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A Comparison of the Adomian and Homotopy Perturbation Methods in Solving the Problem of Squeezing Flow Between Two Circular Plates

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Abstract. The objective of this paper is to compare two methods employed for solving nonlinear problems, namely the Adomian Decomposition Method (ADM) and the Homotopy Perturbation Method (HPM). To this effect we solve the Navier-Stokes equations for the unsteady flow between two circular plates approaching each other symmetrically. The comparison between HPM and ADM is bench-marked against a numerical solution. The results show that the ADM is more reliable and efficient than HPM from a computational viewpoint. The ADM requires slightly more computational effort than the HPM, but it yields more accurate results than the HPM.

Keywords: Adomian decomposition method, homotopy perturbation method, squeezing flow.

AMS Subject Classification: 34B15.

1 Introduction

In recent years, the homotopy perturbation method (HPM) [8] and the Adomian decomposition method (ADM) [1], have been the source of a lot of research activity. These methods have aided in obtaining approximate solutions to a wide class of linear and nonlinear differential equations [1, 9, 10, 11, 14, 23, 24, 25, 27, 29, 30, 38, 40]. However, only a few papers deal with the comparison of these methods [2, 26, 41]. In this paper, we will make a comparative study to examine the performance of the ADM and HPM when applied to squeezing flow between circular parallel plates.

It is important to note that the problem we have chosen to compare the methods is not a trivial problem. Squeezing flow between parallel plates have many applications, for instance in the areas of biomechanics, food industry, chemical engineering, polymer processing, compression and injection molding and hydrodynamic lubrication. Squeezing flows are produced by vertical movements of boundaries or by applying external normal forces. The study of squeezing flows has its origins in the 19th century and continues to receive considerable attention due to its practical applications in physical and biophysical areas [6, 12, 17, 18, 20, 21, 32, 34, 37, 35, 39]. During the formation of foams, bubble boundaries expand biaxially and shrink in thickness in a manner similar to squeezing films, whereas valves and diarthrodial joints are examples for squeeze flows relevant in biology and bioengineering. Finally, some phenomena occurring during food intake can be modelled using squeeze flow: chewing between teeth and/or gums resembles a compression between (irregular) plates. The compression of food between the tongue and the palate can be approximated (for some foods) as a squeeze flow [15, 16, 22].

Due to the mathematical complexity of such types of flows, different methods such as variational, perturbation and numerical techniques have been used for the solution of the Navier-Stokes equations describing the squeeze flow e.g., see [3, 7, 28, 31, 36]. Jackson [13] considered a theoretical study of squeezing Newtonian liquid-flow generated by the unsteady motion of a disc over a plane surface and analysed it by using an iterative method. An explicit solution of a squeeze flow problem taking into account the inertial terms was studied by Thrope [33], but his solution fails to satisfy the boundary conditions [7]. His perturbation solution for the case when the plates approach each other with a constant velocity was erroneous. However, a more accurate solution taking account of the boundary conditions was computed by Gupta [7]. In [28], the problem of squeezing flow between parallel plates have been successfully solved by using HPM.

The current analysis considers the problem of two circular non-rotating plates that are approaching and receding from each other giving rise to the squeezing flow. We further consider the motion of the plates to be symmetric about the axial line. The fluid flowing between the plates is considered to be a Newtonian incompressible viscous fluid. The systems of partial differential equations are reduced to a fourth order non-linear differential equation with appropriate boundary conditions. Here we employ HPM, ADM and the Picard iterative method to solve the problem. The first two methods are based on series expansions and the Picard iteration method transforms the fourth order implicit nonlinear differential equation into a set of algebraic equations that are solved iteratively. One of the major advantages of these methods is that they do not require small parameters and avoid linearization and physically unrealistic assumptions. The comparison between the three methods shows that the ADM is more reliable, and efficient than HPM from a computational viewpoint, although both methods provide solutions in the form of an infinite series. The ADM requires slightly more computational effort than the HPM, but it provides more accurate results and has better convergence properties [4, 19] than the HPM. We have calculated velocity fields for comparison, but other quantities of interest such as volume flux, shear stress distribution, load expression, can easily be determined.

