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ARE THE GLOBAL REAL ESTATE MARKETS CONTAGIOUS?

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ABSTRACT. The aim of this paper is to investigate the contagion across real estate markets of four countries: Hong Kong, China, U.S. and U.K., during the financial tsunami in 2008. We use the Forbes-Rigobon test, the coskewness test and the cokurtosis test. We propose a new cokurtosis test constructed by extending the method of constructing the coskewness test to further higher order moments. It can show additional channels of contagion that other tests fail to show, and hence can provide more information on the direction of contagion, and reflect a more complete picture of the contagion pattern. The coskewness and cokurtosis tests show that contagion exists between the four countries, and the contagion effect is stronger between Hong Kong and China, and between U.S. and U.K. This provides clues for investors on how to diversify their investment to reduce their risk. This paper bridges the gap that previous works on contagion across real estate markets give mixed results, and gives a first insight into the contagion pattern of global real estate markets during the financial tsunami.

KEYWORDS: Contagion; Coskewness; Cokurtosis; Real estate; Financial tsunami

1. INTRODUCTION

In investment science, it is commonly known that diversification reduces risk. Therefore, investors often invest in different types of asset in different countries in order to diversify their risk. However, during a financial crisis, correlation of a type of asset market between two countries usually increases, i.e. they either move up together or, more commonly, move down together. Even correlation between different types of asset markets may increase, too. Therefore, the opportunity of diversification is reduced. This phenomenon is called contagion. The World Bank Group (2011) gives three definitions of contagion:

Broad definition: contagion is the cross-country transmission of shocks or the general cross-country spillover effects.

Restrictive definition: contagion is the transmission of shocks to other countries or the cross-country correlation, beyond any fundamental link among the countries and beyond common shocks.

Very restrictive definition (the definition which was adopted by most of the previous works, and is also adopted in this paper): contagion occurs when cross-country correlations increase during “crisis times” relative to correlations during “tranquil times”.

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Pericoli and Sbracia (2003) also stated five definitions of contagion which are commonly adopted in the literature:

1. Contagion is a significant increase in the probability of a crisis in one country, conditional on a crisis occurring in another country.
2. Contagion occurs when volatility spills over from the crisis country to the financial markets of other countries.
3. Contagion is a significant increase in co-movements of prices and quantities across markets, conditional on a crisis occurring in one market or group of markets.
4. (Shift-)contagion occurs then the transmission channel is different after a shock in one market.
5. Contagion occurs when co-movements cannot be explained by fundamentals.

As mentioned, contagion usually occurs during "crisis times", when shocks transmit from a country to others, causing co-movements (usually downward) of asset prices. There are a number of crises triggering shocks around the world. The most typical one is the Great Depression in the 30's, causing the deepest global recession ever. Other crises include the oil crisis in the 70's, the 1987 U.S. stock market crash, the 1994 Mexico Peso crisis and the 1997-1998 East Asian crisis. The crisis we choose to study is the financial tsunami in 2008 since it happened just recently, its magnitude is significant and it has a great impact over the world.

The financial tsunami in 2008 was the worst global financial crisis since the Great Depression in the 30's. It began on September 15 when Lehman Brothers went bankruptcy, causing the global stock market to plunge sharply. The financial tsunami has a great impact over the world. The most significant impact is the deleveraging effect which persisted until early March in 2009. Investors avoided risky assets such as equities, real estates, commodities and derivative products, causing their prices to fall sharply. On the other hand, the U.S. Dollar and Japanese Yen were considered as "safe havens" and hence appreciated in value. Another important effect of the financial tsunami is the shift of economic power from the west to the east. The U.S. and most of the European countries are badly hurt by the crisis. There influence on the global economy nowadays is much smaller. However, the emerging markets like China, India and Brazil were less hurt. Their economic power grew even stronger after the crisis and had a much greater influence on the global economy than before.

