.
i	Finished goods (FGs) required in the considered committed orders.
b	Existing subtypes of all the FGs in committed orders.
p	Production plants.
l	Production lines in production plants (productive resources).
t	Time buckets.
β	Homogeneous sublots into which a planned production lot is divided.
k	Objective of the reallocation procedure.
Sets	
Os(o)	Set of committed customer orders in the planning horizon.
Ohe(o)	Set of committed orders with a delivery date shorter than or equal to the delivery horizon (i.e., committed due date for customer order<=delivery horizon).
Op(o)	Set of committed high priority orders that should be assigned homogeneous quantities.
Osi(i)	Subset of committed customer orders from *Os(o)* requesting some quantity of FG *i*
Bi(i,b)	Existing subtypes *b* in the actual stock belonging to FG *i*.
I	Set of all the FGs *i* requested in the customer orders of *Os(o.)*
I(o)	Set of the FGs *i* requested in committed customer order *o*.
IL(l)	Set of FGs *i* that can be processed on production line *l*.
Lp(p)	Set of manufacturing lines *l* that belong to production plant *p*.

Parameters	
H	Planning horizon.
he	Delivery horizon.
dd_o	Committed due date for customer order o.
q_{io}	Requested quantity of FG i in customer order proposal o.
ns_o	Number of order lines (FGs) in customer order proposal o.
p_o	Per unit profit of order o.
hc_{io}	Inventory holding costs of quantity q_{io} per time period.
bc_o	Backlogging cost of customer order o per delayed time period.
$rmax_o$	Maximum delay allowed for customer order o in relation to the committed due date (expressed as an integer number of time period length t). It is assumed that $rmax_o \leq T - dd_o$.
nop	Number of priority committed orders (i.e., belonging to set $Op(o)$).
nhe	Number of committed orders with a committed due date in the delivery horizon (i.e., belonging to set $Ohe(o)$)
$stock_{ib}$	Total available stock of subtype b of FG i.
mps_{iplt}	Planned quantity of FG i to be produced on manufacturing line l belonging to production plant p which becomes available during period t (defined in the master production schedule (mps)).
huc_i	Inventory holding cost per unit of FG i and time period.
bci_i	Backlogging cost per unit of FG i and delayed time period.
B_{ilpt}^{β}	Fraction β of each planned lot of FG i produced on manufacturing line l belonging to production plant p which becomes available during period t that can be considered homogeneous. Through these coefficients, splitting lots into homogeneous sublots is modelled, and which depend on the FG, production line and time period. These coefficients are not independent because their total sum should equal to 1 ($\sum_{\beta} B_{ilpt}^{\beta} = 1$).
σ_k	Weight provided by the DM to the k-th objective ($0 \leq \sigma_k \leq 1$; $\sum_k \sigma_k = 1$).

Table 2. Decision variables

AD_{io}	Number of time periods before due date dd_o which is assigned either a planned lot or a stock quantity of FG i to customer order o.
$ATP0_{ib}$	The available to promise quantity of subtype b of FG i after the reallocation process
ATP_{ilpt}^{β}	The available to promise quantity of FG i derived from the homogeneous sublot β of the mps_{iplt} after the reallocation process.
ATP_{iplt}	The available to promise quantity of FG i derived from the mps_{iplt} after the reallocation process.
DDF_{io}	Time period during which the quantity q_{io} of FG i of order o is finally reserved.
DDR_o	Actual due date of customer order o (this due date will be the same as dd_o if there is no delay).
RD_{io}	Number of time periods after due date dd_o which is assigned either an mps or a stock quantity of FG i during period t to customer order o.
RDT_o	Number of time periods after due date dd_o when customer order o is served.

$U0_{iob}$	Binary variable with a value of 1 if the requested quantity of FG i in customer order o (q_{io}) is completely served by $atp0_{ib}$, and a value of 0 otherwise.
UB^{β}_{ioplt}	Binary variable with a value of 1 if the requested quantity of FG i in customer order o (q_{io}) is completely served by the homogeneous sublot β of the mps_{iplt}, and a value of 0 otherwise.
U_{ioplt}	Binary variable with a value of 1 if the requested quantity of FG i in customer order proposal o (q_{io}) is completely served by the mps_{iplt} lot, and a value of 0 otherwise.
US_{io}	Binary variable with a value of 1 if order line with FG I in customer order o is served, and a value of 0 otherwise.
UST_{o}	Binary variable with a value of 1 if customer order o is served, and a value of 0 otherwise.
YA_{io}	Binary variable with a value of 1 if the requested quantity of FG i in customer order o is reserved and the ATP quantity before its due date (i.e., $AD_{io}>0$).
YR_{io}	Binary variable with a value of 1 if the requested quantity of FG i in customer order o is reserved and the ATP quantity after its due date (i.e., $RD_{io}>0$).

2.1.2. Objectives

In the reallocation procedure, considering the optimisation of five objectives is possible. The first objective (1) aims to maximise the gross margin of the served customer orders calculated as the difference of the incomings derived from the profits of the served orders, minus the costs of serving the customer order with delay and the holding costs of reserving the quantities of order lines prior to the actual due date:

$$Max\left[Z1\right] = \sum_{o\in Os(o)} p_o \, ^\star UST_o - \left(\sum_{o\in Os(o)} bc_o \, ^\star RDT_o + \sum_{o\in Os(o)}\sum_{i\in I(o)} hc_{io}(DDR_o - DDF_{io})\right). \quad (1)$$

The second objective (2) attempts to maximise the number of orders served with earlier delivery dates, which is the equivalent to maximising the total sum of the difference between the allocation horizon (T) and the delivery date: as T is a fixed quantity, the lower dd_o, the greater $T-dd_o$ difference. Based on this objective, priority is given to those orders with earlier delivery dates despite their associated profits because it is assumed that there is more flexibility for those with later delivery dates to find other solutions to serve them (i.e., modifying the master plan with additional production lots). In order to avoid the SP-Model not serving those orders with maximum delivery dates ($dd_o = T$), because their contribution to the max-imisation of [Z2] is zero ($T-dd_o = 0$), parameter ε (a positive value that is lower than the unit) is used to induce the model to serve as many orders as possible. Through parameter ε, the above orders also contribute to the maximisation of [Z2]. So if there is enough LHP stock and planned lots, the solution chooses to serve them ($UST_o = 1$).

$$Max\left[Z2\right] = \sum_{o\in Os(o)} ((T - dd_o + \varepsilon)\, ^\star UST_o). \quad (2)$$

The third objective (3) intends to maximise the total number of order lines served. With it, priority is given to customer orders with more order lines because they are more difficult

to serve. This is because a specific customer order is not served unless all its order lines are completely fulfilled.

$$Max[Z3] = \sum_{o \in Os(o)} (ns_o * UST_o). \tag{3}$$

As described in the following section, the SP Model is always feasible unless it is not possible to serve all the priority orders (Constraint 23) or all the orders in the delivery horizon (Constraint 24). In these situations, it makes sense to contemplate objective (4) and objective (5), respectively.

