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DEVELOPMENT OF METHODS FOR DESIGNING RATIONAL TRUSSES

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Abstract. Striving for rationality and long-term reliability is seen in different periods of building activities. Application of linear programming methods has enabled to formalise this striving and to elaborate the necessary mathematical models. But later theoretical and practical investigations have disclosed that not always, when optimising in respect of one criterion, it is possible to obtain solutions rational in other aspects, and this stimulated the application of multicriteria optimization methods. It is useful in this case to apply the ideas of the game theory, game problems solving methods already applied in other building design fields. When adapting methods of the game theory to popular needs for truss designing, a criteria set involving 11 alternatives has been selected. Attempts have been made to find rational truss variants by applying different methods (method of proximity to an ideal point, Wald's and Hurwitz's methods). It has been found when using the method of proximity to an ideal point for rational truss designing that a truss with a sloping brace network and pivoted knots supported by a column and composed of rectangular box shapes is more valuable than other trusses. According to Wald's and Hurvitz's methods, among popular spans of 24 m such a truss is the truss with a lowered bottom chord.

Keywords: rational trusses, multicriteria optimisation, methods of Wald and Hurwitz, method of proximity to an ideal point.

1. Introduction

From different points of view, striving for design and installation of rational building structures is as old as the building construction itself. However, we know that in the second part of the 20th century the concept of rationality in many places was practically supplanted by optimisation (unfortunately, not always sufficiently considered [1]). Minimising the chosen objective function, attempts were made to optimise either expenditure of materials or consumption of labour power and a little later the reliability of structure [2, 3], etc. Succeeding theoretical and practical investigations showed that a structure on optimising it in respect of one criterion cannot always be optimal in respect of other criteria.

Such problems are faced when designing different structures, including metal trusses.

On evaluating the above circumstances, the goal of the present work is to disclose the multicriteria optimality premises for the mostly spread metal trusses, their design possibilities and means.

Much work has been done in this direction. For instance, for optimisation of truss parameters some algorithms concerning materials saving have been suggested [4]. The quantities of materials necessary for trusses are usually minimised by taking a derivative:

$$\frac{\partial G_s}{\partial h} = 0. \tag{1}$$

In this case the truss mass G_S is a function depending on the covered opening l, the moment M corresponding to such an opening in a simple beam, a transverse force Q in the beam, height h, the calculated strength R and correction coefficients. Then for simple triangular trusses it is obtained by

$$h_{opt} = l_j \sqrt{\frac{\frac{2n\sum M}{l}\psi_j}{\sum Q\psi_t} + 1}. \tag{2}$$

Here l_j is the length of truss knot distance; ψ_j is structural coefficient of chord weight; ψ_t is structural coefficient of network diagonal braces weight; n is number of knot distances

$$n = \frac{l}{l_j}.$$

In this case the concept of optimising the truss is directly connected with the mass of web members, as well as with traditional concepts of bending moments and transverse forces. It is supposed that the bending moments developing in the opening l are taken by the truss chords and the transverse forces by the web members. A contradiction arises: the higher the truss, the larger bending moments can be taken by the chord bars of the same cross-section. However, at the same time the length of web members and chords increases, thus their mass becomes larger. It has been suggested to solve this contradiction by assuming some presumptions and, at last, to keep to the ratio

$$\frac{h_{opt}}{l}=\frac{1}{n}\sqrt{\frac{0{,}7n+1}{3}}; \quad (3)$$

for the triangular truss network without rods or, when there are rods in the network of triangular trusses (so much the better when they support small hoists and cranes):

$$\frac{h_{opt}}{l}=\frac{1}{n}\sqrt{\frac{0{,}7n+1}{2}}. \quad (4)$$

Optimisation of trapezium-shape trusses in respect of material consumption minimisation criterion was also analysed by some authors [15–19]. At last it was recommended to consider the optimal height of these trusses as

$$\frac{h}{l}=\frac{\frac{1}{7}}{\frac{1}{9}}. \quad (5)$$

Attention should be paid to the fact that h_{opt} is defined here even more faintly.