So you can see how serious the financial tsunami was. Nearly all risky assets fell together at the same time, causing many people to suffer heavy losses in their investment. This scenario had hardly been seen since the Great Depression. Hence we suspect there might be some sort of contagion occurring during that crisis. If people knew that there is a sign of contagion, they might be able to reallocate their investment to minimize their loss. Therefore, it is important to know the pattern of contagion. This is particularly useful to investors and policy-makers. By observing the indicators of contagion, investors can allocate their investments more appropriately to reduce risk. For example, if the asset prices of many regions fall together during a financial crisis, then this may be an indicator of contagion. Diversification no longer works, so it is better for investors to hold more cash. Policy makers should constantly monitor the correlation of asset returns of different countries. If those statistics increase significantly, indicating that there is a significant evidence of contagion, the policy makers of the countries which are the sources or the recipients of contagion have to strengthen policy coordination to mitigate shocks. For example, during the global financial tsunami in late 2008, governments and central banks of different countries launched bailout plans together in order to rescue the economy. The result was that the global economy began to recover slowly in mid-2009. If any one country did not cooperate in the global bailout plan, the economic recovery would be affected.

In fact, the financial tsunami was caused by the subprime crisis in the U.S., which was a result of the bursting of the U.S. housing bubble. On the other way round, the financial tsunami led to a global stock market slump. Most investors lost money on their investment. This negative wealth effect caused a decrease in demand for housing, so the house price drops. Moreover, as mentioned by Hatemi-J and Roca (2010), the recent globalization and internationalization of real estate markets lead to increasing integration, which is expected to cause more co-movements of prices among global real estate markets. Furthermore, according to Hui and Zheng (2012), real estate is a special commodity which can act not only as consumption goods, but also as an investment tool. Due to this special feature of real estate, the contagion pattern of real estate markets may be different from that of equity markets. Therefore, it is interesting to study contagion of real estate markets during the financial tsunami. However, there are only a few articles on contagion during the financial tsunami since the crisis happened just recently. Most of those articles are on equity markets instead of real estate markets. The lack of previous literature on contagion across real estate markets leads to mixed results (see Section 2). This is the main motivation of our research.

To study contagion across real estate markets, we have to choose appropriate real estate indices. The traditional real estate indices are constructed from real transaction prices recorded in the market (Chau et al., 2005). Samples are taken from all registered transactions available. Since it takes time for transactions to be completed and the index would be weeks lagging behind, on-going transactions are sometimes used. As an example, the Centa-City Leading Index compromises transactions mainly from on-going transactions of the Hong Kong property agent Centaline. Real estate price indices are also constructed from surveying of selected samples by professionals. Such indices are updated monthly or quarterly (Chau et al., 2005). Meanwhile, Hui and Wong (2004) developed the BRE Index which provided an objective tool and a statistical pointer that forecasted future housing price trends. Later, they used the index to further investigate the housing price in Hong Kong (Wong and Hui, 2005).

However, the observations of the traditional real estate indices cannot be done continuously. There is always time lag between price change and observation. Yet, sometimes a change in long-term value of real estate properties cannot be seen on their short-term prices. Stock prices, on the other hand, can be regarded as a continuous evaluation of co-operations by market practitioners. There have been studies about the relationship between real estate price and real estate stock price or the stock market in general (Ong, 1994; Newell and Chau, 1996). Some argue that the stock price is affected by short-term factors like market liquidity (i.e., money supply) and cannot reflect the change in real estate price. However, history tells that the property price is also affected by liquidity. Sing (2001) showed that the demand for condominiums was negatively related to one-quarter lagged stock price change. Recently, researchers used econometric methods to study relationship between real estate and stock markets. For example, Okunev and Wilson (1997) tested whether or not there existed a relationship of co-integration between the REIT and the S&P 500 indices. The results indicated that the real estate and stock markets were fractionally integrated. Okunev et al. (2000) conducted both linear and nonlinear causality tests on the US real estate and the S&P 500 Index and concluded that there exists unidirectional relationship from real estate to stock market when using the linear test, but there is a strong unidirectional relationship from the stock market to the real estate market when using the nonlinear test. Some researchers found a long-term positive correlation between real estate and stock prices. Using unpublished data reported contemporaneously by financial institutions and market watchers, Quigley (2001) investigated how activities in the real estate markets in Southeast and East Asian countries contributed to the Asian financial crisis in 1997. Quan and Titman (1999) examined data from 17 different countries over 14 years, and found a significant positive relation between stock returns and changes in commercial real estate values. Tse (2001) studied the Impact of Property Prices on Stock Prices in Hong Kong from 1974 to 1998, and found that the property and stock prices are cointegrated. Liow (2006) also found long-term positive correlations between real estate and stock prices in general. Similar results were found by Hui et al. (2011), who examined the relationship between real estate and stock markets in U.K. and Hong Kong by the method of data mining. They found not only a positive correlation, but also a co-movement, between the two markets. These suggest that the stock price can actually be a leading indicator of the real estate price.