The plan of the paper consists of Section 2, which develops the equations as well as the boundary conditions governing the squeezing flow. Sections 3 and 4 apply the ADM and HPM to obtain the solutions of the problem, respectively. Section 5 is a comparison of the methods and Section 6 is a summary of the results.

2 Formulation of the Problem

We consider the squeezing flow of an incompressible viscous fluid between two circular plates (see, Fig. 1). The distance between the plates at any time t is $2a(t)$. We select the central axis of the system to be the r-axis while the z-axis is normal to it. It is assumed that the circular plates are non-rotating and move symmetrically with respect to the central region $z = 0$. The flow is axisymmetric about $r = 0$.

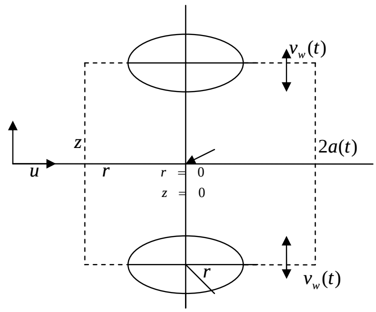

Figure 1. Geometry of the problem.

Now we specify the basic equations for an unsteady axisymmetric flow and assume $\mathbf{v} = \big[u(r,z,t), 0, w(r,z,t)\big]$, where u and w are the velocity components along the radial and axial directions, respectively. Thus the unsteady mass and conservation equations become

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial}{\partial z}(w) = 0, \tag{2.1}$$

$$\rho\Big(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\Big) = -\frac{\partial p}{\partial r} + \mu\Big(\nabla^2 u - \frac{u}{r^2}\Big), \tag{2.2}$$

$$\rho\Big(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\Big) = -\frac{\partial p}{\partial z} + \mu\Big(\nabla^2 w\Big), \tag{2.3}$$

where $\nabla^2 = \dfrac{\partial^2}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial}{\partial r} + \dfrac{\partial^2}{\partial z^2}$. The boundary conditions on $u(r,z,t)$ and

$w(r,z,t)$ are

$$\left.\begin{array}{llcl} \text{at} \quad z=a: & u(r,z,t)=0 & \text{and} & w(r,z,t)=v_w(t), \\ \text{at} \quad z=0: & \dfrac{\partial u(r,z,t)}{\partial z}=0 & \text{and} & w(r,z,t)=0, \end{array}\right\} \tag{2.4}$$

where $v_w(t) = \dfrac{da}{dt}$ denotes the velocity of the circular plates. The conditions (2.4) are due to no-slip conditions at the upper plate $z = a$ and to symmetry at $z = 0$.

If the dimensionless variable $\eta = z/a(t)$ is introduced, equations (2.1)–(2.3) transform to

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{1}{a}\frac{\partial w}{\partial \eta} = 0, \tag{2.5}$$

$$\rho\Big(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{w}{a}\frac{\partial u}{\partial \eta}\Big) = -\frac{\partial p}{\partial r} + \mu\Big[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{a^2}\frac{\partial^2 u}{\partial \eta^2} - \frac{u}{r^2}\Big], \tag{2.6}$$

$$\rho\Big(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + \frac{w}{a}\frac{\partial w}{\partial \eta}\Big) = -\frac{1}{a}\frac{\partial p}{\partial \eta} + \mu\Big[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{a^2}\frac{\partial^2 w}{\partial \eta^2}\Big]. \tag{2.7}$$