When generalising, it should be noted that namely here, in the presented quite simple problems of materials consumption optimisation, it is possible to discern the elements of the minimax problem. It would mean a striving for producing trusses as high as possible in order they become more effective in respect of the moment of external forces taken over; it would also mean a contrary task – that the web members and chords, especially the members under compression, would be as short as possible and of a smaller mass.

By developing the methods of optimisation of metal consumption for trusses [5] attempts were made to formulate a more complex problem in which the objective function is expressed in the following way:

$$C=C_M+C_K+C_S+C_L. \quad (6)$$

Here C_M is the total cost of roof bearing structure;

C_K is the total cost of column intervals in the height limits of roof structure;

C_S is the total cost of wall roofing in the wall height limits;

C_L is the maintenance cost for heating and ventilation.

In this case, on evaluating other expenditures, the optimal height is usually a little smaller, especially in cold regions.

These formulations of optimisation problems and their solving methods were also later improved by other authors (for instance, [3]).

As an optimal criterion was usually taken the total structure mass, sometimes the cost of materials used, rarely the total cost, including the truss manufacture and its erection. For instance, unification of bars decreases truss manufacturing cost but increases considerably the structural mass. Some time later the new mathematical methods were developed and adapted for optimisation of a skeletal structures [5–10].

Also there are well developed mathematical methods successfully used in management engineering and applicable for structural design [11–21].

2. Methods for finding rational solutions

The mathematical methods applied further in this study are, in general, well-known [5–21]. We try to apply them for solving new type problems of structural engineering. It means an adaptation of the known fundamental mathematical achievements in a new field. This aspect expresses the novel character and level of the work.

Under the conditions of indefiniteness, when the probabilities of outward agents are not known, it is expedient to apply methods and rules of the game theory. In the adjoining field of structural engineering, namely, in building technology and management, for finding rational solutions the finite (finite strategies set) games of two agents (of a decision-taking specialist and nature) of zero sums (the profit of a gambler is compensated by the other gambler's loss):

$$\Gamma=\{S_1,S_2;A\}; \quad (7)$$

Here $S_1=\{S_{11},S_{12},\dots,S_{1m}\}$

is the set of the 1st gambler's pure strategies;

$$S_2=\{S_{21},S_{22},\dots,S_{2n}\}$$

is the set of the 2nd gambler's pure strategies;

$$A=[a_{ij}]_{m\times n}$$

is the function of the 1st gambler's gain or of the loss of the second one.

The necessary presumption of the rational solution (of rational game strategy) is the existence of the equilibrium point (saddle point) in the gain function, in the definite set of strategies. In the equilibrium point, the gain function reaches its maximum according to i and its minimum according to j:

$$\max_{S_1}\min_{S_2} A(S_1,S_2)=\min_{S_1}\max_{S_2} A(S_1,S_2)=\nu. \quad (8)$$

It means that the equilibrium strategy of the 1st gambler is the strategy S_1, in which the smallest gain value according to S_2 reaches its maximum; the equilibrium strategy of the 2nd gambler is the S_2 strategy, in which the greatest gain value according to S_1 reaches its maximum.

In practical problems of the 1st gambler's strategy, set S_1 is regarded as the totality of possible (analysed) variants (projects, alternatives). S_2 includes indices (criteria, objective functions) describing the variants efficiency. a_{ij} is values (mostly normalised) under one or another alternative.