Therefore, as an alternative, the indices we used are constructed from stock prices of real estate companies. The frequency of these indices is daily, instead of monthly for most housing price indices. Thus they can reflect the continuous change of value, rather than chasing after the prices. In this paper, we investigate contagion across real estate markets of four countries: U.S., U.K., China and Hong Kong, during the financial tsunami. We use the Forbes-Rigobon test, the coskewness test and the cokurtosis test, which is constructed by extending Fry et al. (2006, 2008, 2010)'s method of constructing the coskewness test to further higher order moments. We compare the results of the three different tests.

The paper proceeds as follows. Section 2 reviews previous literatures on the topic of contagion. In Section 3, we describe the three different tests of contagion. In Section 4, we explained how the crisis period and the indices are selected. Section 5 gives the results of the tests. We draw up conclusion and provide a further discussion of the topic in Section 6.

2. LITERATURE REVIEW

There are a large number of studies about contagion. The following are some of the more important ones. Forbes and Rigobon (2002) showed that correlation coefficients were conditional on market volatility and suggested an adjustment for this bias. They also proposed the Forbes-Rigobon test for contagion. Dungey et al. (2005a) summarized the previous work and made a review of methodologies of modeling of contagion. The main methods they discussed were the latent factor model, the Forbes-Rigobon test and the Chow test (both univariate and multivariate versions). In addition, they also described alternative tests of contagion, including the determinant of the change in the covariance matrix (DCC), probability models (dichotomous and polychotomous classifications), the method of principal components, and also the addition of spillovers and multiple classes of assets into the latent factor model, which was used by Dungey and Martin (2001) before. Later, Dungey and Martin (2007) and Dungey (2009) also used the modified latent factor model in their research. More recently, Dungey et al. (2010) proposed an identified structural GARCH model to test contagion. There are other methods for testing contagion, e.g., the use of the variance-covariance matrices in an ARCH or GARCH context to investigate spillovers between markets (Hamao et al., 1990), the examination of changes in a cointegrating vector between countries (Longin and Solnik, 1995; Granger et al., 2000), the investigation of the determinants of different markets' susceptibility to financial crises (Eichengreen et al., 1996; Forbes, 2004), and wavelet analysis (Zhou, 2010).

However, correlation sometimes may not reflect the pattern of contagion fully. Besides standard deviation and correlation, investors may also be interested in the higher order moments of asset returns. For example, a risk adverse person prefers positive skewness to negative skewness. He also prefers lower kurtosis. Therefore, it is worthwhile studying the changes in higher order moments of asset returns during crisis periods. A different pattern of contagion may be observed. Fry et al. (2006, 2008, 2010) used the generalized exponential class as the framework to construct the coskewness test of contagion and provide an application of the tests. Sometimes the coskewness test is still not enough to reflect the whole picture of the contagion pattern. If we further increase the order of moment by one, additional channels of contagion may be detected. So far, there are still no articles about contagion tests using further higher order moments.

