

REVIEW OF AVAILABLE APPROACHES FOR ULTIMATE BEARING CAPACITY OF TWO-LAYERED SOILS

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Abstract. This paper presents the state of the art report on available approaches to predicting the ultimate bearing capacity of two-layered soils. The article discusses three most popular methods, including the classical method, application of the finite element method and artificial neural network. Various approaches based on these three powerful tools are studied and their methodologies are discussed.

Keywords: review paper, multi-layered soil, ultimate bearing capacity, finite element method, artificial neural network.

1. Introduction

The ultimate bearing capacity of shallow footings has been a challenging area among the researchers and geotechnical engineers for the last three decades. Since early 1950s the investigations on shallow foundation resting on single layer has been carried out and it is still ongoing due to complex nature of the soil. Earlier theories were developed from experimental tests and the analytical solutions had been used, to this day, by engineers for design purpose.

Emergence of computers has definitely made an important change in the way to find solution for soil performance. The classical methods which are deemed unable to accurately model soil or are limited to a particular type of behavior were reinforced by a new computational technique with high capabilities of stress-strain prediction such as Finite Element Method (FEM).

Regardless of merits and demerits of modern numerical approaches, the results are reported to be in close agreement to those of experimental ones while encountered discrepancies and differences seem to be ignored. In fact, the source of these differences comes up from the complex nature of the soil itself whose performance is so complicated and not easy to be assessed. Furthermore, the type of soil may vary from place to place which cause more complexity.

In the presence of sufficient data the Artificial Neural Network (ANN), which is an advanced interpolation tool, is capable of prediction for different soil characteristics. This mathematical system takes advantage of a number of numeric weights in which the relations among

the input data and targets are stored during training and then becomes capable of predicting new data from some new input data which were not experienced during training. Finally these relations can be exploited and utilized to make new equations.

The main objective of this work is to present a state of art report on ultimate bearing capacity of two-layered soil system. The literatures are reviewed and grouped herein as:

- a) Classical methods;
- b) Numerical methods including:
 - i. Finite Element Method (FEM);
 - ii. Artificial Neural Network (ANN).

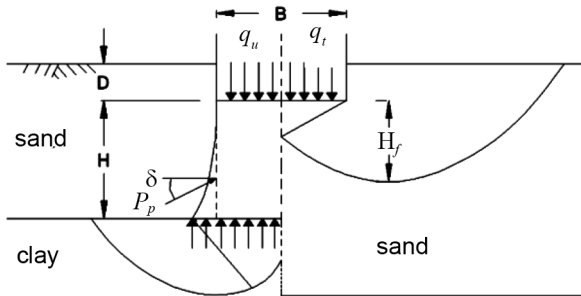
2. Classical methods of ultimate bearing capacity in two-layered soil

Ultimate bearing capacity of two-layered system has been a major concern among the researchers till date due to the discrepancies between developed theoretical approaches and experimental studies.

The bearing capacity of layered soil system for both cases of dense sand over soft clay and loose sand overlying stiff clay has been studied for both strip and circular foundations (Meyerhof 1974). Meyerhof (1974) suggested that for the case of loose sand over stiff clay the bearing capacity is limited to top layer which means that failure surface is also limited to the top layer and pressure does not reach to the bottom layer. Fig. 1 illustrates the failure mechanism adopted by Meyerhof (1974). The ultimate bearing capacity expressions developed by Meyerhof are tabulated in Table 1.

Table 1. U.B.C expression by Meyerhof (1974)

Soil profile	Case no.	Footing profile	Expression for U.B.C
Dense sand on soft clay	1	Strip	$q_u = CN_c + \gamma H^2 \left(1 + \frac{2D}{H}\right) K_s \frac{\tan \phi}{B} + \gamma D \leq q_t^*$
	2	circular	$q_u = 1.2CN_c + 2\gamma H^2 \left(1 + \frac{2D}{H}\right) sK_s \frac{\tan \phi}{B} + \gamma D \leq q_t$
C: undrained cohesion of clay, N_c : bearing capacity factor=5.14, D:depth of embedment H: thickness of top layer, B: footing width, γ : unit weight of top layer, s: shape factor K_s : punching shear coefficient $q_t = 0.5\gamma BN_\gamma + \gamma DN_q$ (for strip footing); $q_t = 0.3\gamma BN_\gamma + \gamma DN_q$ (for circular footing)			
Loose sand on stiff clay	1	strip	$q_u = 0.5\gamma BN'_\gamma + \gamma DN'_q \leq q_b^*$
	2	circular	$q_u = 0.5s'_\gamma \gamma BN'_\gamma + s'_q \gamma DN'_q \leq q_b$
$q_b^* = cN_c + \gamma D$ (for strip footing); $q_b = 1.2cN_c + \gamma D$ (for circular footing)			

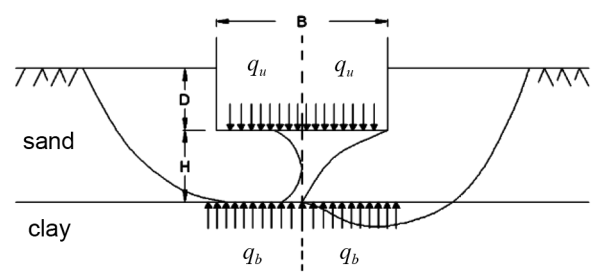
**Fig. 1.** Failure mechanism for dense sand over clay by Meyerhof (1974)

When the loose sand is found to be located on stiff bed of clay it is suggested to assume the clay layer as a rigid base while the shear failure zone is extended to the bottom layer as long as bearing capacity of sand layer is increasing to approach that of clay layer. In this case continuity of shear zone is met at the interface of two layers (as shown in Fig. 1).

Fig. 2 indicates the presumed failure mechanism by Meyerhof in which the left side (with respect to centre line of the footing) represent the case in which bearing capacity of top layer is lower than bottom layer and the right side is for the case whose bearing capacity of both layers are close.

The aforementioned case has been studied by Hanna (1982) for both strip and surface footings and ultimate bearing capacity has been formulated through modification of Terzaghi's equation whose results should not exceed those of lower stiff layer. Modified bearing capacity factors regarding density term and overburden term are introduced as function of (H/B) ratio and internal angle of friction. Table 2 shows the modified expressions proposed by Hanna (1982) in more detail.

Moreover, Hanna (1981b) developed design charts for the case of strong sand over loose sand layer following punching shear theory and expressing the failure zone as roughly truncated pyramid being punched into the bottom layer. This method has been introduced as a solution to improve the bearing capacity of subsoil by replacing the

**Fig. 2.** Failure mechanism for loose sand over stiff clay by Meyerhof (1974)

top loose sand by strong one. Table 3 displays the equations employed for strong sand overlying weak clay.

Similar case has been studied through combination of experimental tests and theoretical procedure based on the classical form of bearing capacity proposed by Terzaghi and it was assumed to be valid by introducing a modified bearing capacity factor which is associated with density as a function of (H/B) ratio as well as passive pressure coefficient and internal angle of friction (Andrawes *et al.* 1996). Eq. (1) expresses the modified bearing capacity factor $N_{\gamma s}$ which is valid for $(H/B) > 1$:

$$N_{\gamma s} = \frac{H}{B} \left[\left(\frac{1 + \sin \phi}{1 - \sin \phi} \right)^2 - 1 \right], \quad (1)$$

in which ϕ is internal angle of friction. The assumed mechanism is based on the formation of a central zone bounded right below the smooth strip footing and the rigid lower layer. The penetration continues till no densification is possible anymore and this is the time that the lateral pressure is exerted horizontally and the central zone bulges. This mechanism is illustrated in Fig. 3.