The solution (rational variant choice; realisation of the highest gain) according to Hurwitz's rule is based on the weighted average with the parameter λ (risk factor) of the worst possible and best possible results:

$$S_1^* = \{S_{1i} \mid S_{1i} \in S_1 \cap \{S_{1i_0} \mid h_{i_0} = \max_i h_i; \\ h_i = (1-e)\cdot \min_j a_{ij} + e \cdot \max_j a_{ij}; 0 \le \lambda \le 1\}\}; \\ (i = 1, 2, ..., m; j = 1, 2, ..., n). \quad (9)$$

It is accepted that by choosing the 1st strategy the 1st gambler wins no less than the weighted average of smallest possible and largest possible gain values in this strategy:

$$h_i = (1-e)\cdot \min_j a_{ij} + e \cdot \max_j a_{ij}; \\ 0 \le e \le 1. \quad (10)$$

Therefore he will select the strategy in which the lowest possible gain h is the largest one:

$$h_{i0} = \max_i h_i. \quad (11)$$

The solution according to Wald's rule is to maximise the guaranteed gain. This has become known as maximum criterion. According to this rule, the optimal strategy is:

$$S_1^* = \{S_{1i} \mid S_{1i} \in S_1 \cap \{S_{1i_0} \mid a_{i_0 j_0} = \max_i \min_j a_{ij}\}\}. \quad (12)$$

The Wald's rule and the minimax principle prescribe, under the saddle point, to the 1st gambler the same optimal strategy.

If the elements of the formed initial solution matrix P are not single-valued (of different units of measurement, and there are maximised and minimised criteria), it is necessary to normalise it by using methods of scale transformation:

1. For values of maximised criteria

$$\overline{a_{ij}} = \left(\frac{a_{ij}}{a_j^*}\right)^2; \quad a_j^* = \max; \quad (i = \overline{1,m}; j = \overline{1,n}) \quad (13)$$

2. For values of minimised criteria

$$\overline{a_{ij}} = \left(\frac{a_j^*}{a_{ij}}\right)^3; \quad a_j^* = \min; \quad (i = \overline{1,m}; j = \overline{1,n}). \quad (14)$$

We determine the existence (or non-existence) of the equilibrium point by the rules of game theory; if two or more equivalent strategies are determined, the gain function does not possess a point of equilibrium in the set defined by strategies. The problem is solved by Simplex method or by introducing an additional criterion.

By applying the method of proximity to the ideal point a generalised criterion is formed and it discloses the deviation of variants being compared from an ideal variant by taking into account only the best indices values. The generalised criterion is calculated for every variant compared, therefore it is possible to determine the priorities of the variants. In addition, contrary to the application of game theory rules, it is possible to evaluate the theoretical, subjective or complex importance of the criteria. Determination of variant priorities is performed by the following stages:

1. The first stage would be the design of initial matrix $[P]$ of compared alternative solutions.
2. The normalisation of the initial matrix $[P]$ into matrix $[\overline{P}]$ is performed by the formula:

$$\overline{x_{ij}} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^2}}; \quad i = \overline{1,m}; \quad j = \overline{1,n}. \quad (15)$$

3. The formation of the weighted decision-making matrix $[\overline{P}^*]$. The matrix is obtained by multiplying matrix $[\overline{P}]$ by the significant test vector $\overline{q}$:

$$\overline{P}^* = [\overline{P}]\cdot[\overline{q}]. \quad (16)$$

4. The formation of weighted normalised matrix when evaluating the importance of individual stages. We consider the erection of trusses as a complex process consisting of separate processes. Expenditures of materials, rigidity resource, manufacturing expenditures, mounting expenses, transport and painting costs – all these processes we regard as stages. Stages are different constituent processes and their influence cannot be of a single meaning in the complex process. Thus when performing the multicriteria analysis of a complex process it is necessary to take into account the importance of both the criteria and every individual stage.

We recommend calculating the weighted normalised matrix when evaluating the importance of stages according to the formula:

$$\overline{P}^* = \sum_{m=1}^{d}\left(\frac{x_{ij}^m \cdot \overline{x}_{ij}^{*(d)}}{x_{ij}^{(d)}}\cdot\left[\overline{q_m}\right]\right), \quad m = \overline{1,d}. \quad (17)$$

Here d is the number of stages; $\overline{q_m}$ is the significance of stages, x_{ij}^m is the jth criterion value characterising the ith alternative of the mth stage; $\overline{x}_{ij}^{*(d)}$ is the normalised weighted value of jth criterion of the ith alternative corresponding to a definite combination; $x_{ij}^{(d)}$ is the value of jth criterion of the ith alternative corresponding to a certain combination.

