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THE BOLT - WITH - A CRACK SHAKEDOWN ESTIMATION BY THE METHOD OF ADDITIONAL LOAD

M. Leonavičius, M. Šukšta

1. Introduction

Within some metal constructions, the bolts dimensions of flange joints and single joints are limited by general compositional requirements. Usually efforts are taken in order to amplify the pre-stress of such joints. Operational reliability is determined by the construction of the joint, selection of the pressing force taking into account the functional purpose of the joint, precision of the pre-stress in the course of putting together, steadiness of pre-stress under cyclic forces. In the case of strong press up to 0,8 σ_y (yielding stress), the working conditions can occur like those shown in Fig. 1.

In cases when plastic deformations take place on the boundary of bolt under cyclic load, its stability may be lost. In such a case the analysis of the shakedown conditions enable to define the limit values of the external effects. The comparison of the given cycle in respect of corresponding limited cycle enables us to estimate the peculiarities of the bolt load in case the infringements of shakedown conditions occur and, consequently, it is possible to specify the approximate danger of deterioration.

Generally, the law of statical shakedown (Melan's law) is formulated as a problem of mathematical programming. The solution of this problem is rather complicated. It becomes easier because of various simplifications, but simplifications do not reduce the precision of the solution [1,2]. The method of "additional load" has been used for the estimation of the bolt shakedown.

2. The shakedown zones of pin

A bolt or a pin (see Fig. 1) as a bar of a circular cross-section is affected by constant axial force and symmetrically variable bending moment - $M^* \le M \le M^*$. Two ways of deterioration can be observed: the bolt may loose the stability of the pressing force, if plastic deformation of variable sign occurs within the cross-section, or one-sided plastic deformation increasing with every new cycle accumulates within the cross-section. Combined plastic deterioration can also be mentioned. The limit state which characterizes the flow of the cross-section with a gap can be described by the following set of equations:

$$N = \frac{d^2}{4}(2\alpha + \sin 2\alpha)\,\sigma_y;$$
$$M = \frac{d^3}{6}\left(\cos^3\alpha - \cos^3\alpha_0\right)\sigma_y. \quad (1)$$

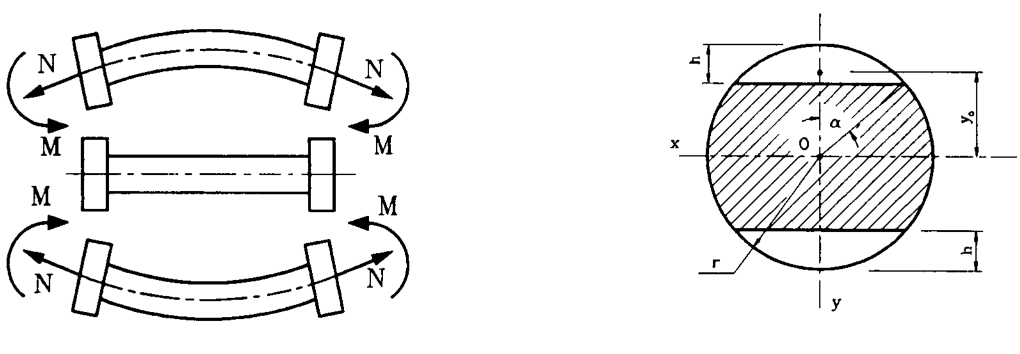

Fig. 1. Bolt and its cross-section with double-sided gap

In this formula the angle α characterizes the position of the neutral line and σ_y means the yield limit of the material.

The limit stresses of the cross-section weakened by the gap, i.e. the stresses causing the full flow of the cross-sections, can be expressed in the following way:

$$N_o = \frac{d^2}{4}\left(2\alpha_o + \sin 2\alpha_o\right)\sigma_y; \qquad M_o = \frac{d^3}{6}\left(1-\cos^3\alpha_0\right)\sigma_y. \tag{2}$$

The limit stresses of the unweakened circular cross-section will be as follows:

$$N_o^{*} = \frac{\pi d^2}{4}\sigma_y; \; M_o^{*} = \frac{d^3}{6}\sigma_y. \tag{3}$$

The calculation of the limit "elastic" bending moment, i.e. the bending moment by which the maximum stress is equal to the yielding stress, is performed in such a way:

$$M_e = W_x\sigma_y = \frac{d^3}{16\sin\alpha_o}\left(\alpha_o - \frac{\sin 4\alpha_o}{4}\right)\sigma_o. \tag{4}$$