The proposed method has shown reasonably good agreement when the angle of friction obtained through tri-axial test is used for 2-D problem is plain strain. It is concluded that the footing roughness has no effect on the bearing capacity while the thickness of top layer is the most effective parameter for the ultimate bearing capacity decreases to a minimum value by decreasing H/B ratio.

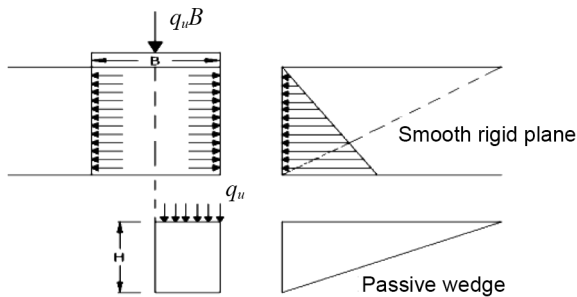


Fig. 3. Failure mechanism by Andrawes *et al.* (1996)

In general both Meyerhof and Hanna believed in punching shear failure for the case of homogeneous thin dense sand on a thick and loose bed layer of clay while different shape of failure surface for the case of thick dense layer of sand underlain by thick layer of clay has been suggested by them. In the later case the critical depth (H_f) which involves the failure surface is estimated as a function of ratio of bearing capacity of bottom layer to that of upper one (Fig. 1).

Both Meyerhof and Hanna theories have been further extended for the case of three-layered soil, i.e. two strong sand layers overlying a loose clay layer (Hanna, Meyerhof 1979).

Mechanism of failure is punching shear penetrating into both upper sand layers while the clay layer follows

the well known Prandtl failure mechanism. Equations formulated by Hanna and Meyerhof (1979) are tabulated in Table 4.

Results obtained through an empirical equation proposed by Satyanarayana and Garg and those of experimental tests for case of strong layer overlying weak layer with an emphasis on bearing capacity as well as load settlement curve (Hanna 1981a).

Hanna has carried out a parametric study on the effect of undrained shear strength of clay, the ratio of depth of top layer on width of surface footing (H/B) and depth of embedment on width of footing (D/B). It is observed that overall bearing capacity of two-layered soil, i.e. strong sand on loose clay, increases by increasing the (H/B) and (D/B) ratios while the opposite trend is found for case of weak layer on strong one when (H/B) increases. The later result is also reported and depicted in Fig. 4 (Hanna 1982).

Considering the punching shear failure mode, compressibility of the soil is then automatically counted for. It is worth mentioning that assumption of general shear failure mode limits the theoretical formulations to incompressible soils following the behavior of a rigid-plastic solid which exhibits no deformation unless shear failure happens (Ismael, Vesic 1981).

Table 2. U.B.C expression by Hanna (1982) (Weak sand on strong layer)

Soil profile	Case no.	Footing profile	Expression for U.B.C
Weak sand on strong layer	1	Strip	$q_u = 0.5\gamma_1 B N_{\gamma}^{\prime**} + \gamma_1 D N_q^{\prime\#} \leq q_b^*$
	2	circular	$q_u = 0.5\gamma_1 B s_{\gamma}^{\prime} N_{\gamma}^{\prime**} + \gamma_1 D s_q^{\prime} N_q^{\prime\#} \leq q_b^*$

$$q_b^* = 0.5\gamma_2 B N_{\gamma 2} + \gamma_1 H N_{q 2} \text{ (for strip footing);}$$

$$q_b^* = 0.5\gamma_2 B s_{\gamma 2} N_{\gamma 2} + \gamma_1 H s_{q 2} N_{q 2} \text{ (for circular footing);}$$

$$N_{\gamma}^{\prime**} : \text{modified bearing capacity factor} = N_{\gamma 2} - (H/H_{f\gamma})(N_{\gamma 2} - N_{\gamma 1});$$

$$N_q^{\prime\#} : \text{modified bearing capacity factor} = N_{q 2} - (H/H_{fq})(N_{q 2} - N_{q 1});$$

$$N_{\gamma}^{\prime**} : \text{modified bearing capacity factor} = N_{\gamma 1} + (1 - H/H_{f\gamma})^2 (N_{\gamma 2} - N_{\gamma 1}); \text{settlement is not added}$$

$H_{f\gamma}, H_{fq}$: depth of failure plane in a thick layer of sand beneath the footing for weight and overburden pressure respectively.

$$N_{\gamma 2} = \left(\frac{c_u N_c}{0.5\gamma B} \right).$$

Table 3. U.B.C expression by Hanna (1982) (Strong sand over weak sand)

Soil profile	Case no.	Footing profile	Expression for U.B.C
Strong sand over weak sand	1	Strip	$q_u = q_b + \gamma_1^{\bullet} H^{\circ 2} \left(1 + \frac{2D^*}{H}\right) K_s^{**} \frac{\tan \phi_1}{B} - \gamma_1 H \leq q_t$
	2	circular	$q_u = q_b + 2\gamma_1 H^2 \left(1 + \frac{2D^*}{H}\right) S_s^{\prime} K_s^{**} \frac{\tan \phi_1}{B} - \gamma_1 H \leq q_t$

D^* : Depth of embedment, H° : thickness of top layer, γ_1^{\bullet} : unit weight of top layer, S_s^{\prime} : shape factor

K_s^{**} : punching shear coefficient

Table 4. Expression of U.B.C for three-layered soil by Hanna and Meyerhof (1979)

Soil profile	Case no.	Footing profile	Expression for U.B.C
Two strong layers of sand over weak bed of clay	1	Strip	$q_u = q_b^* + \frac{2(C_{a1}^{**} H_1 + C_{a2}^{**} H_2)}{B}$ $+ K_{s1} \frac{\bar{\gamma}^{\#} H_1^2 \tan \phi_1^{\ddot{\cdot}}}{B} \left(1 + \frac{2D'}{H_1}\right) + K_{s2} \frac{\bar{\gamma} H_2^2 \tan \phi_2^{\ddot{\cdot}}}{B} \times \left[1 + \frac{2(H_1 + D)}{H_2}\right]$ $-\bar{\gamma}(H_1 + H_2) \leq q_t^*$
	2	Circular	$q_u = q_b + S_s \left[K_{s1} \frac{\bar{\gamma} H_1^2}{B} \tan \phi_1 + K_{s2} \frac{\bar{\gamma} H_2^2}{B} \tan \phi_2 \left(1 + \frac{2H_1}{H_2}\right) \right]$ $-\bar{\gamma}(H_1 + H_2) \leq q_t$

D' : depth of embedment

q_b^* and q_t^* : bearing capacities of footing resting on very thick beds of bottom and top layer respectively,

$\bar{\gamma}^{\#}$: average unit weight of first and second layer = $(\gamma_1 + \gamma_2) / 2$,

C_{a1}^{**} and C_{a2}^{**} : unit adhesions,

$\phi_1^{\ddot{\cdot}}$ and $\phi_2^{\ddot{\cdot}}$: internal angles of friction belong to first and second sand layers respectively.