5. The ideal positive and ideal negative variants are determined by the following formulas:

$$a^{+} = \left\{\left[\left(\max_{i} f_{ij} \, / \, j \in I\right), \left(\min_{i} f_{ij} \, / \, j \in I'\right)\right] / \, i = \overline{1,m}\right\} = \left\{f_1^{+}, f_2^{+}, K, f_n^{+}\right\} \tag{18}$$

$$a^{-} = \left\{\left[\left(\max_{i} f_{ij} \, / \, j \in I\right), \left(\min_{i} f_{ij} \, / \, j \in I'\right)\right] / \, i = 1,m\right\} = \left\{f_1^{-}, f_2^{-}, \dots, f_n^{-}\right\}. \tag{19}$$

Here I is a set of indices (maximised-majorised) the best values of which are the biggest ones; I' is a set of indices (minimised-minorised), the best values of which are the smallest ones; K is the best alternative (the ideal positive variant).

6. The distance between the ideal positive a^{+} (ideal negative a^{-}) and real a_i

variants is defined:

$$L_i^{+} = \sqrt{\sum_{j=1}^{n}\left(f_{ij} - f_{ij}^{+}\right)^2}, \quad \forall_{ij}, i = \overline{1,m}; j = \overline{1,n}, \tag{20}$$

$$L_i^{-} = \sqrt{\sum_{j=1}^{n}\left(f_{ij} - f_{ij}^{-}\right)^2}, \quad \forall_{ij}, i = \overline{1,m}; j = \overline{1,n}. \tag{21}$$

7. As the last stage the conditional proximity of compared variants to the ideal one is defined, ie K_{bit} criterion according to formula:

$$K_{bit} = \frac{L_i^{-}}{L_i^{+} + L_i^{-}}. \quad \forall_i; i = \overline{1,m}. \tag{22}$$

The variants are lined up by priority according to Kbit values. The best variant is the one with the greatest value of this criterion.

3. The variants analysed and their criteria of rationality

From the multicriteria point of view, the variants of rational truss design are discussed for a popular at the present time 24 m long span. For the sake of comparison of the results obtained, the span size is unified, other criteria are varying. Five variants of truss geometry of recently mostly applied trusses are discussed at the present time.

Taking into account [22–24] the character of truss support by columns and the types of truss web members cross-section (angles and box shapes), eleven alternatives for illustrating the method of application have been analysed (Fig 1).

A11 – truss with triangle, additional rods and a network rigidly tied to columns consisting of rectangular box shapes [1];

A12 – truss with triangle, additional rods and a network rigidly tied to columns of two unequal angles [2];

A13 – truss with triangle, additional rod and a network of pivoted knots, supported by columns of rectangular box shapes [3];

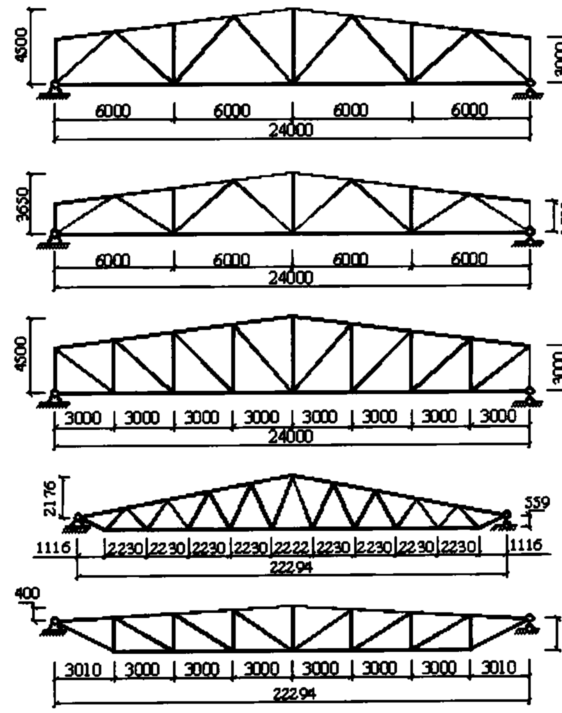

Fig 1. Schemes of the trusses (alternatives) discussed

A14 – truss with triangle, additional rods and a network of pivoted knots, supported by columns of two unequal angles [4];