The plastic flow with different sign under the influence of the bending moment M* ≤ M ≤ M* will take place, if the bending moment reaches the elasticity limit:

$$M = M_e \;\; or \;\; M/M_e = 1. \tag{5}$$

Let us express M_e (5) through M_0 using the expressions (3) and (4):

$$M_e = \frac{3}{8}M_o^{*}\left(\alpha_o - \frac{\sin 4\alpha_o}{4}\right). \tag{6}$$

Hence

$$\frac{M}{M_o^{*}}\frac{8}{3\left(\alpha_o - \frac{\sin 4\alpha_o}{4}\right)} = 1. \tag{7}$$

Introducing the relative coordinate of the bending moment $m = M/M_o^{*}$, we receive from (7):

$$m = \frac{3\left(\alpha_o - \frac{\sin 4\alpha_o}{4}\right)}{8}. \tag{8}$$

This equation describes the condition of variable flow, which can be seen in Fig. 3 as a horizontal line. In order to examine the dependence of the shake-down zone upon the gap depth, let us introduce the factor coefficient $k=h/r$. Then $\sin\alpha_o=(r-h)/r=1-k$.

By setting different depths h of the gap, we can calculate m corresponding to the condition of the flow with a variable sign. The calculation is presented in Table 1 and the results are presented in Fig. 2.

With the change of the moment the normal stresses will increase sometimes on one side of the cross-section, sometimes on the other one and this can be the cause for the accumulation of deformations. In the case when plastic deformation zone intersects the centre of the cross-section within every half-cycle, the plastic deformations will accumulate themselves at the centre and the plastic deterioration will develop progressively.

Adding an extra load ΔN to the remaining stresses which will cause the full flow of the cross-section we obtain:

$$N + \Delta N = N_o. \tag{9}$$

The additional axial force can by expressed through the increase of the bending moment ΔM:

$$\Delta N = \int_A \sigma dA = \int_A \frac{\Delta M}{I_x} y dA = \frac{\Delta M}{I_x}\int_0^{\alpha_o}\frac{d^3}{4}\sin\varphi\cos^2\varphi d\varphi. \tag{10}$$

After the increase $\Delta M = 2M^{*}$ of the maximum possible bending moment is put in and the integration as well as elementary mathematical changes are made, we obtain:

$$\Delta N = \frac{16M^{*}}{d}\;\frac{1-\frac{\cos^3\alpha_o}{3}}{\alpha_o - \frac{\sin 4\alpha_0}{4}}. \tag{11}$$

Table 1. The shakedown zone dependence of the variable sign flow upon the depth of the gap

k	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
m	0.589	0.56	0.511	0.456	0.398	0.341	0.287	0.237	0.191	0.15	0.115

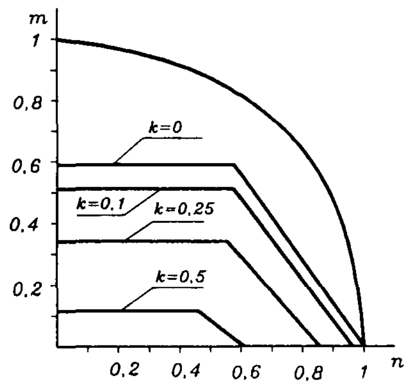

Fig. 2. The dependence of the shakedown zone area upon the depth of the gap

We shall put (11) into (9):

$$\frac{N}{N_o}+\frac{16M^*}{dN_o}\,\frac{1-\dfrac{\cos^3\alpha_o}{3}}{\alpha_o-\dfrac{\sin 4\alpha_0}{4}}=1. \qquad (12)$$

When N_o is expressed through M_o, we obtain:

$$N_o=\frac{3M_o(2\alpha_o+\sin 2\alpha_o)}{2d\left(1-\cos^3\alpha_o\right)}. \qquad (13)$$

and after it is put into (12), the following expression is obtained:

$$\frac{N}{N_o}+\frac{M}{M_o}\frac{32}{3}\frac{\left(1-\dfrac{\cos^3\alpha_o}{3}\right)\left(1-\cos^3\alpha\right)}{\left(\alpha_o-\dfrac{\sin 4\alpha_o}{4}\right)\left(2\alpha_o+\sin 2\alpha_o\right)}=1$$

or

$$n+m\frac{32}{3}\frac{\left(1-\dfrac{\cos^3\alpha_o}{3}\right)\left(1-\cos^3\alpha\right)}{\left(\alpha_o-\dfrac{\sin 4\alpha_o}{4}\right)\left(2\alpha_o+\sin 2\alpha_o\right)}=1. \qquad (14)$$

