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LIMIT ANALYSIS OF REINFORCED CONCRETE CROSS-SECTIONS UNDER CYCLIC LOADINGS

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1. Introduction

The behaviour of reinforced concrete structures subjected to repeated loads of certain histories was investigated in several studies (see, for instance, Refs [1] and [2]). However, the load-carrying structures are exposed to the actions (static, thermal, kinematic, etc) which may vary in random manner. As a result, there are repeated alternating cross-section forces changed arbitrarily within the specified area [3]. At present, only separate design combinations of loads and influences are usually taken into account in analysis and design procedures. In fact, the strength conditions of elements essentially depend on the interaction of variable repeated loads. The strength conditions in terms of generalized forces for sections from homogeneous ideal plastic material for different types of load cycles have been obtained in the studies [4] and [5].

In this paper, an analytical model is presented to analyse reinforced concrete beam and column element cross-sections under low-cyclic loadings. The loads and influences are quasi-static, no dynamic effects and fatigue failure are considered. The cross-section may have any geometrical form, prestressed as well as non-prestressed reinforcement is allowed. The vector of variable repeated forces contains axial force and bending moments about two central axes of cross-section. The torsion and the shear forces are also taken into account but their influences are assumed to be minor. Prestressing forces and thermal actions are considered herein as one of the load types when the vector of resultant internal forces is zero.

The constitutive model for steel reinforcement is bilinear elastic-perfectly plastic without strain hardening. Concrete in compression is presumed to be elastic-plastic and concrete in tension is elastic and then brittle material [6]–[10]. Moreover, tensile strength of concrete may be neglected. In some cases, tensile strength of concrete has to be ignored because of irreversibility of cracking.

2. General relations

Let the cross-section of reinforced concrete element be subjected to the vector of variable repeated forces $S = (N, M_x, M_y, T, V_x, V_y)$, which are changed arbitrarily within the given domain Ω_S. This domain can be simulated by the polyhedron

$$\Omega_S = (S \in \boldsymbol{R}^6 : S \sum_{l \in L} \alpha_l S_l, \quad \sum_{l \in L} \alpha_l = 1, \quad \alpha_l \geq 0, l \in L), \tag{1}$$

where S_l is the vector of design combinations of cross-section forces which are caused by the action of l combination of external loadings (static, thermal and kinematic); α_l is the component of the barycentric coordinate vector, $l \in L$; L is the set of load or force combinations. Note that the thermal action components distributed in the section area may be added to the vector S.

The domain Ω_S contains the coordinate origin or "zero load" $S = 0$ corresponding to initial non-stress state of section with non-prestressed steel or initial stress state of section with prestressed steel. The latter state is considered like a thermal action.

In surfaces dA of concrete area A_c which have coordinates $x = (x, y)$, the stresses $\sigma = (\sigma_z, \tau_{zx}, \tau_{zy})$ appear; the stresses $\sigma_x, \tau y, \tau_{xy}$ are neglected; normal stresses σ_z in reinforcing steel of area A_s are only considered. Subscript "z" for stresses σ_z is omitted and subscripts "c" and "s" for concrete and steel respectively are used below, if necessary.

To check the plasticity of concrete in compression and the strength of concrete in tension a general Balandin-Geniev criterion in terms of principal stresses for three-dimensional stress state is adopted. It can be written as

$$\begin{aligned} &\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) + \\ &+ \left(f_c^c - f_c^t\right)(\sigma_1 + \sigma_2 + \sigma_3) - f_c^c f_c^t \leq 0, \quad x \in A_c, \end{aligned} \tag{2}$$

where f_c^c and f_c^t are the ultimate compressive and tensile concrete stresses, respectively.