A21 – truss with triangle, additional rods and a network of pivoted knots, supported by columns of two equal angles [5];

A31 – truss with sloping brace network rigidly tied to columns consisting of rectangular box shapes [6];

A32 – truss with sloping brace network rigidly tied to columns consisting of two unequal angles [7];

A33 – truss with sloping brace network of pivoted knots supported by columns of rectangular box shapes [8];

A34 – truss with sloping brace network of pivoted knots supported by columns of two unequal angles [9];

A41 – truss with a lowered bottom chord of 24 m long; of rectangular box shapes [10];

A51 – truss with a lowered bottom chord of 22,3 m length, of rectangular box shapes [11].

These different types of trusses are best evaluated by the following technical and economic indices (TEI):

K1 – material expenditures;

K2 – rigidity resource;

K3 – manufacturing expenditures;

K4 – assembling work expenditures

K5 – transport costs (Lt);

K6 – painting costs (Lt).

The presented criteria are of different significance when evaluating the multicriteria rationality of a truss. [25] (Therefore some decision-making methods include an valuation of criteria significance in advance by removing unimportant criteria and by decreasing or increasing the influence of criteria on the efficiency of some alternatives. Justification of criteria is a quite complicated multilateral problem. More detailed evidence of a multiplicity of optimisation criteria with attraction of a heuristic technique will be discussed in a special article to be published).

4. Discussion of the solutions obtained

Independently of the network scheme and the way of support, the K_{bit} values of rectangular box shape trusses achieved by the theoretical, complex and subjective evaluation of criteria significance are higher than those of trusses composed of two unequal angles.

Also, when comparing alternatives (variants) by Wald's and Hurwitz's methods, it has been found that variants minimal values are characteristic of the trusses composed of two unequal angles.

Of different alternatives evaluated in the Figure according to complex and subjective significance of criteria, the truss with a sloping brace network with pivoted knots, supported by column and composed of rectangular box shapes can be considered as a more valuable truss. From the point of view of theoretical significance, it refers to the truss with a lowered bottom chord and composed of rectangular box shapes. But these obtained results differ slightly among themselves.

The best truss constructed by Wald's and Hurwitz's method is the same as that one produced by the theoretical valuation of criteria significance.

Having applied the game theory methods for designing selected trusses, the results obtained in the limits of criteria set are briefly presented in Figs 2–4.

5. Conclusions

1. When optimising the truss to be designed according to one of the selected criteria, for instance, to a structural criterion, the truss can be rather irrational in respect of other criteria (say, criteria of production, maintenance, etc). Therefore when seeking for rational solution it reasonable to apply methods of multicriteria optimisation.

2. For designing rational trusses and other structures it is expedient to adapt the game theory methods. Striving for universality of methods and programmes, it is sensible to introduce for this purpose the principles and