Introducing the functions h and Ω respectively as

$$h = \frac{\rho}{2}(u^2 + w^2) + p, \qquad \Omega = \frac{\partial w}{\partial r} - \frac{1}{a}\frac{\partial u}{\partial \eta},$$

equations (2.6)–(2.7) can be simplified as

$$\begin{aligned} &\frac{\partial h}{\partial r} + \rho\frac{\partial u}{\partial t} - \rho\Omega w + \frac{\mu}{a}\frac{\partial \Omega}{\partial \eta} = 0, \\ &\frac{1}{a}\frac{\partial h}{\partial \eta} + \rho\frac{\partial w}{\partial t} + \rho\Omega u - \mu\Big(\frac{\partial \Omega}{\partial r} + \frac{\Omega}{r}\Big) = 0. \end{aligned}$$

We can eliminate h from these equations by cross differentiation and obtain the system

$$-\rho\frac{\partial \Omega}{\partial t} - \rho\Big[u\frac{\partial \Omega}{\partial r} + \frac{w}{a}\frac{\partial \Omega}{\partial \eta} - \frac{u}{r}\Omega\Big] + \mu\Big[\nabla^2\Omega - \frac{\Omega}{r^2}\Big] = 0 \tag{2.8}$$

along with equation of continuity (2.5). The boundary conditions (2.4) take the form

$$\left.\begin{array}{llcl} \text{at} \quad \eta=1: & u=0 & \text{and} & w=v_w(t), \\ \text{at} \quad \eta=0: & \dfrac{\partial u}{\partial \eta}=0 & \text{and} & w=0. \end{array}\right\}$$

Defining velocity components of the form

$$u = -\frac{r}{2a(t)}v_w(t)f'(\eta), \quad w = v_w(t)f(\eta),$$

we find that the equation of continuity (2.5) is identically satisfied and equation (2.8) yields

$$R\Big[(\eta - f)\frac{d^3 f}{d\eta^3} + 2\frac{d^2 f}{d\eta^2}\Big] + \frac{d^4 f}{d\eta^4} = Q\frac{d^2 f}{d\eta^2}, \tag{2.9}$$

where

$$R = \frac{\rho a v_w}{\mu} \qquad \text{and} \qquad Q = \frac{\rho a^2}{\mu v_w}\frac{dv_w}{dt} \tag{2.10}$$

Both R and Q are functions of t but for the similarity solution R and Q become constants. Integrating the first equation in (2.10) we get

$$a(t) = \left(2\nu R t + a_0^2\right)^{\frac{1}{2}}, \tag{2.11}$$

where $\nu = \mu/\rho$ and $2a_0$ is the distance between the two plates at time $t = 0$.

When $R > 0$, the plates move apart symmetrically with respect to $\eta = 0$ and when $R < 0$, the plates approach each other and squeezing flow exists with similar velocity profiles as long as $a(t) > 0$.

It follows from equations (2.10) and (2.11) that if $Q = -R$ equation (2.9) is reduced to

$$R\Big[(\eta - f)\frac{d^3 f}{d\eta^3} + 3\frac{d^2 f}{d\eta^2}\Big] + \frac{d^4 f}{d\eta^4} = 0. \tag{2.12}$$

The boundary conditions in terms of $f(\eta)$ can be expressed as

$$\left.\begin{array}{lllll} \text{at} & \eta = 1: & f'(1) = 0, & \text{and} & f(1) = 1, \\ \text{at} & \eta = 0: & f''(0) = 0, & \text{and} & f(0) = 0. \end{array}\right\} \tag{2.13}$$

The differential equation (2.12) is nonlinear, we present approximate solutions of this problem using the ADM, HPM and numerical method in the succeeding sections.