For concrete in a state of plane stress, inequality (2) is rewritten as

$$\sigma^2 + 3\left((\tau_{zx})^2 + (\tau_{zy})^2\right) + \left(f_c^c - f_c^t\right)\sigma - f_c^c f_c^t \le 0, \quad x \in A_c. \tag{3}$$

The quadratic inequality (3) may be substituted for linear inequalities for concrete in compression and for concrete in tension, respectively:

$$-\sigma + R_{cl} \le 0, \quad x \in A_c^c, \tag{4}$$

$$\sigma - R_{tl} \le 0, \quad x \in A_c^t, \tag{5}$$

where R_{cl} and R_{tl} are the radicals of functions located in the left side of (3), which depend on shear stresses τ_{zx}, τ_{zy}. They are given by

$$R_{cl} = \left(f_c^t - f_c^c - D_l\right)/2, \tag{6}$$

$$R_{tl} = \left(f_c^t - f_c^c + D_l\right)/2, \tag{7}$$

$$D_l = \sqrt{\left(f_c^t + f_c^c\right)^2 - 12\left(\tau_{zx}^2 + \tau_{zy}^2\right)}, \tag{8}$$

their absolute values are the equivalent strengths of concrete.

The total stresses in compressed concrete of area A_c^c are presented as a sum of elastic σ^e and residual σ^r components:

$$\sigma = \sigma^e(S) + \sigma^r, \quad x \in A_c^c. \tag{9}$$

Concrete in tension is assumed to be a brittle material ($\sigma^r = 0$), hence

$$\sigma = \sigma^e(S), \quad x \in A_c^t. \tag{10}$$

Furthermore, residual shear stresses in concrete are neglected, ie

$$\tau_{zx}^r = \tau_{zy}^r = 0. \tag{11}$$

With referring to Eqs (9)–(10), conditions (4) and (5) take the forms:

$$-\sigma^e - \sigma^r + R_{cl} \le 0, \quad x \in A_c^c, \tag{12}$$

$$\sigma^e - R_{tl} \le 0, \quad x \in A_c^t. \tag{13}$$

The total stresses in reinforcing steel are also presented as a sum of elastic σ^e and residual σ^r components:

$$\sigma = \sigma^e(S) + \sigma^r; \quad x \in A_s. \tag{14}$$

The stress-strain relationship for steel in elastic stage is given by Hook's law $\sigma_s^e = E_s \varepsilon_s$, and conditions of ideal plasticity are given by

$$-f_{sy} \le \sigma_s \le f_{sy}, \quad x \in A_s, \tag{15}$$

where f_{sy} is the steel stress at yield.

Non-ideal elastic-plastic response of materials (with strain hardening or softening) [9] can be considered using approach [6].

It is assumed that dependence $\sigma^e(S)$ of elastic stresses upon external forces at one-pass loading is known. If function $\sigma^e(S)$ is monotonic, the extremal stresses $\sigma_l^{e-}, \sigma_l^{e+}$ are induced by the l dangerous load combination:

$$\sigma^{e-} = min\left\{\min_{l \in L} \sigma^e(S_l); 0\right\}; \quad \sigma^{e+} = max\left\{\max_{l \in L} \sigma^e(S_l); 0\right\}. \tag{16}$$

It is obvious that both inequalities (15) may be actual at the same point x of the steel area of the cross-section. Then, after transformations, we obtain the inequality

$$\sigma_s^{e+} - \sigma_s^{e-} - 2f_{sy} \le 0, \tag{17}$$

which confines the cross-section ultimate capacity by condition of alternating steel yielding.

Referring to Eqs (16), the plasticity conditions (12) and (15) and the strength condition (13) may be written in the following forms:

$$\min_{l \in L}\left(R_{cl} - \sigma^e(S_l)\right) - \sigma^r \le 0; \quad x \in A_c^c, \tag{18}$$

$$\min_{l \in L}\left(\sigma^e(S_l) - R_{tl}\right) \le 0; \quad x \in A_c^t, \tag{19}$$

$$-\sigma^{e-} - f_{sy} - \sigma^r \le 0; \quad x \in A_s^c, \tag{20}$$

$$\sigma^{e+} - f_{sy} + \sigma^r \le 0; \quad x \in A_s^t. \tag{21}$$

Besides, the following equilibrium equations must be satisfied:

$$\int_{A_c^c} \sigma_c^r \, dA + \int_{A_s} \sigma_s^r \, dA = 0, \tag{22}$$

$$\int_{A_c^c} \sigma_c^r \, x dA + \int_{A_s} \sigma_s^r \, x dA = 0, \tag{23}$$

$$\int_{A_c^c} \sigma_c^r \, y dA + \int_{A_s} \sigma_s^r \, y dA = 0. \tag{24}$$

The strength of RC element cross-section is assumed to be ensured if there are fields of residual stresses

$\sigma_c^r(x)$, $x \in A_c^c$, and $\sigma_s^r(x)$, $x \in A_s$ provided that inequalities (17), (18), (20), (21) and equalities (22)–(24) hold.