The fourth objective (4) is defined in order to maximise the number of priority orders to be served. Sometimes, companies give more importance to certain orders, irrespectively of the profit they make. For instance, a company can distinguish between orders with and without priority depending on whether they come from a preferred customer or not, respectively. This objective makes sense when Constraint (23) is removed from the model because not all priority orders can be served.

$$Max[Z4] = \sum_{o \in Op(o)} UST_o. \tag{4}$$

Objective (5) attempts to maximise the number of orders in the delivery horizon to be served. This objective makes sense when Constraint (24) is removed from the model because not all the orders in the delivery horizon can be served due to there is not enough supply to serve them all. The idea is that the reallocation procedure releases the stock reservation for more profitable orders, but with far due dates, in order to assign it to orders with immediate due dates. The reason for this is that it might be the only solution for the orders in the delivery horizon, while it is possible to seek other alternatives for the rest orders, such as master plan modification.

$$Max[Z5] = \sum_{o \in Ohe(o)} UST_o. \tag{5}$$

In the literature, multiple attribute decision making (MADM) and multiple objective decision making (MODM) are two basic multicriteria decision-making approaches (MCDM). MCDM refers to making decisions in the presence of multiple, numerous, and usually, conflicting objectives, which often involve numbers of criteria (Abdullah, Rabiatul 2014), but not always. In our particular case and, depending on the reallocation problem data, the different objectives can be conflictive. When defining the various objectives, our aim was to find a solution that balances them all if this conflict should emerge. For instance, the DM's main objective might be to maximise profit [Z1]. At the same time however, the DM may wish to serve orders with earlier delivery dates [Z2] regardless of their profits because it is more difficult to find alternative solutions to serve them if homogeneous availability is not enough. The same situation is valid for maximising the total number of order lines served [Z3]: for instance, for the same profit, the DM may prefer to serve orders with many lines because it is more difficult to complete the order. An analogous argument is valid for [Z4] and [Z5].

The model contemplates the possibility of integrating all the objectives into a single one by the simple additive weighting (SAW) method (Turskis, Zavadskas 2011), which consists in the sum of the scaled five objectives according to weights σ_1, σ_2, σ_3, σ_4 and σ_5. For our particular case, the five objectives should be maximised. Therefore, in order to obtain the normalisation of the five objectives [Z1], [Z2], [Z3], [Z4] and [Z5], in a range between [0,1], each one is divided by its maximum possible value (see Equation 6): the sum of the profits of all orders for [Z1], all the committed orders belonging to the delivery horizon for [Z2], the sum of all the order lines for [Z3], all the priority orders for [Z4] and all orders in the delivery horizon for [Z5].

The SAW method provides an evaluation score [Z] for each solution by multiplying the scaled value given to the alternative of the objective with the relative importance weights assigned directly by the DM, and then summing the products for all the criteria. Apart from its simplicity, the advantage of the SAW method is that it is a proportional linear transform-ation of raw data. This means that the relative order of magnitude of the standardised scores remains equal (Abdullah, Rabiatul 2014). Weights are assigned by the DM in such a way that the heavier the weight, the greater the importance. However, since these weights might be imprecise, the first DSS functionality was designed to support the DM in defining them by assessing the change in the five objectives when modifying their weights (see Section 3.1):

$$Max\left[Z\right] = \sigma_1 \frac{Z1}{\sum_{o\in Os(o)} p_o} + \sigma_2 \frac{Z2}{T^{\star}\left|Os(o)\right|} + \sigma_3 \frac{Z3}{\sum_{o\in Os(o)} ns_o} + \sigma_4 \frac{Z4}{\left|Op(o)\right|} + \sigma_5 \frac{Z5}{\left|Ohe(o)\right|}. \tag{6}$$

2.1.3. Constraints

One contribution of the present model is that it allows customer order homogeneity re-quirements to be managed. To ensure that customers are served with homogeneous units of the same FG, the present model does not allow an order line to be served from different subtypes or from different homogeneous sublots and time periods. This aspect is modelled through Constraints (7) to (10).

Constraint (7) calculates the existing uncommitted quantity of each subtype b of FG i ($ATP0_{ib}$) by subtracting the quantity assigned to the different order lines of the customer orders that are finally served ($UO_{iob} = 1$) from the existing subtype stock:

$$ATP0_{ib} = stock_{ib} - \sum_{o\in Osi(i)} q_{io} {}^{\star} UO_{iob}, \quad \forall i \in I, b \in B(i). \tag{7}$$

Constraint (8) is analogous to Constraint (7), except for the planned homogeneous sublots in the master production schedule. Through the B_{ilpt}^{β} coefficients in Constraint (8), the homogeneous sublots that are available in the master production schedule (mpsiplt) are anticipated. At this point, it is not necessary to anticipate the specific subtype of each sublot because customers only require homogeneity in the order lines and they do not specify the required subtype. Furthermore, Constraint (8) calculates the remaining homogeneous

quantities of each sublot that are not assigned to any customer order ($ATPB_{iplt}^{\beta}$), which are the input to the order promising process.

$$ATP_{ilpt}^{\beta} = B_{ilpt}^{\beta} * mps_{iplt} - \sum_{o \in Osi(i)} q_{io} * UB_{ioplt}^{\beta}, \quad \forall p, l \in Lp(p), i \in IL(l), \beta, t. \tag{8}$$

Constraint (9) forces a customer order line to be served from only one specific homogeneous sublot, should the order be finally served from the mps quantities.

$$U_{ioplt} = \sum_{\beta} UB_{ioplt}^{\beta}, \quad \forall p, l \in Lp(p), o \in Os(o), i \in I(o), t. \tag{9}$$

Constraint (10) establishes that the order line of FG i belonging to order o is served if it is reserved with a single subtype (real or planned), otherwise it is not served.

$$\sum_{b \in B(i)} UO_{iob} + \sum_{p,l,t} U_{ioplt} = US_{io}, \quad \forall o \in Os(o), i \in I(o). \tag{10}$$

Constraints (7) to (10) ensure homogeneity in the reserved units. It is important to highlight that even when all the units of the same lot are homogeneous, the model considers that the lots manufactured in different resources and time periods are not likely to be homogeneous. For this reason, the model forces a customer order line to be served with only one subtype in stock or with units from only one lot (10).

Constraint (11) indicates that for order o to be served, it is necessary for all its order lines i to be served. Constraint (11) acts in the opposite way; that is, if the order is not served, it is senseless to reserve any of its order lines independently.

$$\sum_{i \in I(o)} US_{io} = ns_o * UST_o, \quad \forall o \in Os(o). \tag{11}$$

Constraint (12) calculates the delay (RD_{io}) or the advance (AD_{io}) of reserving the requested amount of each order line of FG i in relation to the committed due date of order o (dd_o). If the order is not served, then none of its lines is served given Constraint (11) and, consequently, neither delays nor advances are calculated. When an order line of FG i in order o is served from the stock ($stock_{ib}$), that is $t = 0$, then the advance is dd_o.

$$AD_{io} - RD_{io} = dd_o * UST_o - \sum_{p,l,t} U_{ioplt} * t, \quad \forall o \in Os(o), i \in I(o). \tag{12}$$

Constraint (13) indicates that the advance cannot be longer than the due date and forces the associated binary variable to take a value of 1 if an advance exists. Besides, Constraint (14) obliges binary variable YA_{io} to be 0 when there is no advance.

$$AD_{io} \le dd_o * YA_{io}, \quad \forall o \in Os(o), i \in I(o); \tag{13}$$

$$YA_{io} \le AD_{io}, \quad \forall o \in Os(o), i \in I(o). \tag{14}$$

Constraint (15) indicates that the final order line delay cannot be longer than the maximum permitted for this order ($rmax_o$). As observed, this parameter depends on the order, and it is possible to define it for each order according to the importance the company attaches to it. Simultaneously if there is a delay to an order, associated binary variable YR_{io} takes a value of 1. Constraint (16) makes the binary variable YR_{io} take a value of 0 if there is no delay.

$$RD_{io} \leq rmax_o * YR_{io}, \quad \forall o \in Os(o), i \in I(o); \tag{15}$$

$$YR_{io} \leq RD_{io}, \quad \forall o \in Os(o), i \in I(o). \tag{16}$$

Constraint (17) is employed to ensure that there is a delay or an advance, or neither, in the delivery of an FG i in a specific order, but never both at the same time.