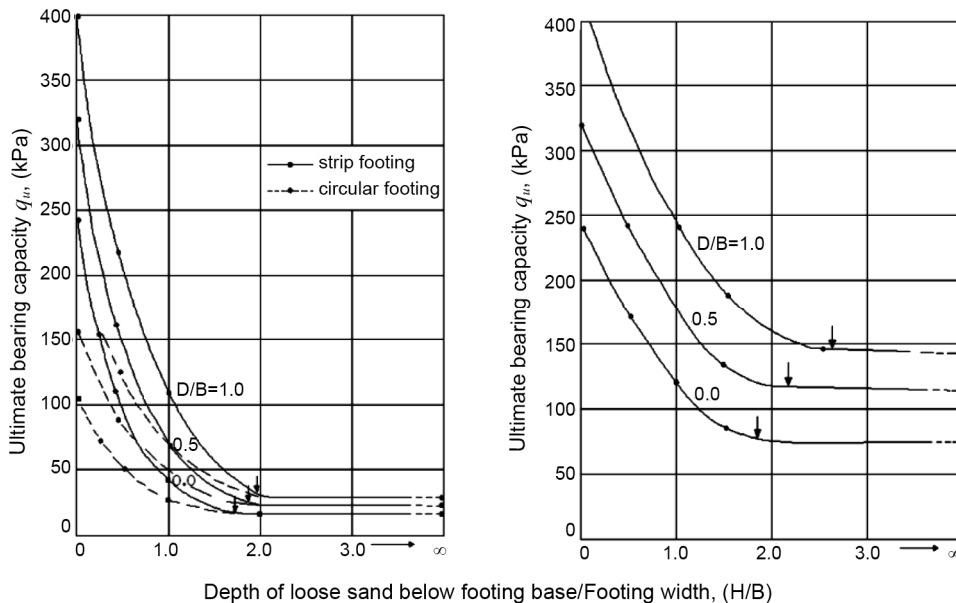


Fig. 4. Bearing capacity of two layer soil by Hanna (1982): strip and circular footings in loose sand overlying a dense sand (left); strip footing resting on compact sand underlain by dense sand (right)

However, as long as external manifestation of failure mode is concerned it is easier to find the peak load from the load-settlement curve under general shear failure during performing experimental tests. The difficulty of finding the peak load is pronounced in case of high values of (H/B) ratios in weak layer underlain by strong one, and also low (H/B) ratios for strong layer overlying weak strata since the failure mode tends to change from general shear failure mode to local shear one (Hanna 1981a, 1982).

It is observed from this study that any parameter which affects the mode of failure may have influence on

ultimate bearing capacity as well. These factors may be such as size and shape of the footing, configuration of the soil and load application, strength of both layer and arrangement of layers (Hanna 1981a, 1982).

Meyerhof's equations are also involved the key properties of soil layers such as internal angle of friction (ϕ), undrained shear resistance C_u , D/B , H/B , etc.

Another significant factor in Meyerhof's equations is punching shear coefficient (K_s) that is function of internal angle of friction and passive pressure which is

mobilized during the punching of sand layer basically because of lateral resistance of sand layer through the force which lies on a direction making angle of δ (δ) to horizontal direction (refer to Fig. 1).

Calculation of punching shear coefficient is discussed comprehensively and final results are presented in the form of design charts for ultimate bearing capacity of foundations on sand overlaying soft clay strata (Hanna, Meyerhof 1980). The developed equation by Hanna and Meyerhof is said to overestimate bearing capacity compared to experimental tests especially for values of D/B ratio greater than 3 as well as for large depths of sand however may lead to more accurate results provided that the local shear failure mode is taken into the account (Kenny, Andrawes 1997).

Since the proposed semi-empirical method by Hanna is based on the punching shear which assumes a vertical cylinder beneath the strip footing while the sides tend to bend outward as the cylinder moves down in an actual case, the assumed passive pressure which is also function of strength of bottom layer, lies on an upward direction with lower magnitude of internal angle of friction compared to those of actual values which is also confirmed by Hanna (1981b).

Due to different failure strain of top and bottom layer failure cannot take place in both layers simultaneously, hence the mobilized angle of shear resistance of the sand layer is less than its peak value.

Fig. 5 presents few random results reproduced here from Hanna's equation described in Table 3. The overall trend of equation, for three values of footing width is illustrated hereafter. As expected, the bearing capacity of two-layered soil increases by incrementing (H/B) and (D/B) ratios.

The increment of bearing capacity is up to a certain value of (H/B) after which it goes constant. This is because the overall ultimate bearing capacity of two-layered system is not allowed to exceed that of top layer. This criterion is provided through an inequality presented in Table 3.

As it was mentioned before, punching shear coefficient which is given by Hanna and Meyerhof is function of (δ/ϕ) ratio which itself is function of bearing capacity of top clay layer on that of bottom layer ratio (q_2/q_1) in which:

$$q_1 = 0.5\gamma_1BN_\gamma \quad (2)$$

$$q_2 = cN_c \quad (3)$$

The presence of thin layer of clay in a thick bed of sand called interstratified layers undergoing strip footing has been experimentally studied while the major concern was to see the effect of neglecting non-uniformity of the soil (Oda, Win 1990). Presence of critical depth (only for the case of thin clay layer) at which the clay layer has the most descending effect on the overall bearing capacity has been confirmed by Oda and Win and its corresponding magnitude is found to be two times of footing width.

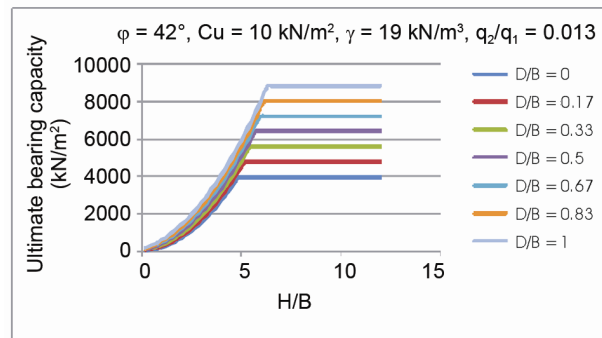
This conclusion does not seem to exist for thick interstratified clay layers while for all cases (regarding the

thickness of clay layer) plastic strains tend to be limited to shallow depth under strip footing.

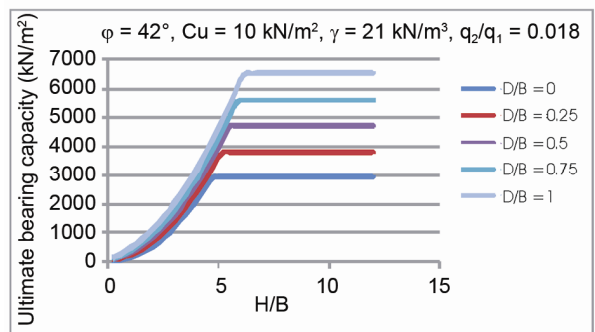
When the presence of clay layer is encountered in practice underlying a sand layer it is said that lateral plastic strains of clay negatively affect the bearing capacity of upper sand layer which consequently decreases overall bearing capacity.