3. Mathematical models of the problem

The **primal problem** about the ultimate capacity of the element cross-section can be formulated in case when vectors S_l of the section force combinations consist of constant S_{ol} and variable S_{vl} terms, where S_{vl} depends only on the one parameter of load F_o:

$$S_l = S_{ol} + F_o S_{vl}, \quad l \in L. \qquad (25)$$

Thus, the following infinite-dimensional non-linear programming problem is derived: the parameter of load should be maximised,

$$F_o \to max, \qquad (26)$$

while constraints (17)–(24) depended on F_o are satisfied.

The variables of this problem are the fields of optimal control variables $\sigma_c^r(x)$, $x \in A_c^c$, $\sigma_s^r(x)$, $x \in A_s$, and parameter F_o.

Similarly, the **inverse (design optimisation) problem** can be formulated, if the vectors S_l are known, and ultimate concrete stresses f_c^c, f_c^t and steel yield stress f_{sy} (unknowns) depend on parameter λ; $\left(f_c^c, f_c^t, f_{sy}\right) = \lambda\left(\overline{f_c^c}, \overline{f_c^t}, \overline{f_{sy}}\right)$, where $\overline{f_c^c}, \overline{f_c^t}, \overline{f_{sy}}$ are some positive constants: parameter λ should be minimised,

$$\lambda \to min, \qquad (27)$$

while constraints (17)–(24) depended on λ are satisfied.

This problem has the same variables as previous problem, if substitute F_o for λ.

In order to obtain the numerical solutions of these problems they have to be reduced to the finite-dimensional problems by division the cross-section area $A = A_c \cup A_s$ into the elementary areas $\Delta A_i, i \in I$, where I is the set of elementary areas. Then the vector of variables (residual stresses σ^r) will have dimensions of value $|I|$, and problems formulated can be solved by the conventional methods of optimisation.

It is possible to use other simple and accurate computer aided numerical procedures based on the approach [5].

The technique for solving the primal problem can be realised by applying the following iterative scheme:

Assume a value for parameter of load F_o corresponding to the cross-section ultimate capacity derived without considering cyclic load interactions.

1. Determine the extremal elastic stress distributions on the areas of cross-section and check for condition (17).
2. Take location of neutral axis.
3. Determine the stresses in steel and in concrete in the limit state.
4. From (18), (20), (21) as from equalities obtain the residual stresses σ^r.
5. Substitute σ^r into Eqs (22)-(24) and obtain out-of-balance values.
6. Check convergence: if out-of-balances do not exceed the tolerances, the solution is found; in the other case go to the next step.
7. Repeat steps 4 through 8 changing the location and inclination of neutral axis until the neutral axis does not intersect the section area.
8. Correct F_o and go to step 2.

To solve the inverse problem the scheme of procedure may be sketched as follows:

1. Determine the extremal elastic stress distributions on the cross-section areas.

2. Assume a value for parameter λ (adopt from results of analyses carried out without considering cyclic load interactions).

3-8. See the same steps of the previous scheme.

9. Correct λ and go to Step 2.

4. Numerical examples

On the basis of described analytical model, the computer program has been developed. Some numerical results obtained by using this program are presented below.

Example 1. In this example, the primal problem is solved. The cross-section considered has rectangular form and non-prestressed reinforcing bars as shown in Fig 1.