$$YA_{io} + YR_{io} \leq 1, \quad \forall o \in Os(o), i \in I(o). \tag{17}$$

Constraint (18) establishes that the delay in order o equals the maximum delay of the order lines composing it because the order cannot be served until all the order lines are reserved. Constraint (19) ensures the impossibility of delaying order o if any of its order lines are delayed. Constraint (20) forces a situation in which a delay in an order cannot exceed the maximum delay established for this order (should this order be served). If the maximum delay permitted is equal to 0 for all the orders, this is a specific case in which serving with delays is not allowed.

$$RD_{io} \leq RDT_o, \quad \forall o \in Os(o), i \in I(o); \tag{18}$$

$$RDT_o \leq \sum_{i \in I(o)} RD_{io}, \quad \forall o \in Os(o); \tag{19}$$

$$RDT_o \leq rmax_o * UST_o, \quad \forall o \in Os(o). \tag{20}$$

Constraint (21) defines the real date (DDR_o) on which order o is to be delivered, which is the due date plus the delay in order o. Through Constraint (22), the real date of the reservation of order line i of committed customer order $o$is defined. The difference between the order delivery date (DDR_o) and the reservation date (DDR_o-DDF_{io}) provides the number of time periods during which the order line quantity (q_{io}) is stored, which allows a precise calculation of the holding costs.

$$DDR_o = dd_o * UST_o + RDT_o, \quad \forall o \in Os(o); \tag{21}$$

$$DDF_{io} = dd_o * UST_o - AD_{io} + RD_{io}, \quad \forall o \in Os(o), i \in I(o). \tag{22}$$

Irrespectively of the order due date and/or profit, the company may wish the strategic orders to be served with the actual stock and the future master plan quantities. Constraint (23) ensures that all the defined priority orders are served.

$$\sum_{o \in OP(o)} UST_o = nop. \tag{23}$$

Constraint (24) forces orders with due dates in the delivery horizon to be served. That is, priority is given to assign the current available LHP quantities for these orders, because otherwise, an inevitable delay occurs.

$$\sum_{o \in Ohe(o)} UST_o = nhe. \tag{24}$$

It is worth stressing that the model may prove infeasible because of the two Constraints above (23 and 24). In this case, the DM is immediately informed about the impossibility of serving all the orders in the delivery horizon and the priority ones with the actual homogeneous supply. To achieve a feasible solution, either Constraint (23) is removed from the model or the company should reconsider priority orders. It is possible that, even after removing Constraint (23), the model remains infeasible, which means that there is no possibility of serving all the orders in the delivery horizon. As before, Constraint (24) should be eliminated or the orders in the delivery horizon should be redefined.

Finally, Constraint (25) defines the nature of the variables.

$$ATP0_{ib}, ATP_{ilpt}^{\beta}, ATP_{iplt}, \quad \text{continuous;}$$

$$AD_{io}, DDF_{io}, DDR_o, RD_{io}, RDT_o, \quad \text{integer;}$$

$$U0_{iob}, UB_{ioplt}^{\beta}, U_{ioplt}, US_{io}, UST_o, YA_{io}, YR_{io}, \quad \text{binary.} \tag{25}$$

2.2. Auxiliary Shortage Planning Model (ASP-Model)

The reallocation problem is deterministically solved by the SP-Model. Inherent LHP uncertainty is approached by the designed DSS user functionalities. Based on these functionalities, the DM can generate several solutions to the problem with different scenarios. The role of the Auxiliary Shortage Planning Model (ASP-Model) is to evaluate the quality and robustness of a specific SP-Model solution, that is optimal for a specific scenario, either if other scenarios occur or the data of the initial scenario used for solving the SP-Model are disturbed due to LHP uncertainty (projections of reality). For instance, the DM can estimate, based on historical data, that the percentages of the homogeneous sublots in a lot are 0.7, 0.2 and 0.1, and can solve the SP-Model with this information (scenario 1). However, the DM might be interested in knowing what the values of the different objectives are if the real homogeneous sublots are in the 0.6, 0.3 and 0.1 proportions (scenario 2) instead of the initially planned one, and if the optimal solution of scenario 1 is implemented. In doing so, part of the optimal SP-Model solution in scenario 1 (values of the decision variables) can be passed as the input data for the ASP-Model, along with the scenario 2 data. The ASP-Model solution will provide real orders, which can be finally served from the initial ones, as well as the new values of the objectives achieved. If the new values of the objectives of a specific solution differ slightly from the initial ones in different scenarios, it can be stated that the solution is robust.

As the following section 3 describes, the DSS allows the DM to edit and modify the ASP-Model according to the decisions chosen to be evaluated. For example, let's assume that the DM needs to know what orders can be really served from those selected by the SP-Model if the input data for homogeneous sublots are 0.6, 0.3 and 0.1 instead of 0.7, 0.2

and 0.1. Then new data, consisting in the value of the orders to be served from the SP-Model ($USTsp_o$) and a new Constraint (26), should be added to the ASP Model for each order o. Through this Constraint (26), the ASP-Model may not serve ($UST_o = 0$) the previously committed orders by the SP-Model ($USTsp_o = 1$) given the impossibility of accomplishing the homogeneity requirements because of the variation in the homogeneous sublots. It might also be impossible to serve those orders that were not previously chosen to be served by the SP-Model ($USTsp_o = 0$).

$$UST_o \leq USTsp_o, \quad \forall o \in Os(o). \tag{26}$$

If, in addition, the DM wishes to maintain the allocation of subtypes and lots to promised customer orders (i.e., not allowing reallocation), the value of the decision variables specifying the subtype ($UOsp_{iob}$) or sublot from which to serve the order Usp_{ioplt} should be passed to the ASP Model. Then Constraints (27) and (28) should be added to the ASP-Model. In Section 3, the inclusion of the ASP-Model in the DSS and its use are further described.

$$UO_{iob} \leq UOsp_{iob}, \quad \forall o \in Os(o), i \in I(o), b \in B(i); \tag{27}$$

$$U_{ioplt} \leq Usp_{ioplt}, \quad \forall p, l \in Lp(p), o \in Os(o), i \in I(o), t. \tag{28}$$

2.3. Model validation

In this section, the SP-Model is used to solve the reallocation problem of a ceramic company. The obtained results reveal model validity and provide some insights into its resolution aspects. The SP-Model input data of this case study were obtained from a ceramic tile company, but they were partially modified for confidentiality reasons. This case was also used as an example to illustrate the DSS functionalities of Section 3.

In ceramic companies, units of the same first quality FGs (subtypes) can differ in tone (degree of colour) and gage (thickness). Customers require these attributes to be homogeneous among the units of the same FG in the same order because ceramic pavings and coverings are placed and presented together.

The present case study considers a planning horizon of 31 periods, and a set of 400 committed orders with 41 priority orders and with 24 orders in the delivery horizon. The orders in the delivery horizon are all those whose delivery date are the first period of the planning horizon. Ten FGs, produced by two plants following a Make-To-Stock manufacturing strategy, are considered. The first production plant has two production lines, while the other has just one. Finally, each FG can be classified into three subtypes.

The characteristics of the 400 orders entering the model were generated according to the historical data provided by the company. Of all the company's orders, 99.5% are composed of 1 to 10 lines. Therefore in our example, each order was randomly assigned a number of order lines, from 1 to 10, which gives 945 order lines. The sale prices (pvp_i), backlog costs (bc_i) and inventory holding costs (huc_i) per m² appear in Table 3. Parameters p_o and bc_o for each order were calculated by multiplying quantity q_{io} by the corresponding unitary parameter pvp_i and bci, respectively. Then all the FGs i included in the customer order proposal were summed.