Most of the previous works are on the equity market. There are only a few studies of contagion across real estate markets. This leads to mixed results. Bond et al. (2006) investigated the contagion across real estate markets during the 1997-98 East Asian crisis using the latent factor model, and found that contagion among the markets existed. On the contrary, Wilson and Zurbruegg (2004), who used the Forbes-Rigobon test to examined contagion from the Thai real estate market to other East Asian real estate markets during the 1997 Asian crisis, found only little evidence of contagion. Most of the previous work studied the 1997-98 East Asian crisis. There are only a few articles about contagion across asset markets during the global financial tsunami in 2008, like Fry et al. (2008) and Dungey (2009). The contagion pattern across real estate markets of different countries during the financial tsunami is yet to be explored.

As mentioned before, there are a lot of studies on the topic of contagion. In the next section, we describe two of the contagion tests: the Forbes-Rigobon test and the coskewness test. We also extend Fry et al. (2006, 2008, 2010)'s method to further higher order moments, and construct the cokurtosis test of contagion.

3. TESTS OF CONTAGION

3.1. The Forbes-Rigobon test

The most common method of testing contagion is to compare the correlation of asset returns of two countries in the crisis period with that in the pre-crisis period. However, according to Forbes and Rigobon (2002), when testing contagion from country *i* to country *j*, if there are no changes in the fundamental relationship between the asset returns of the two countries, then an increase in the volatility of the asset return of country *i* would cause an increase in correlation between the asset returns of the two countries, i.e., $\sigma_{y,i} > \sigma_{x,i}$ implies that $\rho_y > \rho_x$, where $\sigma_{x,i}$ and $\sigma_{y,i}$ denotes the standard deviation of the asset return of country *i* during the pre-crisis period (low volatility period) and the crisis period (high volatility period) respectively, while ρ_x and ρ_y denotes the unadjusted correlation between the asset returns of countries *i* and *j* during the pre-crisis period and the crisis period respectively. Hence the unadjusted correlation coefficient is, in fact, biased. To get a more reliable test for contagion of asset return from country *i* to country *j*, we have to use the adjusted (unconditional) correlation coefficient. The formula for the adjusted correlation for testing contagion from country *i* to country *j* is (Forbes and Rigobon, 2002; Dungey et al., 2005b).

$$\nu_{y/x} = \frac{\rho_y}{\sqrt{1+\left(\frac{\sigma_{y,i}^2}{\sigma_{x,i}^2}-1\right)(1-\rho_y^2)}}. \tag{1}$$

The null hypothesis for the test for contagion of asset return from region i to region j is

$$H_0 : \nu_{y/x} = \rho_x \tag{2}$$

against the alternative hypothesis of

$$H_1 : \nu_{y/x} > \rho_x . \tag{3}$$

The Forbes-Rigobon statistics is given by (Dungey et al., 2005b)

$$FR_1(i \to j) = \frac{\ln\left(\frac{1+\hat{\nu}_{y/x}}{1-\hat{\nu}_{y/x}}\right) - \ln\left(\frac{1+\hat{\rho}_x}{1-\hat{\rho}_x}\right)}{2\sqrt{\frac{1}{T_y - 3} + \frac{1}{T_x - 3}}} . \tag{4}$$

where: ^ denotes the sample estimator, and T_y and T_x denote the sample sizes of the crisis period and the pre-crisis period respectively.

Under the null hypothesis,

$$FR_1 \xrightarrow{d} N(0,1) . \tag{5}$$

3.2. The coskewness test

Focusing on correlations alone may not capture the whole picture of contagion. To obtain more details about the contagion pattern, one method is the extension to higher order moments. This was discussed by Harvey and Siddique (2000). One important outcome is the interaction between the first and second moments of the joint distribution of returns, i.e. coskewness. Hence by testing the changes in coskewness, we can detect additional contagious channels.

The fundamental of Fry et al. (2006, 2008, 2010)'s framework is the generalized exponential class, with the multivariate normal distribution as a special case, thus providing a framework that constructs Lagrange multiplier tests of contagion with the multivariate normal distribution defined under the null. This framework can be generalized to other tests of contagion based on different combinations of higher ordered moments like cokurtosis (which will be discussed in the next sub-section).