3 Solution of the Problem by ADM

The ADM has been used effectively, easily, and accurately for a large class of linear and nonlinear, ordinary or partial, deterministic or stochastic differential equations to obtain approximate solutions, which converge rapidly to accurate solutions. Following ADM, we define the highest order linear operator L_η for equation (2.12) as $L_\eta = \dfrac{d^4}{d\eta^4}$.The inverse L_η^{-1} is an integral operator given by

$$L_\eta^{-1} = \int_0^\eta \int_0^\eta \int_0^\eta \int_0^\eta dw\, dw\, dw\, dw.$$

Thus, (2.12) can be written as

$$L_\eta[f] = -\Big[R\eta\frac{d^3 f}{d\eta^3} + 3R\frac{d^2 f}{d\eta^2}\Big] + R\Big[f\frac{d^3 f}{d\eta^3}\Big].$$

Taking L_η^{-1} on both sides of the above equation gives

$$L_\eta^{-1} L_\eta[f] = -L_\eta^{-1}\Big[R\eta\frac{d^3 f}{d\eta^3} + 3R\frac{d^2 f}{d\eta^2}\Big] + RL_\eta^{-1}\Big[f\frac{d^3 f}{d\eta^3}\Big] \tag{3.1}$$

$$f = -\frac{\eta^3}{2} + \frac{3\eta}{2} - L_\eta^{-1}\Big[R\eta\frac{d^3 f}{d\eta^3} + 3R\frac{d^2 f}{d\eta^2}\Big] + RL_\eta^{-1}\Big[f\frac{d^3 f}{d\eta^3}\Big]. \tag{3.2}$$

As required by the Adomian decomposition method we express the solutions $f(\eta)$ and nonlinear term $f(\eta)f'''(\eta)$ by the infinite series

$$f=\sum_{n=0}^{\infty} f_n, \quad ff'''=\sum_{n=0}^{\infty} A_n, \tag{3.3}$$

where $A_0, A_1, A_2, A_3, \ldots$ are the Adomian polynomials defined as

$$A_n=-\frac{1}{n}\frac{d^n}{d\lambda^n}\Big\{F\Big(\sum_{n=0}^{\infty} f_n\Big)\Big\}.$$

So using this relationship we get

$$A_0=f_0\frac{d^3f_0}{d\eta^3}, \quad A_1=f_0\frac{d^3f_1}{d\eta^3}+f_1\frac{d^3f_0}{d\eta^3}$$
$$A_2=3\Big\{\Big(\frac{df_0}{d\eta}\Big)^2\frac{df_1}{d\eta}+\Big(\frac{df_1}{d\eta}\Big)^2\frac{df_0}{d\eta}\Big\}.$$

Substituting (3.3) into (3.2) we obtain the recursive formulae

$$f_0=-\frac{\eta^3}{2}+\frac{3\eta}{2}, \quad f_{n+1}=-L^{-1}\left[R\eta f_n^{'''}+3Rf_n^{''}\right]+RL^{-1}A_n. \tag{3.4}$$

The first few components of f_n follow immediately upon setting:

$$f_1{=}-L^{-1}\left\{R\eta f_0'''+3Rf_0''\right\}+L^{-1}RA_0, \; f_2{=}-L^{-1}\left\{R\eta f_1'''+3Rf_1''\right\}+L^{-1}RA_1,$$
$$f_3=-L^{-1}\left\{R\eta f_2'''+3Rf_2''\right\}+L^{-1}RA_2,$$

which implies

$$f_1=\frac{37}{560}R\eta-\frac{73}{560}R\eta^3+\frac{1}{16}R\eta^5+\frac{1}{560}R\eta^7,$$
$$f_2=9.86\times10^{-3}\eta R^2-0.13490\frac{\eta^3}{6}R^2+\frac{41}{2800}\eta^5R^2$$
$$-\frac{51}{39200}\eta^7R^2+\frac{1}{1440}\eta^9R^2-\frac{3}{123200}\eta^{11}R^2,$$
$$f_3=0.030216R^3\eta-0118029R^3\eta^3+\frac{97347}{5017600}R^3\eta^4-0.004497R^3\eta^5$$
$$-\frac{52739}{5017600}R^3\eta^6-\frac{41}{19600}R^3\eta^7+\frac{1869351}{351232000}R^3\eta^8+\frac{17}{117600}R^3\eta^9$$
$$-\frac{609123}{351232000}R^3\eta^{10}+\frac{1}{15840}R^3\eta^{11}+\frac{3103}{10752000}R^3\eta^{12}+\frac{3}{1601600}R^3\eta^{13}$$
$$-\frac{8241}{717516800}R^3\eta^{14}-\frac{21}{26624000}R^3\eta^{16}-\frac{1}{104448000}R^3\eta^{18}.$$