Table 3. Selling prices and costs for each FG

Finished Good	Selling price €, $pvp(i)$	Backlogging cost, $bci(i)$	Holding cost, $huc(i)$
FG1	7	0.75	0.064
FG2	18	0.65	0.052
FG3	12	0.5	0.04
FG4	10	0.45	0.036
FG5	5	0.45	0.036
FG6	11	0.7	0.052
FG7	13	0.65	0.04
FG8	12	0.5	0.036
FG9	6	0.5	0.052
FG10	15	0.45	0.045

Generation of supply (initial stock and mps quantities) is based on previously defined orders. As the SP-Model deals with shortage situations, supply was defined in such a way that only 70% of the committed quantities are covered. For this example, the probable scenario as regards the distribution of the homogeneous sublots was considered ($\beta 1 = 0.2$, $\beta 2 = 0.8$, $\beta 3 = 0$).

With all the above data, the SP-Model was executed. A PC was used with an "Inter(R) Core™ i5 CPU 650 @ 3.20GHz processor" and 4 GB of Ram memory. The SP-Model was translated into a computer readable form by the MPL v4.2, data stored in Access2010. The GUROBI 5.0.2.solver was used.

The SP-Model execution provides an infeasible solution when considering Constraints (23) and/or (24), which means that there is not enough supply to serve all the priority orders and/or orders in the delivery horizon. Hence, in order to obtain a feasible solution, both constraints were removed and the SP-Model was re-solved by considering the weights (0.5, 0.2, 0.1, 0.1, 0.1) for the five objectives ($Z1$, $Z2$, $Z3$, $Z4$, $Z5$), respectively. The results for each individual objective are found in Table 4: a total profit of €806,925.49 was made from the served orders, which summed 5,148,367 periods; 861 of 945 committed order lines were completed, 37 of 41 priority orders and 23 orders in the delivery horizon of the 24 existing ones were served. Obviously, the definition of the different weights for each objective provides different solutions and, therefore, different values for each objective. Section 3 describes how the DSS can assist the DM in the hard task of assigning weights to each objective in a logical and partially automated manner.

Table 4. Values of the five objectives

Weights for each objective function	Z1 (€)	Z2 (periods)	Z3 (orderlines)	Z4 (priority orders)	Z5 (orders in delivery horizon)
(0.5, 0.2, 0.1, 0.1, 0.1)	806925.49	5148.367	861	37	23

Table 5 provides information about the SP-Model size, its solution time and quality. As seen, the model achieved a good solution with a GAP of 0.535% from the optimal one in only 240 sec. However, achieving a GAP nearer to zero (0.170%) was much more time-consuming

(3 hours, 45 min and 8 sec). As the solution time was severely affected by the admissible GAP, the DSS allowed the DM to define either the quality of the solution to be obtained by specifying the minimum GAP or the maximum solution time (see Section 3).

Table 5. Solution characteristics

Constraints	Integer Var.	Binary Var.	Continous Var.	Mean Solution Time (sec)	GAP
398024	12800	1668400	4140	240	0.535

3. The Model-Driven DSS architecture

The architecture of the Model-Driven DSS includes dialogue, modelling and data components (Fig. 2). Java v7 and the ECLIPSE platform were used to develop dialogue components. MPL v4.2 was selected to translate the mathematical programming models into a readable machine format. The GUROBI5.0.2. solver was chosen for the contrasted quality of the solutions obtained. An Access database was used to store the corresponding data. All the solution tables and results of the DSS can be exported to an .xls or .txt file to confer the DM more flexibility when analysing the results.

The DSS was designed to provide the DM with a variety of functionalities to facilitate the generation of multiple solutions and the selection of the most satisfactory one after considering the uncertainty of the environment. The two first DSS functionalities are based on the SP-Model and allow the generation of multiple solutions by modifying model options and defining scenarios, respectively. The third DSS functionality evaluates the quality and
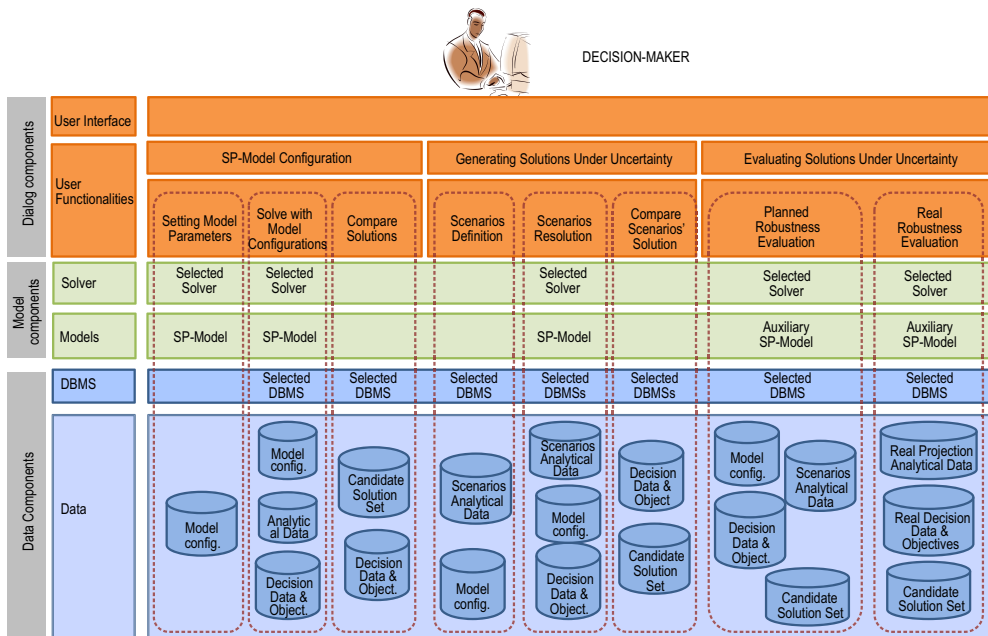

Fig. 2. The Model-Driven DSS architecture

robustness of the previous solutions and compares them according to the ASP-Model. While seeking the optimal or the most satisfactory solution, the DM can add or remove solutions to the candidate solution set (CSS). The CSS contains solutions that are interesting for the DM and are, therefore, candidates to be finally chosen to be implemented. Before choosing the final solution, it might be necessary to solve the mathematical models as many times that the DM wishes. However as presented in Subsection 2.3, slightly improving the quality of a solution (i.e., reducing the GAP) can prove very time-consuming. Thus when solving MILP models, the DSS allows the DM to adjust solver parameters, such as GAP (quality) and solution time, among others, using the options button. The following paragraphs describe the DSS functionalities according to the case presented in Subsection 2.3.

3.1. SP-Model configuration

Through this DSS functionality, the DM can generate multiple solutions to the reallocation problem by varying some model parameters that do not depend on the environment (uncertainty), and which the DM has complete control over. To do this, the DM should define the different model configurations, solve the SP-Model as the same number of times as configurations defined, and compare the obtained solutions in order to choose the most satisfactory ones to be saved in the CSS.

a) Setting model parameters:

The DM must choose the analytical data input into the model using the "Select DB" option. If a shortage situation occurs, the SP-Model allows the reallocation of the available supply to previously committed orders in different ways, which must be selected by the DM within this functionality (Fig. 2): a) if reallocation is to be made by considering only the available stock and/or the planned lots in the master production schedule; b) if a delay is allowed when serving orders; c) if Constraints (23) and (24) are active or not (and therefore, if the corresponding objectives [$Z4$] and [$Z5$] are optimised); and d) the weight assigned by the DM to each objective. Assigning weights to objectives can be an extremely hard task. Usually it is a case of the heavier the weight assigned to an objective, the greater the importance that the DM attaches to it. However, these weights are often imprecise and vague for the DM, so different values are admissible. The DSS can assist the DM in selecting these weights when setting model parameters. By this functionality, the DM can define multiple combinations for the weights of the five objectives and save them. Then the SP-Model is solved for each combination of weights and the resulting solutions are compared based on the value of the five objectives. Through this DSS functionality, it is possible to assess the sensitivity of the solutions to the weights of the previously configured objectives in a semi-automated manner by comparing the performance of the solutions generated by distinct weights. Furthermore, it is also possible to generate different solutions. In short, different model parameter values (SP-Model Configuration) provide distinct solutions to the reallocation problem. All the SP-Model Configurations saved by the DM are subsequently evaluated.