The coskewness statistics for testing the null hypothesis of no contagion from country i to country j are (Fry et al., 2008)

$$CS_1(i \to j; r_i^1, r_j^2) = \left(\frac{\hat{\psi}_y(r_i^1, r_j^2) - \hat{\psi}_x(r_i^1, r_j^2)}{\sqrt{\frac{4\hat{\nu}_{y/x}^2 + 2}{T_y} + \frac{4\hat{\rho}_x^2 + 2}{T_x}}} \right)^2 , \tag{6}$$

$$CS_1(i \to j; r_i^2, r_j^1) = \left(\frac{\hat{\psi}_y(r_i^2, r_j^1) - \hat{\psi}_x(r_i^2, r_j^1)}{\sqrt{\frac{4\hat{\nu}_{y/x}^2 + 2}{T_y} + \frac{4\hat{\rho}_x^2 + 2}{T_x}}} \right)^2 , \tag{7}$$

where:

$$\hat{\psi}_y(r_i^m, r_j^n) = \frac{1}{T_y} \sum_{t=1}^{T} \left(\frac{r_{i,t} - \hat{\mu}_i}{\hat{\sigma}_{y,i}} \right)^m \left(\frac{r_{j,t} - \hat{\mu}_j}{\hat{\sigma}_{y,j}} \right)^n , \tag{8}$$

$$\hat{\psi}_x(r_i^m, r_j^n) = \frac{1}{T_x} \sum_{t=1}^{T} \left(\frac{r_{i,t} - \hat{\mu}_i}{\hat{\sigma}_{x,i}} \right)^m \left(\frac{r_{j,t} - \hat{\mu}_j}{\hat{\sigma}_{x,j}} \right)^n . \tag{9}$$

For details of derivation of the formulae (6) and (7), see Appendix. Under the null hypothesis of no contagion, the above testing statistics are asymptotically distributed as χ_1^2.

3.3. The cokurtosis test

As mentioned in Section 1, sometimes the coskewness test is still not sufficient to show the complete pattern of contagion (See Section 5 for the results). If we further increase the order of moment by one, which was never done in previous work, we may be able to find additional channels of contagion. In this sub-section, we extend Fry et al. (2006, 2008, 2010)'s framework to tests based on further higher order moments. We introduce the cokurtosis test of contagion, which tests the interaction between the first and third moments of the joint distribution of returns.

The coskewness statistics for testing the null hypothesis of no contagion from country i to country j are (see Appendix for details)

$$CK_1(i \to j; r_i^1, r_j^3) = \left(\frac{\hat{\psi}_y(r_i^1, r_j^3) - \hat{\psi}_x(r_i^1, r_j^3)}{\sqrt{\dfrac{2(7+13\hat{v}_{y/x}^2 - 8\hat{v}_{y/x}^4)}{T_y} + \dfrac{2(7+13\hat{\rho}_x^2 - 8\hat{\rho}_x^4)}{T_x}}} \right)^2, \quad (10)$$

$$CK_1(i \to j; r_i^3, r_j^1) = \left(\frac{\hat{\psi}_y(r_i^3, r_j^1) - \hat{\psi}_x(r_i^3, r_j^1)}{\sqrt{\dfrac{2(7+13\hat{v}_{y/x}^2 - 8\hat{v}_{y/x}^4)}{T_y} + \dfrac{2(7+13\hat{\rho}_x^2 - 8\hat{\rho}_x^4)}{T_x}}} \right)^2, \quad (11)$$

where: $\hat{\psi}_y(r_i^m, r_j^n)$ and $\hat{\psi}_x(r_i^m, r_j^n)$ are defined by (8) and (9) respectively. Under the null hypothesis of no contagion, the above testing statistics are asymptotically distributed as χ_1^2.

4. DATA SOURCE

In the previous section, we described three tests of contagion: the Forbes-Rigobon test, the coskewness test and the cokurtosis test. We apply these three tests to test contagion across real estate markets during the financial tsunami. Before conducting the tests, we have to select the data for the tests first.