The comparative study has confirmed the overall trend of stress-settlement curves to be the same as the case in which subsoil is considered as uniform. Curves are found to be convex downward which has arisen from non-uniformity of subsoil. The peak value of ultimate bearing capacity is reported to be hard to be placed when clay layer approaching the surface footing.

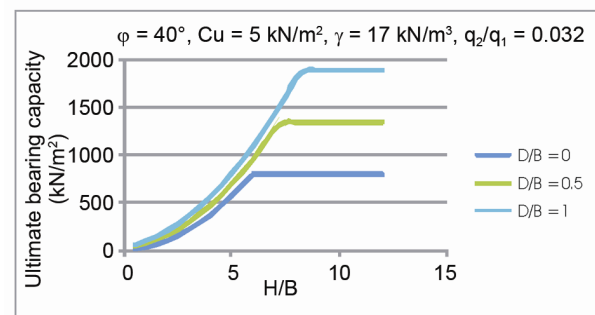
In similar way a theoretical method based on punching shear mode is developed assuming a strip element of soil beneath a strip footing located on a two layer soil system (Al-Shenawy, Al-Karni 2005). Analysis is done through equating summation of forces mobilized against the external load to zero. The failure zone in upper sand layer is the same as that of Hanna and Meyerhof while



a)



b)



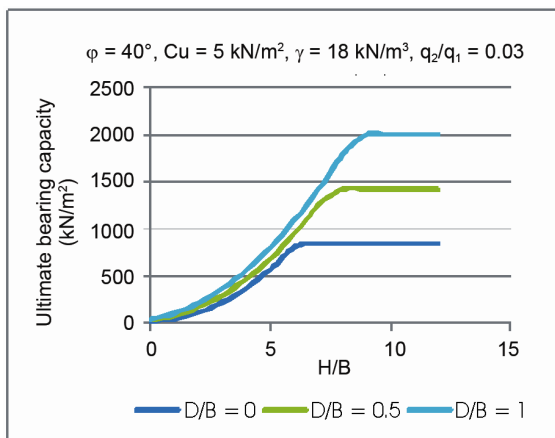
c)

Fig. 5. Ultimate bearing capacity of two-layered soil for: (a) B = 3 m; (b) B = 2 m and (c) B = 1 m

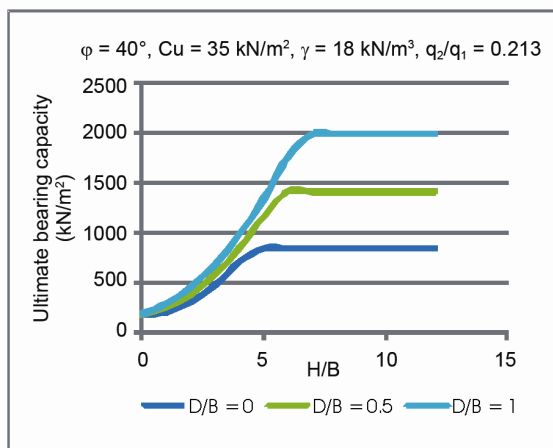
the failure mode of lower clay layer is following the prandtl type.

In addition to the key parameters that are normally found in all theoretical equations the developed equation is also a function of angle of mobilized passive pressure which should be extracted from design charts prepared by Hanna and Meyerhof (1980). Equation is presented here for ease of reference (Eq. 4). The results are presented in their paper as design charts in a dimensionless form for ease of use. The overall performance of equation above is depicted in Fig. 6 for few random cases with varying (H/B) and (D/B) ratios:

$$q_u = 5.14C_u + \gamma B \left(\frac{D}{B} + K_p \tan \delta \left(\frac{H}{B} \right)^2 + 2K_p \tan \delta \left(\frac{D}{B} \right) \left(\frac{H}{B} \right) \right) \leq q_t \quad (4)$$



a)



b)

Fig. 6. Ultimate bearing capacity by Alkarni's equation

Application of load spreading method known as projected area method has been studied by some researchers for two-layered soil system (Carlos 2004; Kenny, Andrawes 1997; Okamura *et al.* 1998). In this approach external load is supposed to spread linearly from either edges of footing to a larger area of sand as pressure penetrates deeply into the top layer through a constant angle

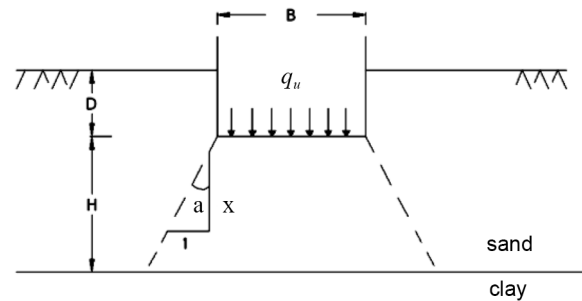


Fig. 7. Projected area mechanism by Kenny and Andrawes (1997)

therefore the intensity of the load decreases along the depth. This mechanism is presented in Fig. 7.

Kenny and Andrawes (1997) have found that better and more reliable results can be obtained by employing lower values of load spread angle. Carlos (2004) has developed an equation for strip footing resting on two-layered soil employing punching shear mode following projected area method which is more similar to actual shape of failure therefore closer value of δ can be selected to that of internal angle of friction (ϕ).

The suggested equation is developed through summation of forces induced and mobilized against exerted pressure at a selected strip element located in upper sand layer (Carlos 2004):

$$\frac{q_u}{\gamma_1 B} = \frac{q_b}{\gamma_q B} + \frac{K_p \sin \delta}{\tan \alpha} \left[\frac{DF}{B} + \frac{H}{B} - \frac{F}{2 \tan \alpha} \right] - \frac{H}{B}, \quad (5)$$

in which α is the angle of assumed failure plane with the vertical direction originating from edge of footing as indicated in Fig. 7. There was good agreement with experimental results for small values of (H/B) ratio while there is wide discrepancy for higher values of (H/B) ratio.

The projected area method, theoretical equation developed and applied by Hanna and Meyerhof and centrifugal method are compared and the punching shear factor which is offered by Hanna and Meyerhof (1980) is discussed in their paper (Okamura *et al.* 1998).

The following conclusions are drawn through comparison of Hanna and Meyerhof's equation with centrifugal test:

- Since the proposed punching shear by Hanna and Meyerhof is independent of depth of footing embedment those values of punching shear for footing with embedment are said to be less than the actual ones which leads to underestimation of ultimate bearing capacity of two layer system;
- Effect of increasing α is more pronounced for circular footings than strip ones;
- In case of small undrained shear resistant and high (H/B) ratio the equation underestimates the bearing capacity;
- The suggested punching shear coefficient are in good agreement with those observed for circular footing, however some discrepancies in the mentioned case are due to the assumption

- of $\alpha = 0$ by Hanna and Meyerhof. The equation overestimates those observed in centrifugal test through increment of undrained shear strength of clay and overburden pressure;
- v. Basically the existing differences among projected area method and that of Hanna and equation are because of the assumed shape of sand blocks and mobilized forces on the sides of block;
 - vi. It can be concluded that the Hanna and Meyerhof's equation underestimates the bearing capacity when there is no embedment and overestimates as the (H/B) ratio increases.