For our example, the database in Section 2.3 was selected. Figure 3 shows the different options chosen. The reallocation option selected for all the configurations was used to reassign
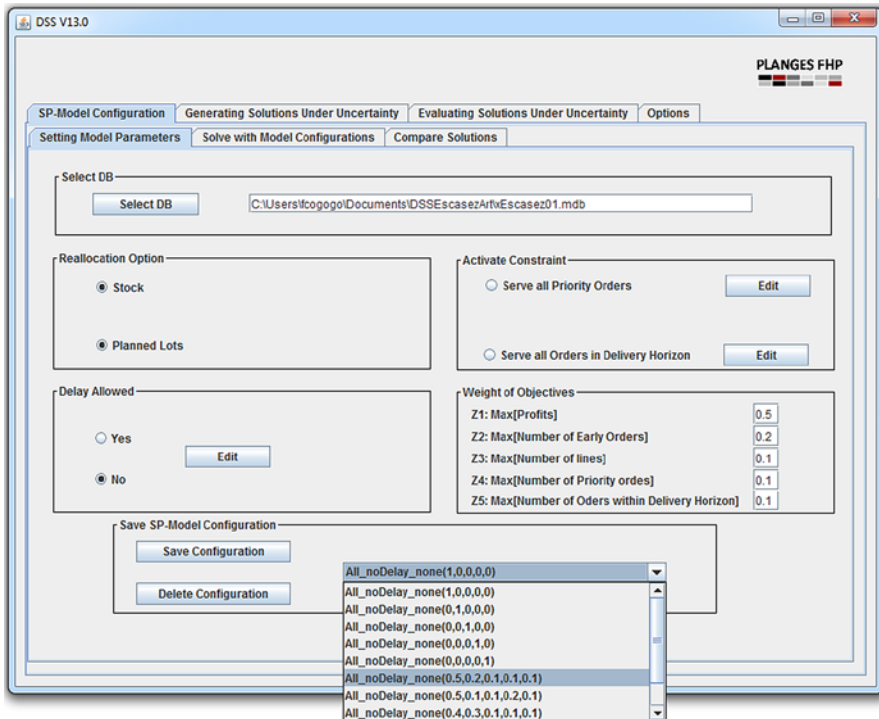
Fig. 3. The "Setting Model Parameters" DSS screen

the stock and planned lots to committed orders (All). Delay was not allowed for any customer order (no Delay). As regards the activate constraint option, the SP-Model was used in an attempt to provide a solution by serving all the priority orders and/or orders in the delivery horizon first. The solution was infeasible for all cases, which means that there was not enough supply to serve all the above customer orders (as in Section 2.3). Therefore, these two constraints were deactivated. Seventeen different weight combinations were defined for the five objectives. Those weights for [$Z4$] and [$Z5$] that differed from zero were defined because the two corresponding constraints were not activated.

    b) Solving with model configurations:

     With the input data and previously configured options, the model is solved the same number of times as configurations saved. For each solution, the DSS provides the DM with the value of each objective, as well as the possibility of obtaining more details about the decision variables (i.e., orders to be served; allocation of stocks and homogeneous sublots to each order). In this step, if the DM deems it suitable, the solutions whose different objectives have acceptable values can be added to the CSS. However as shown below, it is possible to conduct a more profound analysis by comparing the generated solutions before saving them in the CSS.

    c) Comparing solutions:

     The DM can change the model configuration (e.g., weights assigned to each objective) and/or the analytical input data, re-solve the model and analyse its solution as many times

as he/she wishes. Finally, the DM can compare the solutions obtained with different model configurations to choose the most suitable one (Fig. 4). Solutions are compared based on the MACROS defined in the .mpl file which, in this case, coincides with the five objectives (1–5) and the weighted sum of them all (6).

Figure 4 shows the results obtained when solving the SP-Model for the seventeen configurations saved for our example. The best and the worst values for each objective are marked in green and yellow, respectively. As seen, the best values for each objective [Z_i] were achieved when the value of its weight (σ_i) was set to 1 and the others were set to 0. Therefore, the first five rows in Figure 4 provide the variation range of the objectives. For our example, it was assumed that the DM chooses the three following solutions to be saved in the CSS: All_noDelay_none (0.5,0.2,0.1,0.1,0.1), All_noDelay_none (0.5,0.3,0.2,0.0,0.0) and All_noDelay_none (0.4,0.3,0.1,0.1,0.1). Selection was made based on the fact that the different objectives for the three solutions came very close to the maximum values of each objective. In this step, the DM can also remove solutions from the CSS.
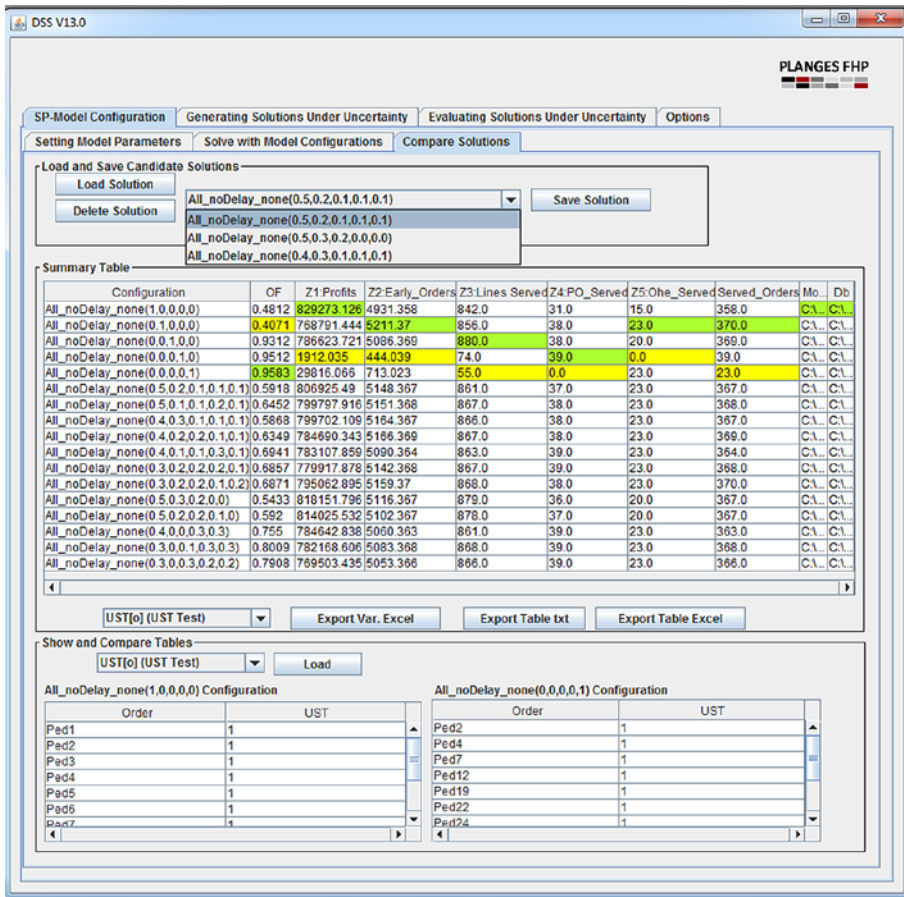
Fig. 4. The "Compare Solutions" screen of the DSS

3.2. Generating solutions under uncertainty

The DSS deals with uncertainty in any model input data (parameters). To do this, the DM can generate different scenarios by changing the input data, and finding the most satisfactory and robust solution for a specific model configuration among all the CSSs.

d) Scenarios definition and resolution:

The DM can define different scenarios to simulate various situations and carry out "what-if" analyses in two ways: 1) by uploading an existing database ("Add DB"/ "Add Scenario") or 2) by modifying any input data from a selected scenario previously loaded by an existing database ("Edit Scenario"). The new scenarios can be saved in the set of scenarios. Then the DM should select the scenarios to be solved from among those generated to be compared. Retrieved databases can be available from the beginning or can be created at any time during the process to be subsequently loaded.