In view of the above equations, the solution in series form is

$$
\begin{aligned}
f \cong & -\frac{\eta^3}{2}+\frac{3\eta}{2}+\frac{37}{560}R\eta-\frac{73}{560}R\eta^3+\frac{1}{16}R\eta^5 \\
& +\frac{1}{560}R\eta^7+9.86\times 10^{-3}\eta R^2-0.13490\frac{\eta^3}{6}R^2+\frac{41}{2800}\eta^5R^2-\frac{51}{39200}\eta^7R^2 \\
& +\frac{1}{1440}\eta^9R^2-\frac{3}{123200}\eta^{11}R^2+0.030216R^3\eta-0118029R^3\eta^3+\frac{97347}{5017600}R^3\eta^4 \\
& -0.004497R^3\eta^5-\frac{52739}{5017600}R^3\eta^6-\frac{41}{19600}R^3\eta^7+\frac{1869351}{351232000}R^3\eta^8+\frac{17}{117600}R^3\eta^9 \\
& -\frac{609123}{351232000}R^3\eta^{10}+\frac{1}{15840}R^3\eta^{11}+\frac{3103}{10752000}R^3\eta^{12}+\frac{3}{1601600}R^3\eta^{13} \\
& -\frac{8241}{717516800}R^3\eta^{14}-\frac{21}{26624000}R^3\eta^{16}-\frac{1}{104448000}R^3\eta^{18}.
\end{aligned}
\tag{3.5}
$$

Thus we have the solution of the problem using the Adomian decomposition method.

4 Solution of the Problem Using HPM

The HPM approach requires that we first start by defining a homotopy $F(\eta,q): \Omega * [0,1] \to \Re$ for (2.12) which satisfies the equation

$$L[F]-L[f_0]+qL[f_0]+q\Big[R(\eta-F)\frac{d^3F}{d\eta^3}\Big]=0, \tag{4.1}$$

where $L=\dfrac{d^4}{d\eta^4}+3R\dfrac{d^2}{d\eta^2}$ is the linear operator, $q\in[0,1]$ is the embedding parameter, and f_0 is the initial guess approximation. We assume that subject to the boundary conditions (2.13), the initial guess approximation of (2.12) is

$$f_0=\frac{1}{S}\Big[\eta\cos k-\frac{\sin k\eta}{k}\Big], \tag{4.2}$$

where $k=\sqrt{3R}$, $S=\cos k-\sin k/k$. We further assume that the solution of (2.12) can be expressed as a power series in q, i.e.,

$$F(\eta,q)=F_0(\eta)+qF_1(\eta)+q^2F_2(\eta)+\cdots, \tag{4.3}$$

where the F_i's are independent of q. Substituting (4.3) into (4.1), and (2.13) and equating powers of q gives rise to a set of problems that we will now specify and solve in the succeeding sections.

4.1 The Zeroth-Order Problem

The differential equation of the zeroth-order problem is

$$L[F_0]-L[f_0]=0, \tag{4.4}$$

subject to

$$\left.\begin{array}{llllll} F_0' = 0, & \text{and} & F_0 = 1, & \text{at} & \eta = 1, \\ F_0'' = 0, & \text{and} & F_0 = 0 & \text{at} & \eta = 0. \end{array}\right\}$$

Since L is a linear operator, therefore the solution of the zeroth-order problem is

$$F_0(\eta) = \frac{1}{S}\left[\eta\cos k - \frac{\sin k\eta}{k}\right] = f_0(\eta),$$

where k and S are defined in (4.2).