Furthermore it was reported that projected area method underestimates the bearing capacity of strip footing and overestimates those of circular footings since the highest magnitude of α has been chosen over the range of 0° to 30° by Okamura *et al.* (1998). This is the angle for which reactions at clay surface plays an important role at overall bearing capacity.

The relationship between the base area of sand block on that of footing ratio and angle α is plotted for two cases of $(H/B) = 1$ and 2 and presented in the following for ease of refer (Fig. 8).

A theoretical equation is also developed by Okamura following same basic as projected area method and it is illustrated in Fig. 9 while the bearing capacity of bottom clay layer is supposed to be the same as the applied vertical stresses at the interface of two layers (at the base of sand block). The expressions suggested by Okamura are shown in Table 5 in detail.

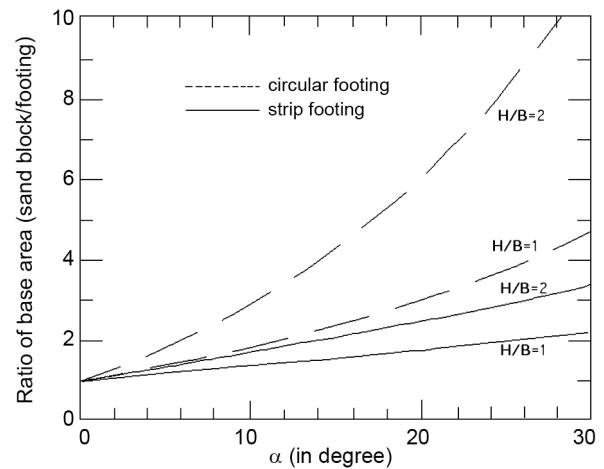


Fig. 8. Variation of base area ratio with angle of the side of sand block developed by Okamura *et al.* (1998)

The ultimate bearing capacity of two layer of purely cohesive soil employing linear nonhomogeneity is investigated by Reddy and Srinivasan (1971).

This nonhomogeneity has been counted by linear variation of undrained cohesion while no angle of friction has been considered (saturated condition). The aforementioned analysis has come up with different types of anisotropy while the vertical strength on horizontal strength ratio, known as anisotropy index, is said to be constant for any type of clay Reddy and Srinivasan (1971).

Table 5. Developed equations by Okamura *et al.* (1998)

Soil profile	Case no.	Type of Footing	Expression for U.B.C
sand over clay	1	Strip	$q_u = (1 + 2 \frac{H}{B} \tan \alpha_c^*) (c_u N_c + p'_o + \gamma' H) + \frac{k_p \sin(\phi' - \alpha_c)}{\cos \phi' \cos \alpha_c} \times \frac{H}{B} (p'_o + \gamma' H) - \gamma' H (1 + \frac{H}{B} \tan \alpha_c)$
	2	circular	$q_u = (1 + 2 \frac{H}{B} \tan \alpha_c)^2 (c_u N_c s_c + p'_o + \gamma' H) + \frac{4k_p \sin(\phi' - \alpha_c)}{\cos \phi' \cos \alpha_c} \{ (p'_o + \frac{\gamma' H}{2}) \frac{H}{B} + p'_o \tan \alpha_c (\frac{H}{B})^2 + \frac{2}{3} \gamma' H \tan \alpha_c (\frac{H}{B})^2 - \frac{\gamma' H}{3} \{ 4(\frac{H}{B})^2 \tan^2 \alpha_c + 6 \frac{H}{B} \tan \alpha_c + 3 \} \}$

$$\alpha_c^* : \text{angle of the side to the vertical direction} = \tan^{-1} \left(\frac{\sigma_{mc}/c_u - \sigma_{ms}/c_u (1 + \sin^2 \phi')}{\cos \phi' \sin \phi' \sigma_{ms}/c_u + 1} \right);$$

$$\sigma_{mc}/c_u = N_c s_c (1 + \frac{1}{\lambda_c} \frac{H}{B} + \frac{\lambda_p}{\lambda_c});$$

$$\sigma_{ms}/c_u = \frac{\sigma_{mc}/c_u - \sqrt{(\sigma_{mc}/c_u)^2 - \cos^2 \phi' ((\sigma_{mc}/c_u)^2 + 1)}}{\cos^2 \phi'};$$

$$\lambda_c = c_u N_c / \gamma' B;$$

$$\lambda_p = p'_o / \gamma' B.$$

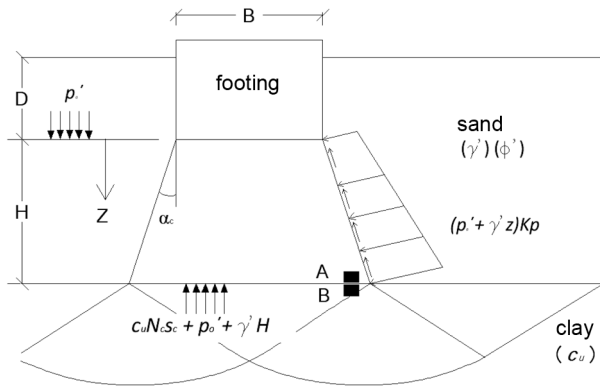


Fig. 9. Failure mechanism developed by Okamura et al. (1998)

A different rupture surface is proposed that penetrates to the second layer also and the whole failure surface is a part of a circle whose origin is located over the top surface layer in a symmetrical form about the vertical center line of footing.

The difference between the major principal stresses and failure surface, called inclination, is presumed to be constant. Keeping the linear relationship of undrained cohesion with depth of layer the proposed bearing capacity factor, related to the term which expresses the cohesion, is a function of radius of presumed circle, rate of variation of cohesion, coefficient of anisotropy and various angles obtained from proposed arc including inclination angle.

The nonhomogeneity and anisotropy is found to have a governing effect on the bearing capacity of the two-layered clayey soil. It is concluded that considering same undrained cohesion in all directions leads to very high overestimation. For the case in which the actual anisotropy index is less than unity if the two-layered system is assumed homogeneous then the results will be conservative.

Zhang and Luan (2008) have presented an equation for two-layered homogeneous clayey soil, applicable for both horizontal and vertical load, as a function of thickness of upper layer and undrained shear strength ratio. The critical depth, at which the bearing capacity reaches the minimum, is found equal to $0.75(H/B)$. The following equation is developed by Zhang and Luan (2008):

$$\frac{V}{BS_{u1}} = \xi_V \left\{ \left(1 + \frac{\pi}{2}\right) + \cos^{-1} \left(\frac{H}{BS_{u1}} \right) + \sqrt{1 - \left(\frac{H}{BS_{u1}} \right)^2} \right\},$$

$$\frac{V}{BS_{u1}} \geq \xi_V \left(1 + \frac{\pi}{2}\right); \tag{6.1}$$

$$\frac{H}{BS_{u1}} = \mp 1, \quad 0 \leq \frac{V}{BS_{u1}} \leq \xi_V \left(1 + \frac{\pi}{2}\right), \tag{6.2}$$

where: ξ_V is modified factor of failure envelope

$$= \left[1 + \frac{S_{u2} - S_{u1}}{S_{u1}} \exp \left((22.5 \exp(-0.825 \frac{S_{u2}}{S_{u1}}) - 8.5) \frac{H_1}{B} \right) \right];$$

S_u – soil strength; H_1 – thickness of upper layer; V – vertical bearing capacity; H – horizontal bearing capacity.