In our example, all the results reported above were generated for the probable scenario ($\beta1 = 0.2$, $\beta2 = 0.8$, $\beta3 = 0$). Due to LHP, these beta parameters were uncertain. Therefore, the DM might be interested in defining different scenarios based on them (Mundi *et al.* 2013): Optimistic scenario ($\beta1 = 1$, $\beta2 = 0$, $\beta3 = 0$), Probable scenario ($\beta1 = 0.2$, $\beta2 = 0.8$, $\beta3 = 0$) and Pessimistic scenario ($\beta1 = 0.1$, $\beta2 = 0.4$, $\beta3 = 0.5$) (Fig. 5). The Optimistic and Pessimistic scenario definitions were made by clicking on the "Add Scenario" button for loading the probable scenario database followed by "Modify Scenario". When modifying a scenario, the DM can edit any table in the database to modify it manually, or to add, subtract or multiply it by a constant. In our case, the table of the beta parameters in the Probable scenario were
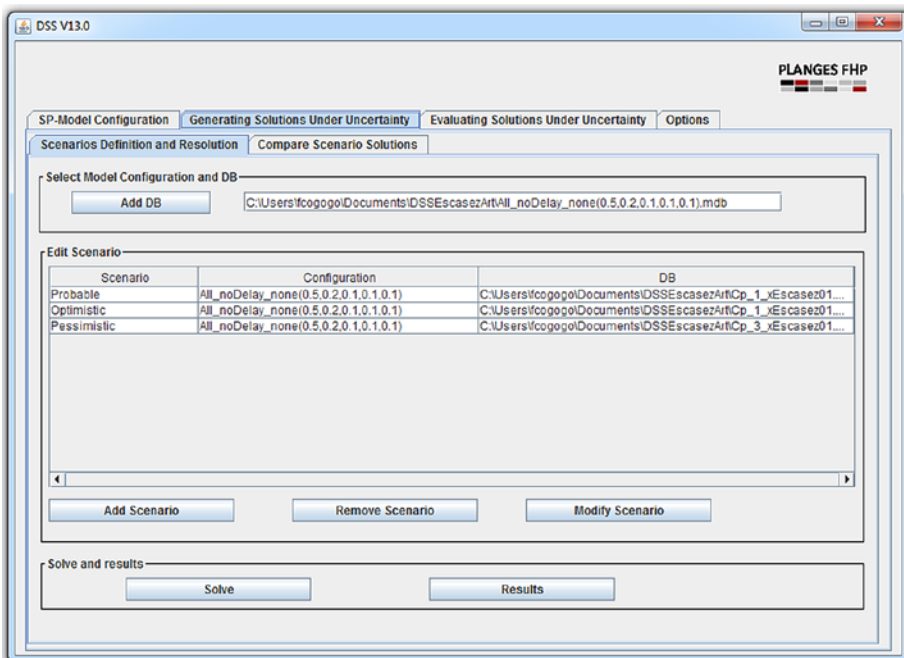

Fig. 5. The "Scenario Definition and Resolution" DSS screen

manually modified twice and saved under the new names Optimistic scenario and Pessimistic scenario. Once saved, all the defined scenarios were solved by the "Solve" button. Their results are shown and compared in the next option.

e) Comparing solutions of scenarios:

For the solution of each scenario (specific input data), the DSS provides the value of the five objectives (1–5) and their weighted sum (6), similarly to the previous DSS functionality: "Comparing Solutions from different Model Configurations". The DM can perform an in-depth analysis of a selected solution with the "Show and Compare Tables" option, which provides a view of the orders to be served and other decision variables. As a result of this analysis, the DM can eliminate solutions (Remove Solution) or add satisfactory ones to the candidate solution set (Save Solution).
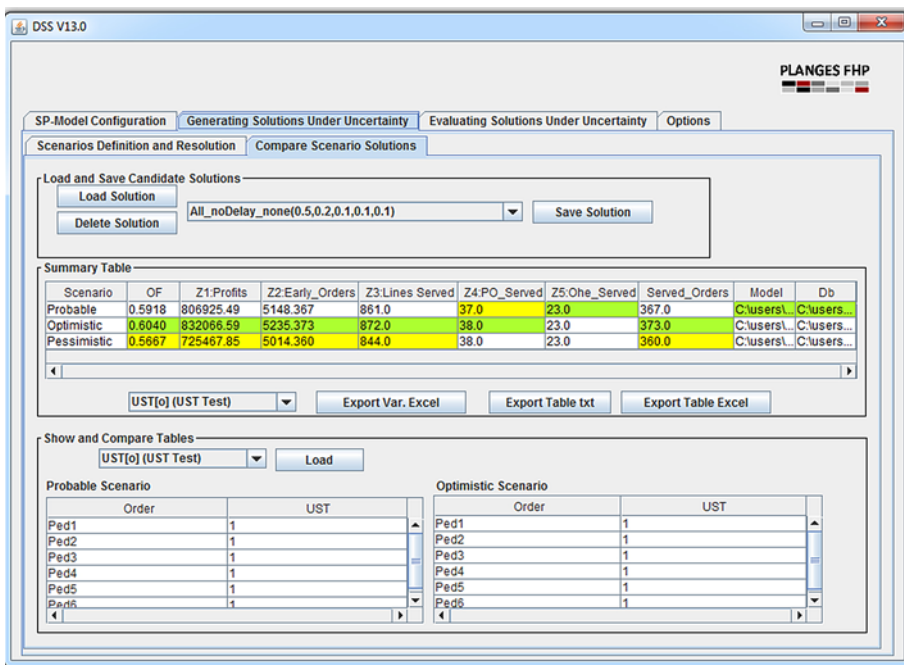
Fig. 6. The "Compare Scenario Solutions" DSS screen

Figure 6 provides the results obtained for the three scenarios of our example. As seen, the best and worst values for the majority of the objectives were achieved for the Optimistic scenario and the Pessimistic scenario, respectively. This means that the greater the number of homogeneous sublots in a lot, the worse the results obtained because fewer orders can be served with homogeneous quantities, and vice versa.

3.3. Evaluating solutions under uncertainty

An evaluation according to the uncertainty of the obtained solutions can be made in a planned or real environment, as described in the following paragraphs.

f) Planned robustness evaluation:

It is worth stressing that for a specific scenario, the values of the five objectives of each solution can be achieved only if the corresponding scenario really occurs and this solution is implemented. Due to uncertainty, the DM does not actually know which scenario will finally occur, and he/she may be interested in evaluating the behaviour of the solutions generated in a specific scenario (Scenario i) in other scenarios (Scenario j). This analysis provides the DM with information about the robustness of each solution (changes in the value of the objectives) when other planned scenarios occur.