4.2 The First-order Problem

The differential equation of the first-order problem is

$$L\left[F_1\right] + L\left[F_0\right] + \frac{k^2}{3}(\eta - F_0)\frac{dF_0^3}{d\eta^3} = 0,$$

subject to

$$\left.\begin{array}{lllll} \text{at} & \eta = 1: & F_1' = 0, & \text{and} & F_1 = 0, \\ \text{at} & \eta = 0: & F_1'' = 0, & \text{and} & F_1 = 0. \end{array}\right\}$$

The solution of the first-order boundary value problem is given by

$$\begin{aligned} F_1(\eta) = & \frac{1}{36\ kS^2}\left(\frac{10\sin(k)^2}{S} + 15\cos(k)\right)\sin(k)\eta \\ & + \frac{1}{36\ kS^2}\left(\cos(k) + 3k\sin(k) - \frac{10\sin(k)^2}{S}\right)\sin(k\eta) \\ & - \frac{\sin(k)}{12S^2}\eta^2\sin(k\eta) - \frac{5\ \sin(k)}{12\ kS^2}\eta\cos(k\eta) - \frac{1}{72\ kS^2}\sin(2k\eta). \end{aligned}$$

4.3 The Second-Order Problem

The differential equation of the second-order problem is

$$L\left[F_2\right] + \frac{k^2}{3}\eta F_1''' - \frac{k^2}{3}\left[F_0F_1''' + F_1F_0'''\right] = 0,$$

subject to

$$\left.\begin{array}{lllll} \text{at} & \eta = 1: & F_2' = 0, & \text{and} & F_2 = 0, \\ \text{at} & \eta = 0: & F_2'' = 0, & \text{and} & F_2 = 0. \end{array}\right\}$$

The solution of the second-order boundary value problem is given by

$$
\begin{aligned}
F_2(\eta)=&\eta^4 \sin k\eta\Big[-\frac{1}{288}\frac{k\sin^2 k}{S^3}\Big]+\eta^3\cos k\eta\Big[-\frac{13}{432}\frac{\sin^2 k}{S^3}\Big]+\eta^3\Big[-\frac{k\sin k}{72\ S^3}\Big]\\
&+\eta^2\sin k\eta\Big[-\frac{A}{4\ k^3}+\frac{41}{288}\frac{\sin^2 k}{k\ S^3}\Big]+\eta^2\sin 2k\eta\Big[-\frac{\sin k}{216\ S^3}\Big]\\
&+\eta\cos k\eta\Big[\frac{113}{288}\frac{\sin^2 k}{k^2\ S^3}-\frac{5A}{4\ k^4}-\frac{1}{864\ S^3}\Big]+\eta\cos 2k\eta\Big[-\frac{17}{648}\frac{\sin k}{k\ S^3}\Big]\\
&+\eta\Big[\sin^4 k\Big(-\frac{287}{2592}\frac{}{k^2\ S^4}+\frac{1}{108\ S^4}\Big)+\sin^3 k\Big(-\frac{143}{1944}\frac{\cos k}{k\ S^4}+\frac{61}{1944\ k\ S^3}\\
&-\frac{H}{6\ k^4\ S}\Big)+\sin^2 k\Big(\frac{3A}{4\ S\ k^4}-\frac{113\cos k}{288\ k^2\ S^3}-\frac{5}{432\ S^4}+\frac{\cos k}{72\ S^3}\Big)\\
&+\sin k\Big(\frac{k}{72\ S^3}+\frac{17}{648\ k\ S^3}-\frac{\cos k}{864\ k\ S^4}\Big)+\frac{5A\cos k}{4\ k^4}+\frac{1}{864\ S^4}\Big]\\
&+\sin k\eta\Big[\sin^3 k\Big(-\frac{11}{432\ S^4}+\frac{2431}{7776\ k^2\ S^4}\Big)+\sin^2 k\Big(-\frac{95}{3888\ k\ S^3}-\frac{281\cos k}{1944\ k\ S^4}\\
&+\frac{k}{288\ S^3}+\frac{H}{6\ k^4\ S}\Big)+\sin k\Big(-\frac{3A\sin k}{4\ k^4\ S}-\frac{169\cos k}{1944\ k^2\ S^3}+\frac{23}{864\ S^4}+\frac{\cos k}{108\ S^3}\Big)\\
&+\frac{A}{4\ k^3}-\frac{H\cos k}{6\ k^4}+\frac{1}{10368\ k\ S^3}\Big]+\sin 2k\eta\Big[\frac{169}{3888}\frac{\sin k}{k^2\ S^3}+\frac{H}{12\ k^4}\Big]\\
&+\sin 3k\eta\Big[-\frac{1}{31104kS^3}\Big],
\end{aligned}
$$