Employing various mechanisms leads to different methods while these differences can be only few simple changes in the original mechanism offered by earlier researchers. Following the Prandtl-Terzaghi mechanism with changing the wedge angles, Purushothamaraj et al. (1974) have formulated a method applicable for any combination of key properties of homogeneous two-layered soil system.

Two cases have been studied regarding location of central wedge, i.e. on the top layer and extending to bottom layer, determining both the external and internal work done, including that of weight of soil, and eventual summation of them the equation is developed similar to that of Terzaghi but different expressions for classical bearing capacity factors. Fig. 10 illustrates the proposed failure mechanism.

Final expression of ultimate bearing capacity is as the following:

$$q = c_1 N_c + \gamma_1 D_f N_q + 0.5 \gamma_1 b N_\gamma. \tag{7}$$

Unlike the classical bearing capacity factors that are only functions of internal angle of friction, in the method offered by Purushothamaraj et al. (1974), in addition to internal angle of friction, they are assigned to be functions of cohesions of both layers, different angles presented through their assumed mechanism including, angle of middle cone, $\theta = (\pi/2 - \alpha + \phi)$, wedge angles α and β and etc.

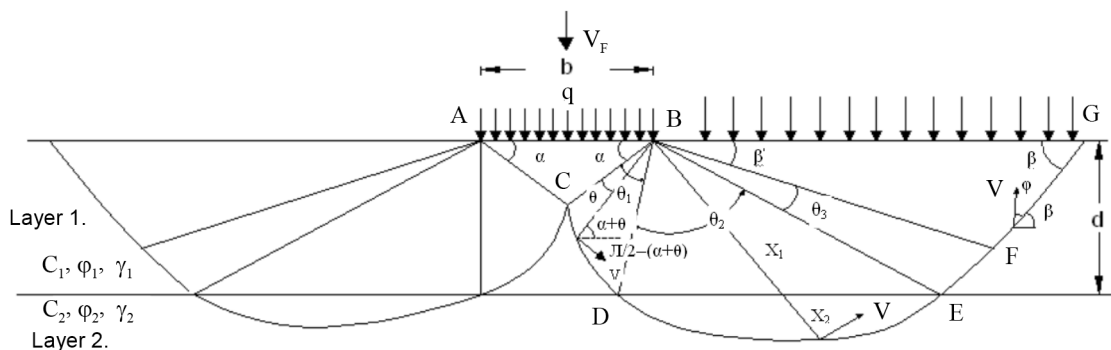


Fig. 10. Failure mechanism by Purushothamaraj et al. (1974)

Purushothamaraj has presented these factors and plotted in his paper for various ranges of geometric configuration and soil specifications.

Study on kinematic approach following limit analysis is applied for any structure of two-layered soil with attention paid to sand-on-clay case (Michalowski, Shi 1995). Kinematic approach is strictly dependent of assumed failure mechanism observed through experimental tests which is not the case for static approach.

Anticipated failure mechanism assumes the velocity discontinuities to bend at interface of two layers which is set to be the function of difference between internal angles of friction of two considered layers. External forces and internal energy dissipation are equated through upper bound theorem which is minimized to the least through the proposed failure mechanism.

Michalowski and Shi (1995) have used Mohr-Coulomb as the constitutive law to simulate the behavior of the soil through associated flow rule in addition to considered incompressibility of clay. Associated flow rule sets the dissipation rate inside the granular layer to be zero and that of clay equal to product of undrained shear strength.

From above solution it is concluded that the so called average pressure beneath the footing is function of thickness of sand layer, width of footing, undrained shear strength of clay, unit weight and internal angle of friction of sand and overburden pressure as follows:

$$\frac{\bar{p}}{\gamma B} = \frac{c_u}{\gamma B} N_c + \frac{q}{\gamma B} N_q + N_\gamma \quad (8)$$

It is concluded that bearing capacity factors are not only functions of internal angle of friction of sand but also they are dependent of undrained shear resistance of clay as well as thickness of top sand layer.

Critical depth of clay is defined as the depth for which it has no effect on overall bearing capacity and its variation is studied according to the variation of other parameters like undrained shear resistance and internal angle of friction which present the strength of top layer and the more stronger the sand the larger the critical depth.

Application of non-associated flow rule, employing angle of dilatancy, is argued that yields better estimations of settlement for granular sand.

An earlier researcher has proposed a numerical approach in which the main concern is the effective domain of the pressure transferred through the footing respective of layers profiles and subsequently the number of soil layers involved in mobilization, against the external horizontal, vertical and shear forces, by the forces along the failure surfaces (Georgiadis 1985).

The resisting forces are assumed to actuate along the sides of three conical shaped failure surfaces, one under footing with upward apex and other two next ones upside down located under the area which are bearing the overburden pressure. Fig. 11 illustrates the developed failure mechanism.

For the case of which failure may deeply affect different sub-layers of the soil the analysis will be as the following:

- a) the reaction forces and effective width are calculated as functions of the safety factor also called material factor;
- b) the vertical force is then equaled to the actual one while the horizontal force is to be calculated, both with the same safety factor of one;
- c) the resisting forces along the failure surfaces are obtained through a trial and error process on all possible failure surfaces till the equilibrium of actual horizontal force and that of internal one is met.

For the shallow failure cases due to the presence of inter-block forces shallow sliding takes place and the rest will be done same as procedure above.

The mobilized shear forces in single layer of sand and clay layer are formulated as functions of internal angle of friction and undrained shear strength respectively. However, when the pressure domain exceeds more than one or two layers having different properties the total shear forces along the sliding block is then affected by this profile variation and it will be a resultant of summation of all shear forces belonging to involved layers.

The assumed failure mechanism does not seem to be following any of three well known failure modes, i.e. general, local and punching shear failure modes, and compressibility of the soil is not discussed.

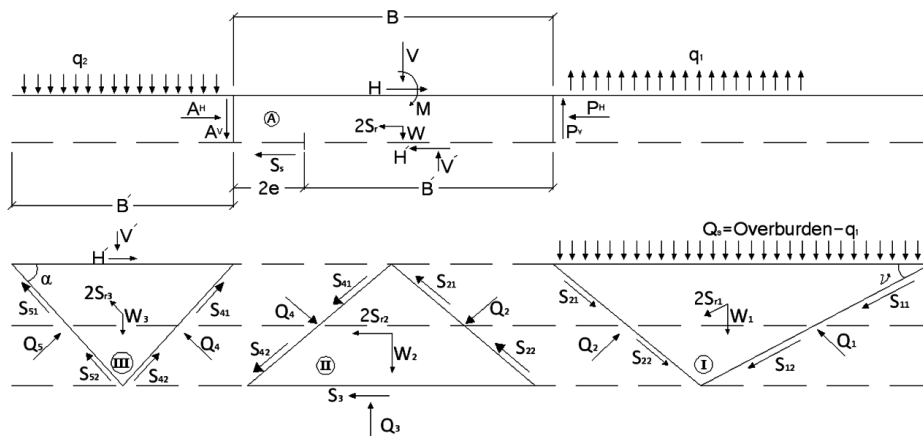


Fig. 11. Failure mechanism by Georgiadis (1985)

This method has shown good agreement with other semi-empirical ones and is said to be applicable for any combination of soil layers, soil profiles and loads although it is not developed through the failure manifestation of the soil as it is usually done for other classical approaches and as a consequence this method owes its accuracy to the iterations through which the optimum failure is found.