4.1. Selection of data

We select the following real estate indices:

1. Hong Kong-DS Real Estate (Hong Kong).
2. China-DS Real Estate (China).
3. US-DS Real Estate (U.S.).
4. UK-DS Real Estate (U.K.).

All indices are daily equity indices obtained from Datastream. The returns are computed as the difference of the natural logarithms of daily price indices.

4.2. Selection of crisis period

The whole period of observation is set to be from January 2, 2006 to March 31, 2009, a total of 847 observations. We divide the whole timeline into pre-crisis and crisis periods. It is often difficult to select the crisis period. Usually, we use the presence of a speculative attack associated with stock market turmoil as the criterion to determine the crisis period. In the case of the financial tsunami in 2008, we use September 15, 2008 as the starting date of the crisis period because the bankruptcy of Lehman Brothers occurred on that day, triggering a global stock market slump. Using this criterion, we determine the pre-crisis period and crisis period as follows:

- Pre-crisis period: January 2, 2006 to September 12, 2008 (705 observations).
- Crisis period: September 15, 2008 to March 31, 2009 (142 observations).

Now we can apply the three tests described in Section 3 to the data selected over the period of observation. The test results are shown in the next section.

5. RESULTS

5.1. Preliminary statistics

We obtain the daily real estate indices of the four countries from December 30, 2005 to March 31, 2009. We index the time t by denoting $t = 0$ for December 30, 2005, $t = 1$ for January 2, 2006, and so on. We index the four countries as follows:

1. Hong Kong (Note: Hong Kong is a Special Administrative Region of China, not a country).
2. China.
3. U.S.
4. U.K.

Let $r_{i,t}$ be the daily index of country i at time t. The continuously compounded daily return of the index of country i at time t, $s_{i,t}$, is:

$$s_{i,t} = \ln\left(\frac{r_{i,t}}{r_{i,t-1}}\right) \tag{12}$$

We calculate the mean, standard deviation and correlation coefficients of $s_{i,t}$ throughout the whole period, and in the two separate periods as specified in Section 3, as shown in the Table 1.

Table 1. The mean, standard deviation and correlation coefficients of $s_{i,t}$ throughout the whole period, the pre-crisis period and the crisis period

Whole period	HK	China	US	UK
Mean	–0.01%	0.08%	–0.10%	–0.13%
Standard Deviation	0.0217	0.0337	0.0318	0.0210

Correlation	HK	0.7509	0.0554	0.3268
	0.7509	China	0.0053	0.1968
	0.0554	0.0053	US	0.2455
	0.3268	0.1968	0.2455	UK

Pre–crisis period	HK	China	US	UK
Mean	0.02%	0.08%	0.00%	–0.04%
Standard deviation	0.0175	0.0290	0.0166	0.0172

Correlation	HK	0.7375	0.0477	0.2001
	0.7375	China	0.0301	0.0808
	0.0477	0.0301	US	0.3500
	0.2001	0.0808	0.3500	UK

Crisis period	HK	China	US	UK
Mean	–0.15%	0.10%	–0.65%	–0.62%
Standard deviation	0.0361	0.0510	0.0680	0.0337

Correlation	HK	0.7756	0.0614	0.4803
	0.7756	China	–0.0105	0.3678
	0.0614	–0.0105	US	0.1968
	0.4803	0.3678	0.1968	UK

From Table 1, we can see that during the crisis period, the standard deviation of the daily return of the real estate indices of all four countries increased sharply, i.e., they became much more volatile. The correlation between the indices of the four countries increased, except for the correlation between China and U.S., and between U.S. and U.K. The increase in correlation meant that the opportunity of diversification was reduced. However, as mentioned in Section 3, the unadjusted correlation is biased and leads to inaccurate results. To get a more reliable test of contagion, we need to use the adjusted correlation instead, and conduct the Forbes-Rigobon test, of which the results are shown in the next sub-section.

5.2. Results of the three tests

Here we apply the Forbes-Rigobon test, the coskewness test and the cokurtosis test to the data $s_{i,t}$ we obtained. We prefilter the data to remove the common shock factors. The following tables (the formats of the tables are similar to that of Fry et al., 2006) show the results of the tests.