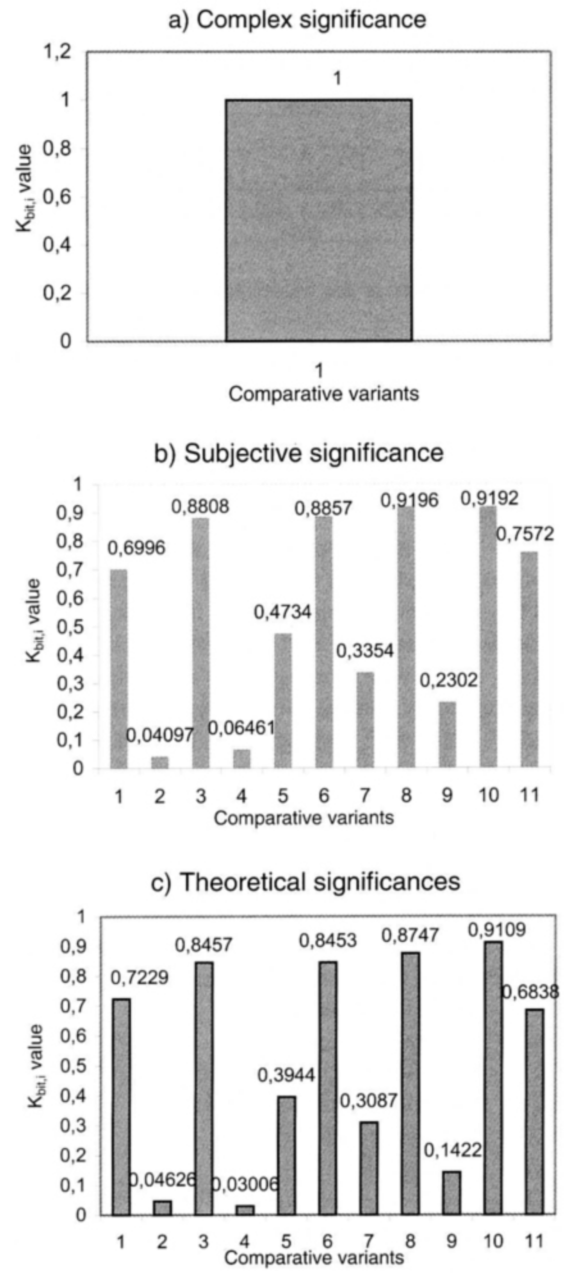

Fig 2. Valuating by the proximity to ideal point method the truss rationality results obtained by using complex valuation (a), subjective valuation (b) and theoretical valuation (c) of criteria significance

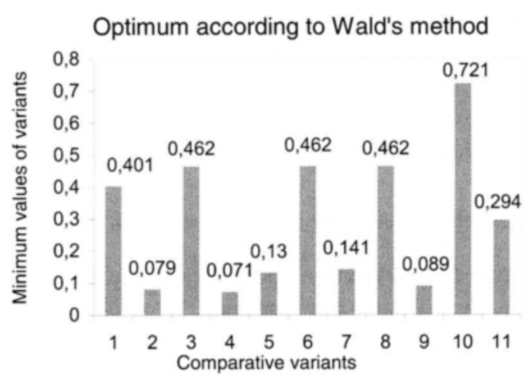

Fig 3. Valuation of truss rationality by Wald's method

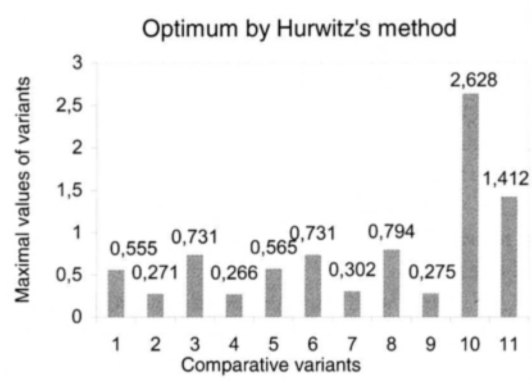

Fig 4. Valuation of truss rationality by Hurwitz's method

methods of two agents (man and nature) null sums game of finite strategy set (these principles and methods are applied in the technology and management of building operations).

3. When designing rational (from the multicriteria point of view) trusses, it is expedient to form a criterion set which includes the subsets of structural, manufacturing, service rationality criteria.

4. Following the method of proximity to the ideal point, it has been found that the truss with sloping brace network and pivoted knots supported by a column and made of rectangular box shapes may be considered as a more valuable structure. According to Wald's and Hurwitz's methods, the structure of such a kind is the truss with a lowered bottom chord and of rectangular box shapes. But the results obtained differ slightly.

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