Application of multi-rigid-block solution was shown for strip footings resting on two-layered soil, applicable for any profile combination, has been reported by Huang and Qin (2009). The failure mechanism proposed is discretized into some rigid blocks whose edges, i.e. discontinuities, are located in one of the layers (Fig. 12).

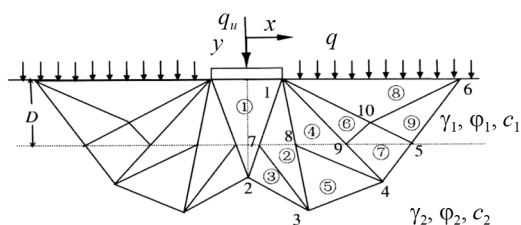


Fig. 12. Proposed failure mechanism by Huang and Qin (2009)

Analysis process developed by Huang and Qin (2009) is consisting of two main sub-process named as determination of compatible velocity field and determination of critical failure through which the minimum magnitude of bearing capacity is obtained. The later one takes advantage of Monte Carlo technique for determination of the least magnitude of bearing capacity.

Results are compared with those of Hanna and Meyerhof for two-layered soil sand-on-clay profile and it is observed that by increasing the top layer thickness discrepancy rises through overestimation of Huang's method. Conclusion from nominated method has shown good agreement for two-layered clayey soil compared to some other proposed approaches.

3. Ultimate bearing capacity through finite element analysis

The formulation and implementation of finite element method was carried out to predict UBC and model the condition of tests such as boundary condition and size of footing (Hanna 1987).

Hanna has focused on two-layer homogeneous sandy soils, dens sand over loose one and dense sand over compact one, under plain strain condition. Modulus of elasticity, stress-strain relationship, footing settlement and ultimate bearing capacity are major concerns of this research and results from finite element approach and those of experimental ones are found noticeably close while finite element has overestimated the experimental results with small difference up to $(H/B) = 4.5$ throughout the incremental trend of ultimate bearing capacity through increasing (H/B) ratio. The finite element discretization created by Hanna (1987) is shown in Fig. 13.

Yin *et al.* (2001) studied effect of the soil nonassociativity on bearing capacity of a strip foundation by finite-difference method, the 2-D FLAC 1998 was used to simulated soil behavior through an elastic-plastic model associated with Mohr-Coulomb failure criterion. The dilation angle was found to have a significant influence on the bearing capacity factors.

Evaluation of behavior of 2-layered cohesive-frictional soils under shallow foundation has been carried out and these investigations are continued in present time while involving detailed information and introduced the bearing capacity as a function of parameters like (H/B) ratio (height of the top layer to width of the footing), angle of friction, dilatancy and cohesion coefficient considering strip footing (Zhu 2004).

Zhu (2004) has focused on application of Finite Element Method (FEM) in calculation of ultimate bearing capacity for the case of rough strip footing resting on two-layered weightless clay soil and subsequent cohesion coefficient (N_c) while comparing the results with upper and lower bound solutions.

Results obtained through displacement control method in FEM are compared with those of upper and lower bound solutions. It is observed that when a weak clay layer is overlaying a strong one N_c increases as (H/B) rises.

Magnitude of N_c has been confirmed to approach 5.146 for any case of weak clay on strong one that coincides to the presumption of failure surface limited to top layer. The critical depth is defined the depth for which N_c approaches 5.146 that is associated with $(H/B) = 2$. Outcomes from FEM are found to lie within the limit of upper and lower bound solutions.

Above analysis is done for circular surface footing (Szypcio, Dołżyk 2006). Szypcio discussed some results that are limited to the case of subsoil with a weak cohesion lower layer having small angle of friction while proposing that there is no much difference to use average angle of friction in case of multi-layered cohesive-friction soils.

Among the calculations that have discussed the multi-layered soil, employing the associated and non-associated flow rules, one has demonstrated the ability of the linear matching model in defining the limit bearing capacity of strip footing on multi-layered soils.

The major variable parameters were undrained shear strength of soft clay layer and the friction angle of sand layer to investigate the layering effects while considering the effect of dilation angle (Boulbibane, Pontar 2005). This study has focused on two-layered subsoil involving three different possible layering conditions and has been concluded that key geometrical and material parameters affect the bearing capacity factors of strip footings.

Three dimensional evaluation of bearing capacity for square and rectangular footings resting on two-layered clay subsoil employing Tresca yield criteria has been conducted. Clayey layers are assumed to be undrained (incompressible) (Zhu, Michalowski 2005). Bearing capacity factor N_c is reported by Zhu and Michalowski (2005)

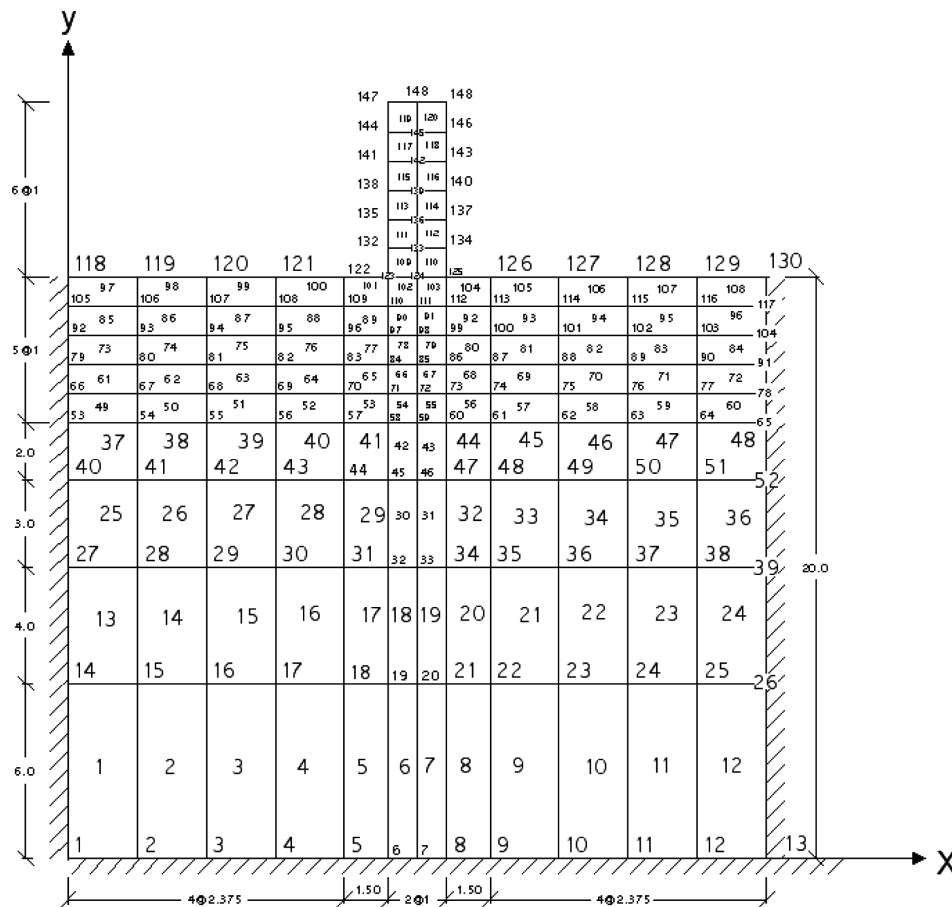


Fig. 13. Numerical modeling of structure and soil medium by Hanna (1987)

to be governed by strength ratio (c_1/c_2) of two layers rather than the depth while variations of these two parameters are more pronounced for overall bearing capacity.