where

$$
\begin{aligned}
A&=\frac{k^2}{3\ S^3}\left[\frac{7}{18}k\sin k\cos k+\frac{5}{9}k\sin^3 k+\sin^2 k\left(\frac{3}{4}-\frac{k^2}{12}\right)\right],\\
H&=\frac{k}{6\ S^3}\left[\frac{1}{18}k^2\cos k-\frac{5}{9S}k^2\sin^2 k+\frac{k^3}{6}\sin k+\frac{3}{4}k\sin k\right].
\end{aligned}
$$

Finally, the homotopy perturbation solution of the problem up to 2nd order is

$$
f(\eta)=\lim_{q\to 1}F(\eta,q)=F_0(\eta)+F_1(\eta)+F_2(\eta)+\cdots. \tag{4.5}
$$

Using (3.4), (3.5) and (4.4) with (4.5) we get the required approximate solution in terms of $f(\eta)$.

5 Comparison of the Adomian and HPM

In order to compare the two methods we first need to set up a bench mark numerical solution as a guide. We do this by employing the well established Picard Iteration Method [5] for solving nonlinear problems. In Figure 2 we note the comparison between the two methods and the numerical solution of (2.12) along with the boundary conditions (2.13). We can note here that the Adomian is closer to the numerical solution than the HPM. In addition, it is worth noting that as R increases from 0.2 to 0.7 the HPM solution gets progressively worse, whereas the Adomian solution maintains its accuracy.