This equation expresses the condition of one-sided accumulation of plastic deformations for the cross-section, within which symmetrically located gaps have develop. Low let us bring the stresses of the circular cross-section received from (2) and (3) into the formula (14):

$$N_o=N_o^\bullet(2\alpha_o+\sin 2\alpha_o)/\pi\,;$$
$$M_o=M_o^\bullet\left(1-\cos^3\alpha_o\right). \qquad (15)$$

After they are put into the formula (14), we receive:

$$\frac{N}{N_o^\bullet}\frac{\pi}{\left(2\alpha_o+\sin 2\alpha_o\right)}+\frac{M}{M_o^\bullet}\frac{32}{3}\frac{\left(1-\dfrac{\cos^3\alpha_o}{3}\right)}{\left(\alpha_o-\dfrac{\sin 4\alpha_o}{4}\right)\left(2\alpha_o+\sin 2\alpha_o\right)}=1. \qquad (16)$$

After corresponding marking $n={N}/{N_o^\bullet}$ and $m={M}/{M_o^\bullet}$ is brought into this expression, finally we receive an equation which characterizes the condition of one-sided accumulation of plastic deformations:

$$na+mb=1. \qquad (17)$$

Following the data presented in Table 2, Fig. 2 shows the kinetics of the shakedown zone.

3. The safety reserve of bolt

The received data enable to identify the reserve according to the shakedown diagram. Let us take one of them, for instance, when $k=0$ (Fig. 3). The point D with the coordinates m_d and n_d in the diagram is obtained in those cases when the bar (the bolt) is loaded with the axial force N and bending moment M. The limit point L with the coordinates m_{lim} and n_{lim} is obtained by means of the radius of similar cycles, the inclination angle of which

$$tg\beta=\frac{m_{lim}}{n_{lim}}\ \text{or}\ tg\beta=\frac{m_d}{n_d}. \qquad (18)$$

Table 2. The dependence of the factor a and b upon the depth h of the gap

$k=h/r$	0.000	0.050	0.100	0.150	0.200	0.250	0.300	0.350	0.400	0.450	0.500
a	1.000	1.013	1.039	1.073	1.116	1.169	1.232	1.307	1.398	1.508	1.642
b	0.721	0.745	0.791	0.853	0.933	1.033	1.158	1.316	1.517	1.777	2.121

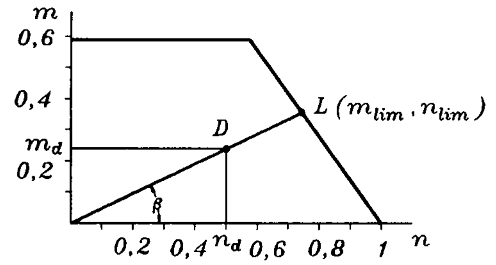

Fig. 3. Limit diagram for case, when k=0

The reserve coefficient is identified from quotient

$$\eta = \frac{m_{lim}}{m_d} \quad \text{or} \quad \eta = \frac{n_{lim}}{n_d}. \tag{19}$$

In case the bolt contains a crack with the depth *h*, another limit diagram shall be drawn up, depending upon the ratio *k*.

4. Conclusions

The simplified "additional load" method can be successfully used for the solution of shakedown problems. The results obtained by this approach coincide with the results obtained by the use of the kinematic shakedown theorems which have been earlier developed by the authors. This fact shows the effectiveness of the proposed method. The obtained shakedown diagram enables to determine safety reserve of the bolt as the function of appeared crack depth.

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VARŽTO SU DVIPUSIU PLYŠIU PRISITAIKOMUMO TYRIMAS PAPILDOMOS APKROVOS METODU

M. Leonavičius, M. Šukšta

S a n t r a u k a

Straipsnyje analizuojamas varžto (smeigės) prisitaikomumas veikiant pastoviai ašinei jėgai ir simetriškai kintančiam lenkimo momentui. Uždavinys sprendžiamas atsižvelgiant į dvipusio, simetriškai išsidėsčiusio plyšio atsiradimą ir vystymąsi. Medžiaga idealiai tampriai plastiška. Sprendimui taikomas papildomos apkrovos metodas. Išnagrinėti vienpusio plastinės deformacijos kaupimosi ir kintamo ženklo plastinio tekėjimo atvejai. Gautos varžto su plintančiu dvipusiu plyšiu prisitaikomumo zonų ribos. Nustatyta varžto atsarga naudojantis prisitaikomumo diagrama įvairių plyšių gylių atvejais.

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