To go about this, the DM should select the solutions to be evaluated for planned robustness, as well as the scenarios in which they are to be evaluated, with the "Inputs" option (Fig. 7). The DM should also specify the part of the solution (value of what decision variables) is to be passed as input to the ASP-Model ("Variables to Evaluate" button). To evaluate the decisions in Scenario i in different planned Scenarios j, the ASP-Model is solved using the data corresponding to Scenario j and the selected solution part of Scenario i as input data. With the "Model" option, the ASP-Model can be edited in the .mpl format and adapted according to the part of the solution to be evaluated. For our example, five solutions were chosen to be evaluated in three scenarios (Fig. 7). The value of the decision variable passed to the ASP-Model is the binary variable UST_o, which indicates if an order o is served or not.

Finally, the ASP-Model solution provides the orders served initially from the solution of Scenario i, which can be finally served if Scenario j occurs. The "Analysis" option allows the comparison of the results obtained for the defined performance parameters. As seen in Figure 8, for each solution generated in Scenario i, the real value for the selected
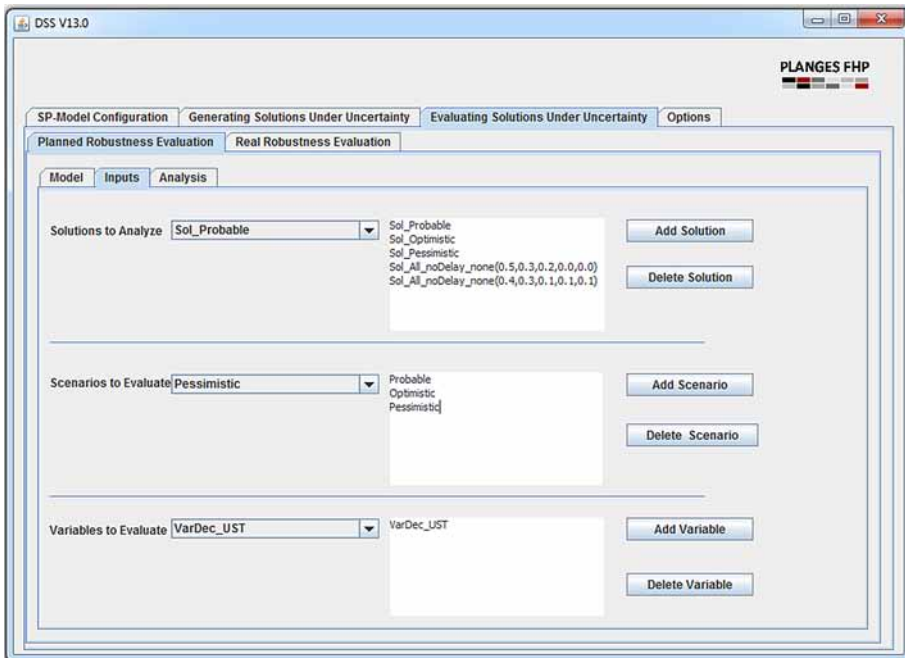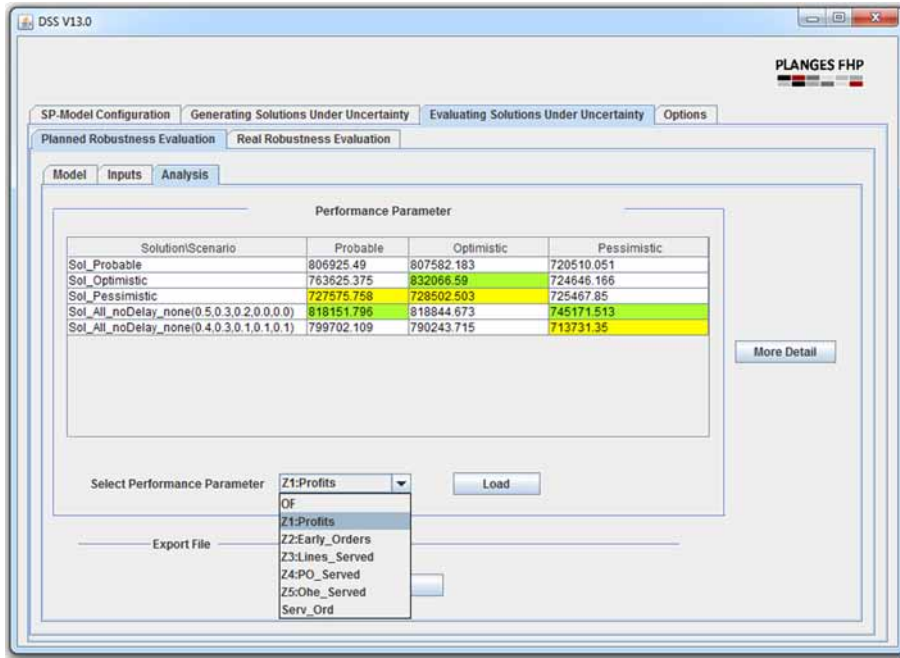

Fig. 7. The "Planned Robustness Evaluation" screen

Fig. 8. The "Analysis" Option for the "Planned Robustness Evaluation" screen

performance parameter (objectives) in other Scenarios j is reported. Green and yellow respectively represent the best and the worst value of the performance parameter for a given scenario. For the specific example shown in Figure 8, the Sol_All_noDelay_none (0.5, 0.3, 0.2, 0, 0) solution seems good for the objective [Z1] (profits of served orders) in all three scenarios compared to other solutions. The DM can analyse the performance of the remaining four objectives by selecting them from the drop-down menu "Select Performance Parameter".

The DSS allows a more detailed comparison and analysis of the performance parameters using the "More Detail" button. With this button, the maximum, minimum and average values of the five objectives are obtained for a specific solution in several scenarios. The DSS can also define the probability of occurrence of each scenario by providing the expected value. From these results, a comparison of different solutions performance and their robustness can be made, which facilitates the choice made by the DM of the most satisfactory one after taking into account inherent LHP uncertainty.

g) Real robustness evaluation:

As the nature of the SP-Model is deterministic, all the parameters are also assumed deterministic. From the above functionality, different scenarios can be defined for those parameters considered uncertain. However, the DM might be interested in knowing the robustness of a specific solution with real projections of uncertainty data. To facilitate this task, the DSS allows the definition of each uncertain parameter, its distribution probability and the number "*n*" of the real projected values of these parameters to be

made. Each projection can be interpreted as a possible real situation in the future; e.g., for the "Probable Scenario", the DM can assume that the beta values are $\beta1 = 0.2$, $\beta2 = 0.8$, $\beta3 = 0$; this means that each lot is divided into two homogeneous sublots with the corresponding proportions. However, although the probable scenario actually occurs, these are mean values, which for some lots will be $\beta1 = 0.27$, $\beta2 = 0.73$, $\beta3 = 0$, and will be $\beta1 = 0.18$, $\beta2 = 0.82$, $\beta3 = 0$ for others. That is, following a specific probability function, beta values are variable for each lot.

The DSS allows the DM to select uncertain parameters, choose the probability distribution among different predefined ones and specify the number "n" of the real projections to be made. Each projection of reality is obtained by generating new values of uncertain parameters based on the probability distribution defined and saved in a new database (similarly to a new scenario). A projection of reality can be assumed to be the possible final values of the beta parameters (portion of the homogeneous sublots in a lot) when lots are produced and classified. For our particular example (Fig. 9), $n = 10$ projections of reality were generated for the beta parameters (number of columns in the table apart from the initial one). To generate the beta values for each lot, a triangular distribution was selected.