Kumar and Kouzer (2007) investigated the influence of the footing roughness on the bearing capacity factor (N_γ) by a 2D finite element approach assuming associated flow rule to be governing the soil behavior although this leads to higher values of collapse loads compared to that of nonassociated flow rule. Different domains for soil mass were studied and various finite element meshing were employed. The former one corresponds to the vertical and horizontal sides of the cohesionless soil layer beneath the rigid footing and latter one refers to the size of finite element, i.e. coarse, medium and fine.

An upper bound approach is employed to formulate the collapse load through equating work done by external loads and internal dissipated energy within the elements through which the collapse load has been expressed in terms of nodal velocities and multiplier rates and eventually integrated to a linear program. The selected failure criterion Mohr-Coulomb has been linearized to achieve an upper bound solution. It is reported that the finer is the element size the lower is the N_γ . It is also observed that increment of soil-footing roughness leads to increase the bearing capacity factor N_γ and also found that when the roughness δ approaches the internal angle of friction in magnitude the footing is analogous to the perfectly rough status and plastic zone gets larger when both increase. It

is also argued that assumption of perfectly rough footing is unsafe when the roughness is less than internal angle of friction.

4. Application of Artificial Neural Network as a solution for ultimate bearing capacity

The development of Artificial Neural Network and its application to predict the ultimate bearing capacity of multi-layered systems as a numerical method has been reported in the literature but there are little research work earned in Artificial Neural Network prediction of ultimate bearing capacity.

This numerical tool is of course applied to many other geotechnical fields and has shown good capability for estimation of stress-strain relationship, settlement and classification of soils and further things but little study is done for the case of ultimate bearing capacity of two-layered soil.

Neurofuzzy model was employed by Padmini *et al.* (2008) for shallow foundations as an alternative approach to calculate the bearing capacity of cohesionless soils. The applied model, i.e. adaptive neurofuzzy inference system (ANFIS), is basically a fuzzy inference system which takes the advantage of the adaptive neural network framework and has five layers through which a linear relationship among so-called premise parameters is developed.

Ninety seven data sets have been used to train the system which were divided into two parts namely training and calibration set. Each data set was comprised of footing width, depth of embedment, length-footing width ratio, unit weight of soil and internal angle of friction. The developed model has been validated by two more models artificial neural network (ANN), fuzzy network and three classical approaches. Feedforward multilayer ANN system with three hidden nodes and sigmoidal transfer function trained by Levenberg-Marquardt (LM) learning algorithm. Performance of numerical methods ANFIS, ANN and fuzzy network were generally found to be better than traditional ones while among numerical ones ANFIS has performed better than ANN and ANN better than fuzzy network.

$$q_u^{4layer} = \left[\frac{45.53}{\{1 + \exp(-1.994T_1 - 4.232T_2 + 3.295T_3 + 3.992T_4 + 4.758T_5 - 1.396)\}} + 5.27 \right] a ; \quad (9.1)$$

$$T_{i=1, \dots, 5} = [1 + \exp(a^{-1}(w_1c_1 + w_2c_2 + w_3c_3 + w_4c_4) + w_5h_1 + w_6h_2 + w_7h_3 + w_8h_4 + C)]^{-1} ; \quad (9.2)$$

$$q_u^{10layer} = \left[\frac{46.11}{\{1 + \exp(-5.183T_1 + 3.834T_2 - 1.845T_3 + 4.158T_4 - 0.311T_5 - 1.704T_6 + 3.787T_7 + 1.402)\}} + 5.15 \right] a ; \quad (10.1)$$

$$T_{i=1, \dots, 7} = [1 + \exp(a^{-1}(w_1c_1 + w_2c_2 + w_3c_3 + w_4c_4 + w_5c_5 + w_6c_6 + w_7c_7 + w_8c_8 + w_9c_9 + w_{10}c_{10}) + w_{11}h_1 + w_{12}h_2 + w_{13}h_3 + w_{14}h_4 + w_{15}h_5 + w_{16}h_6 + w_{17}h_7 + w_{18}h_8 + w_{19}h_9 + w_{20}B + C)]^{-1}. \quad (10.2)$$

In which c_i is shear strength of each layer and a is a scalar quantity defined as:

$$c_{\min} \leq ac_i \leq c_{\max} , \quad (11)$$

while

$$1.0 \leq c_i \leq 10.0kpa . \quad (12)$$

This parameter (a) has been introduced due to the wide range of cohesion available for different soils to improve the prediction of bearing capacity of developed ANN system.

Kalinli *et al.* (2011) employed ant colony optimization (ACO) to improve the classical equation of bearing capacity for granular soils in addition to proposing an artificial neural network for ultimate bearing capacity of granular soils. Similar to Padmini *et al.* (2008), the selected ANN architecture is a MLP with single hidden layer but with 10 hidden nodes taking advantage of tangent hyperbolic function and LM learning algorithm. The ANN model developed by Kalinli *et al.* (2011) has been found to be superior to that of Padmini *et al.* (2008) while same input data has been used by both researchers.

5. Conclusions

The literature related to ultimate bearing capacity of two-layered soil with special attention to strip footing resting on sand-on-clay profile soil was reviewed. Most of applicable

Ultimate bearing capacity of strip footings resting on multi layer soil profile in practice is often faced, but not studied as much as single- or two-layer conditions (Kuo *et al.* 2009). An ANN based model is proposed taking the advantages of Multi Layer Perceptron (MLPs) being trained by backpropagation learning algorithm for 4-layer and 10-layer subsoil conditions.

This calculation is done for ultimate bearing capacity of multi layer cohesive soils being function of soil cohesion coefficients, thickness of layers and footing width. Results have shown good and satisfactory accuracy in predictions of bearing capacity of strip footing.

Calculation of ultimate bearing capacity of 4- and 10-layered cohesive soil is formulated as following:

equations developed by researchers were compiled in this paper for ease of use in addition to their applications.

Various failure mechanisms adopted by the researchers were discussed further to their characteristics and their ability to predict the soil strength.

There are too many works found in the classical field regarding bearing capacity of two-layered soil while the number of investigations decreases in numerical field.

Moreover, parametric studies done by researchers regarding effect of different parameters on ultimate bearing capacity of the two-layered soil with any combination of soil properties was discussed.

There are many researches carried out to bring new and more accurate solutions following classical and theoretical approaches however number of studies regarding application of finite element method and artificial neural network is much less.

This indicates that there is still open field for employing ANN systems for UBC prediction.

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