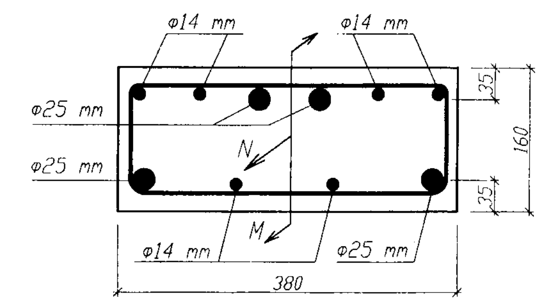

Fig 1. Cross-section for example 1 (dimensions in mm)

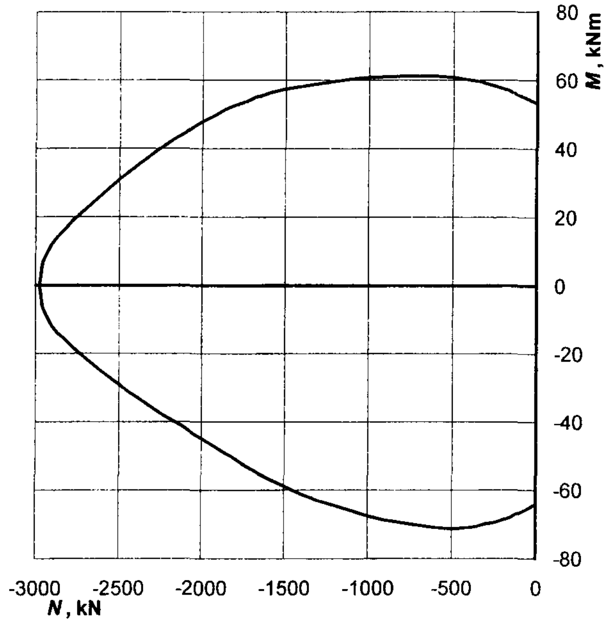

Fig 2. Ultimate strength surface for cross-section at one-pass loading

The ultimate stresses are: $f_c^c = 30$ MPa, $f_{sy} = 400$ MPa, and the modules of elasticity are: $E_c = 20 \cdot 10^3$ MPa, $E_s = 200 \cdot 10^3$ MPa. The load forces are reversing bending moment M and compressive axial force N. Fig 2 represents an ultimate strength surface for cross-section in case when the influences of interactions of variable repeated forces are ignored.

Let the components of variable term S_{vl} of vector of section forces S_l are within the surface shown in Fig 2, and constant term S_{ol} is equal to zero. Then, the value of parameter of load $F_o = 0.89$ was obtained by analysis. In other words, in our particular example, the cross-section ultimate capacity degradation of value of 11 percent due to the influence of cyclic load interactions is derived.

Example 2. Let's solve the inverse problem for cross-section shown in Fig 3 a.

The inverted tee-shaped section has non-prestressed lower A_s and upper A_s' longitudinal reinforcement. The Young's modulus of steel and initial modulus of elasticity of concrete are taken to be $200 \cdot 10^3$ MPa and $30 \cdot 10^3$ MPa, respectively.

Let the cross-section be subjected to the bending moment M about the horizontal axis and axial force N. These forces may be changed within the hatched areas Ω_1, Ω_2 and Ω_3 as shown in Fig 4. In Fig 3 b, c, d, the corresponding distributions of the extremal elastic stresses are given.

The analysis results indicated that steel yield stress $\overline{f_{sy}} = 450$ MPa and concrete strength $\overline{f_c^c} = 30$ MPa are large enough to ensure sufficient cross-section capacity within the full area Ω_3 if the interaction of repeated forces is ignored. The following values of parameter λ (see the inverse problem statement in the Section 3), when the interaction is considered, were derived: $\lambda = 1.32$ if forces varied within the area Ω_1 and $\lambda = 1.26$, and $\lambda = 1.44$ if forces varied within the areas Ω_2 and Ω_3, respectively. Thus, in this example, the influence of variable cyclic load interactions of value up to 44 percent is observed.