Then the planned orders to be served, obtained from running the SP-Model once with the planned betas values and for a specific configuration (i.e., a specific solution), are substituted in the Auxiliary SP-Model. The Auxiliary SP-Model is solved "n" times after considering one of the corresponding "n" real projections of the uncertain parameters to be inputs for each
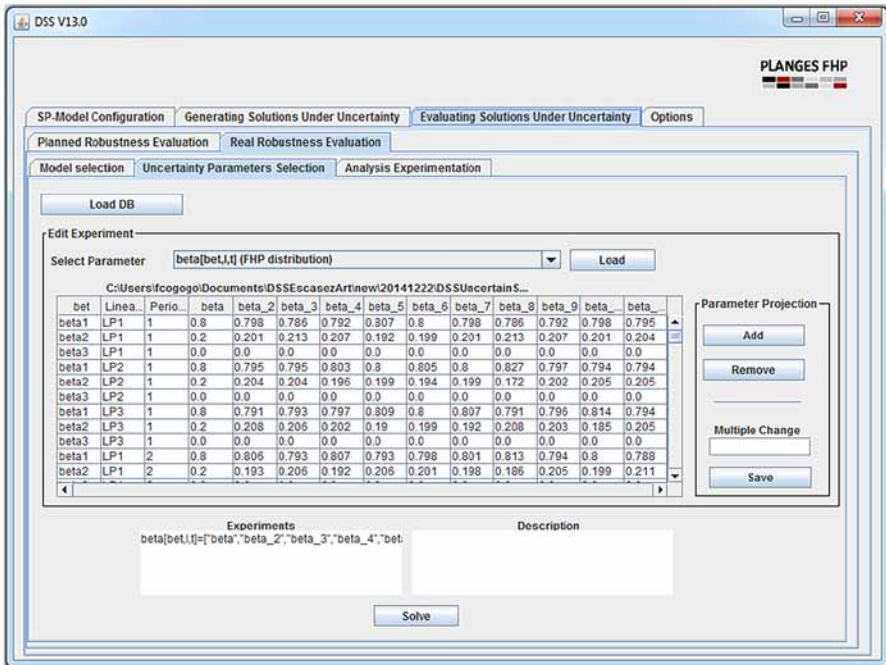

Fig. 9. The "Uncertain Parameters Selection" option for the "Real Robustness Evaluation" DSS screen
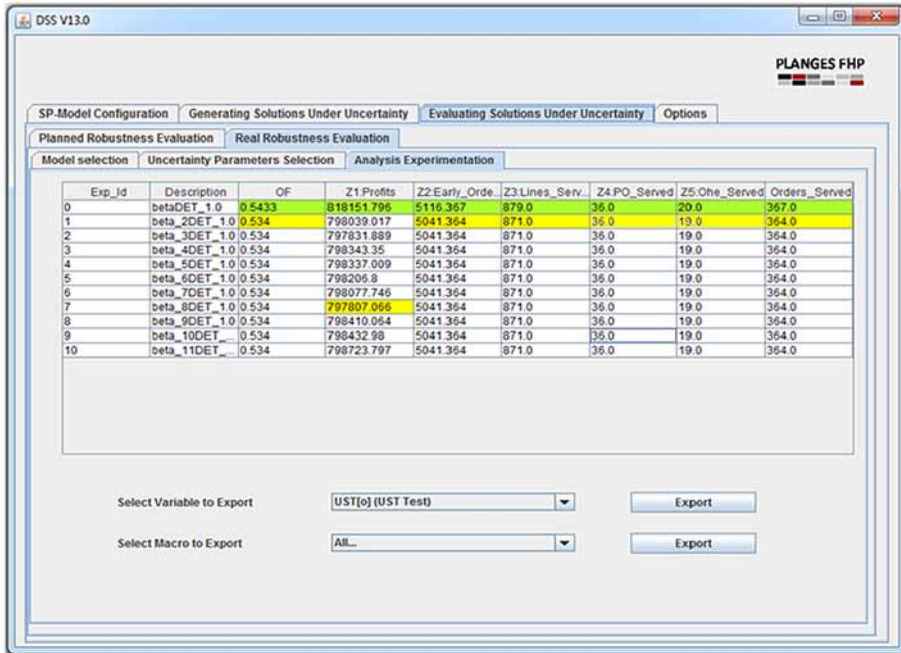
Fig. 10. The "Analysis Experimentation" option for the "Real Robustness Evaluation" DSS screen

execution. Then the values of the orders actually served and the real value of the objectives are derived from the Auxiliary-SP solution, and are compared with their planned ones. Smaller differences between the planned values and the real ones obtained from the "*n*" executions of the Auxiliary-SP Model imply a more robust solution. Figure 10 shows the value of all five objectives for each projection of reality of the beta parameters for the Sol_All_noDelay_none(0.5, 0.3, 0.2, 0.0, 0.0) solution. As observed, the difference between the best and the worst values for each objective is not that significant. This means that the solution is not very sensitive to the changes in the beta values for the probable scenario (i.e., it is robust).

This analysis can be carried out for different solutions in distinct scenarios and projections of reality. Finally, the DM should choose a specific solution from the CSS. This solution represents the set of orders to be served in a shortage situation, which provides the desired balance between the performance of the different objectives and their robustness.

As it can be seen, the SP-Model was solved the same number of times as configurations defined (17 combinations of weights for our example), plus the different scenarios created (3 for our example): 20 times in total. The ASP-Model was solved the same number of times as the number of solutions selected for the "planned robustness evaluation" per number of scenarios (5*3 for our example), plus number of solutions to be selected for the "real robustness evaluation" per number of real projections of reality (1*10 for our example): 25 times in total. Because the models should be solved several times, it is important during the process of selecting the solution to be implemented that the DM reasonable adjusts the GAP and the permissible solution time.

Conclusions

LHP is a relevant matter since it is encountered in several sectors. LHP introduces novel aspects when matching supply and demand, 1) customer homogeneity requirements and 2) uncertainty in the real size of each homogeneous sublot, which is only known once produced and classified. Due to LHP uncertainty, discrepancies between planned and real homogeneous quantities exist. More often than not, this means that certain customers cannot be served with the right homogeneous quantities. If anything, LHP companies achieve a poor service level with very high levels of fragmented stocks of different subtypes. Therefore, LHP companies need tools to obtain the flexibility required to efficiently achieve commitments made to customers when a shortage situation occurs. Reallocation of stocks and supply seems a suitable solution. Nevertheless, given the vast volume of information, problem constraints and the existing uncertainty to find not only an optimal solution, but also a feasible one, are very difficult and time-consuming.

In line with this, the motivation behind this study was to build a Model-Driven DSS for the reallocation of real stocks and uncertain future homogeneous planned production among already committed customer orders when a shortage situation occurred. The DSS is based on two MILP deterministic models: the SP-Model and the ASP-Model. The SP-Model, which is one of the contributions of this paper, provides the orders that can actually be served after the reallocation procedure with a specific data set (scenario). The ASP-Model helps evaluate the robustness of the previous solution by computing the real orders to be served if another scenario comes into play. The DSS is integrated by three main functionalities which generate multiple solutions, consider inherent LHP uncertainty by scenario definitions, and evaluate the quality and robustness of each selected solution in other scenarios or projections of reality.

Although the Model-Driven DSS centres on ceramic SCs, both mathematical models and the DSS architecture and functionalities can be adapted to companies from other LHP sectors to improve stock management, reduce costs, and increase incomes and customer satisfaction.

As a further research work, LHP inherent uncertainty will be modelled by Fuzzy Sets, which is another way of generating solutions. Finally, additional functionalities, like linking models such as the Order Promising Model and the SP-Model on a rolling horizon basis, will be developed.

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