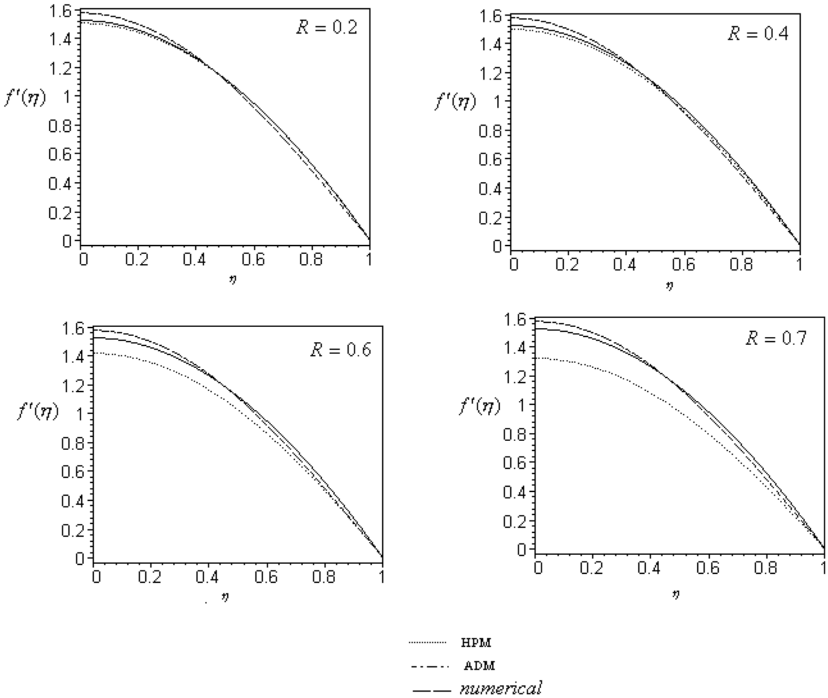

Figure 2. Comparison of solution profiles $f(\eta)$ by ADM and HPM for different values of R.

We further note in Figure 3 that the derivative of the solution reflects similar behaviour; once again as expected the Adomian is a better solution than the HPM solution. Of course the HPM is a good method in solving nonlinear problems, but we note that in this particular instance the Adomian is clearly a better choice. In addition, we need to note that the Adomian has an established support for its convergence (cf. [4, 19]), this is an important point since both methods provide infinite series solutions.

6 Summary

In this paper, the homotopy perturbation and the Adomian decomposition have been successfully applied to solve the non-linear equation (2.12) along with the boundary conditions (2.13) arising in the case of squeezing flow of an incompressible viscous fluid between two circular plates. It was shown that HPM and ADM are efficient in attaining solutions. The comparison between HPM and ADM with the numerical solution when applied to solve the equation (2.12) showed that the ADM is more reliable and efficient than HPM from a computational viewpoint, although both methods provide solutions in the form

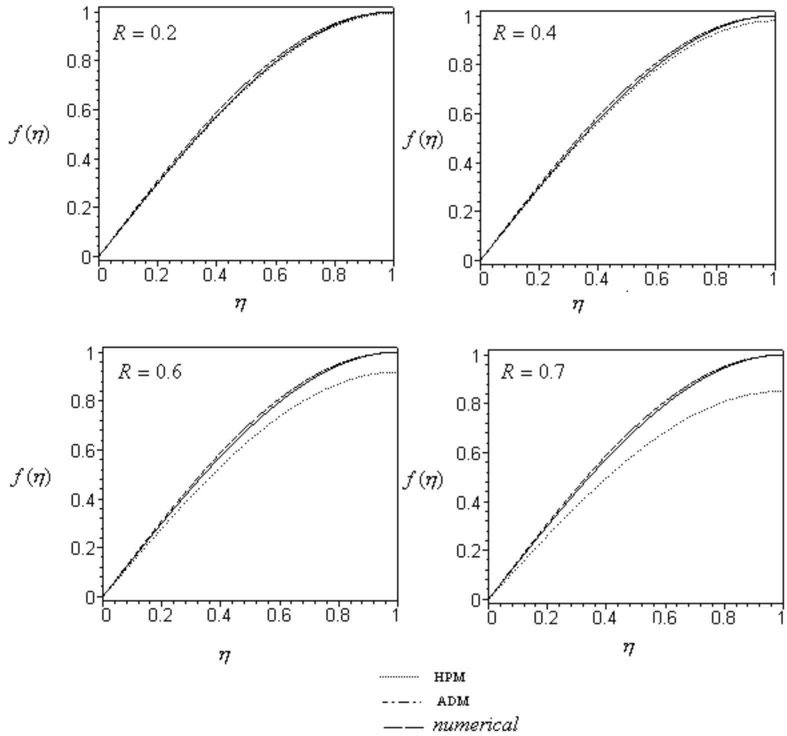

Figure 3. Comparison of velocity profiles $f'(\eta)$ by ADM and HPM for different values of R.

of an infinite series. The ADM requires slightly more computational effort than the HPM, but it yields more accurate results than the HPM. We also note that the solutions using the HPM loose accuracy as we change the value of R, whereas the Adomian maintains its accuracy. In addition, noting that the Adomian has better convergence support makes it a better choice.

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