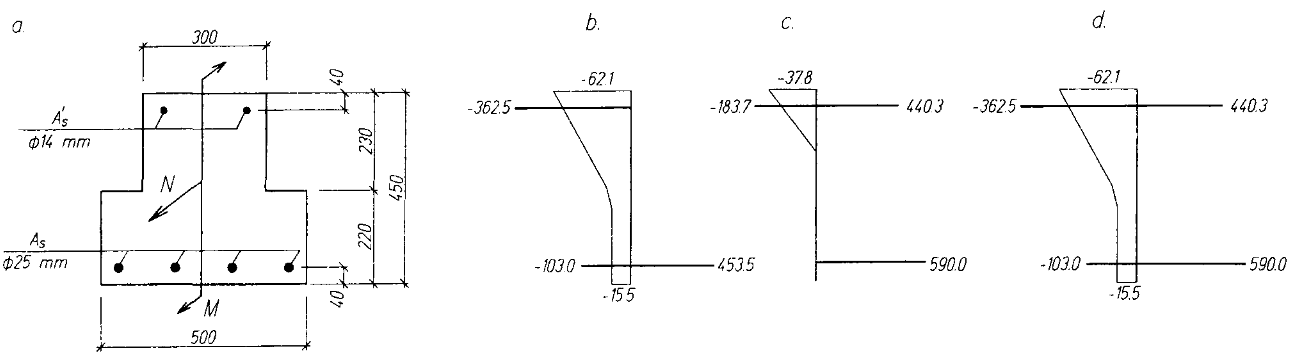

Fig 3 a. Cross-section for example 2 (dimensions in mm) b, c, d. Distributions of extremal elastic stresses, MPa, for the force action areas Ω_1, Ω_2 and Ω_3, respectively

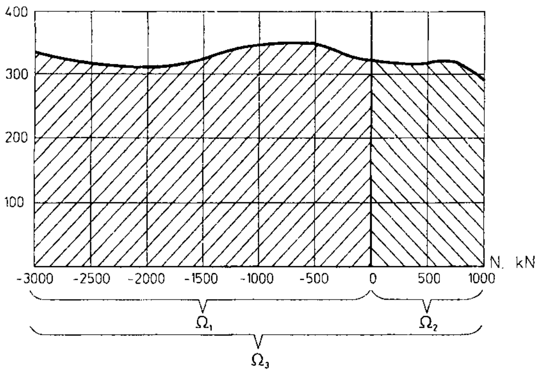

Fig 4. Areas of action of section internal forces: axial force *N* versus bending moment *M*

5. Conclusions

In this study, an analytical model is formulated and calculation methods are proposed to carry out a limit analysis of the cross-sections of RC elements subjected to low-cyclic loadings. The results of the numerical examples given in the paper indicated that the analysis may overestimate the ultimate carrying capacity of cross-section if the influences of repeated forces interaction are neglected. Therefore, to ensure the safety of RC structures subjected to variable repeated loads the effects mentioned have to be taken into account in design practice.

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CIKLINĖS APKROVOS VEIKIAMŲ GELŽBETONINIŲ ELEMENTŲ STIPRUMO NUSTATYMAS

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Santrauka

Nagrinėjamas kartotinės mažaciklės apkrovos veikiamų gelžbetoninių konstrukcijų strypinių elementų skerspjūvių laikomosios galios nustatymo uždavinys. Skerspjūvių forma neapibrėžta, gali būti įvairi. Armatūra gali būti paprasta arba iš anksto įtempta. Pjūvio įrąžų vektoriuje įeina ašinė jėga ir lenkimo momentai. Atsižvelgiama ir į sukimo momentą ir skersines jėgas, tačiau ją įtaka laikoma antraeile. Išankstinis įtempimas ir temperatūros poveikiai laikomi kaip viena iš skerspjūvį veikiančių apkrovų.

Laikomasi prielaidos, kad betonas gniuždymo srityje ir armatūra visur deformuojasi kaip idealiai, tampriai plastiškos medžiagos; betonas tempimo srityje dirba kaip idealiai trapi medžiaga. Remiantis prisitaikymo teorija suformuluotas gelžbetoninių skerspjūvių ribinės analizės optimizacijos uždavinys. Aptarta galimybė apibendrinti šią formuluotę neidealaus plastiškumo atveju esant medžiagos sustiprėjimui ar susilpnėjimui. Pateikti tiesioginis ir atvirkštinis sudaryto uždavinio sprendimo būdai. Pateikti pavyzdžiai rodo, kad apkrovos pakartotinis poveikis mažina gelžbetoninių elementų skerspjūvių laikomąją galią ir armavimo optimalumo parametrą.

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