From the above tables, we can see different patterns of contagion between the real estate markets of the countries. The result of the Forbes-Rigobon test is shown in Table 2. We can see that all entries in the table are smaller than the 5% critical value of 1.645, showing that there is no significant evidence of contagion from any country to others at 5% significance level.

The result of the coskewness test is shown in Table 3. From the table, we can see that the overall effect of contagion is much larger than the result shown by the Forbes-Rigobon test in Table 2. Using the 5% critical value of 3.84, there are 8 entries in each table greater than this critical value. In particular, the contagion effect between U.S. and U.K. (in both directions) is the most significant. We can see that the coskewness test can detect additional channels of contagion that the Forbes-Rigobon test fails to show. Another important feature of the coskewness test is that the $CS_1(i \rightarrow j; r_i^1, r_j^2)$ and $CS_1(i \rightarrow j; r_i^2, r_j^1)$ statistics show opposite directions of contagion. For example, the $CS_1(i \rightarrow j; r_i^1, r_j^2)$ statistic shows significant contagion from U.S. to Hong Kong, but very little contagion in the opposite direction. However, the $CS_1(i \rightarrow j; r_i^2, r_j^1)$ statistic shows that the contagion from Hong Kong to U.S. is much more significant than that in the opposite direction. If we look closer into the formulae (6) and (7), we can see that the $CS_1(i \rightarrow j; r_i^1, r_j^2)$ and $CS_1(j \rightarrow i; r_j^2, r_i^1)$ statistics have the same numerator $(\hat{\psi}_y(r_i^1, r_j^2) - \hat{\psi}_x(r_i^1, r_j^2))$. Their difference in denominator is relatively small, so their values are similar to each other. The values of the statistics $CS_1(i \rightarrow j; r_i^2, r_j^1)$ and $CS_1(j \rightarrow i; r_j^1, r_i^2)$ are similar for the same reason. Therefore, the tables of $CS_1(i \rightarrow j; r_i^1, r_j^2)$ and $CS_1(i \rightarrow j; r_i^2, r_j^1)$ show opposite directions of contagion. In fact, the table of $CS_1(i \rightarrow j; r_i^2, r_j^1)$ has the effect of "reflecting" the table $CS_1(i \rightarrow j; r_i^1, r_j^2)$ through the diagonal line.

Table 2. The results of the Forbes-Rigobon test on $s_{i,t}$. The 5% critical value is 1.645.

$FR_1(i \rightarrow j)$

Recipient

		HK	China	US	UK
Source	HK		–4.0949	–0.1938	0.6403
	China	–3.1701		–0.3893	1.5267
	US	–0.3533	–0.3524		–3.4103
	UK	0.7916	1.2907	–2.8334	

Table 3. The results of the coskewness test on $s_{i,t}$. The 5% critical value is 3.84.

$CS_1(i \rightarrow j; r_i^1, r_j^2)$

Recipient

		HK	China	US	UK
Source	HK		56.1396	0.7879	28.7980
	China	54.0738		1.9523	2.2183
	US	18.5882	2.8479		98.1325
	UK	78.7229	34.6840	189.3625	

$CS_1(i \rightarrow j; r_i^2, r_j^1)$

Recipient

		HK	China	US	UK
Source	HK		57.7264	18.5677	79.5252
	China	52.5875		2.8477	34.2174
	US	0.7887	1.9524		191.7856
	UK	28.5075	2.2485	96.8927	

Table 4. The results of the cokurtosis test on $s_{i,t}$. The 5% critical value is 3.84.

$CK_1(i\to j;\ r_i^1, r_j^3)$

Recipient

Source

	HK	China	US	UK
HK		1162.9588	1.3715	522.4729
China	1731.5095		371.4659	286.5529
US	126.1763	0.3558		200.6902
UK	660.4182	287.9455	1136.9412	

$CK_1(i\to j;\ r_i^3, r_j^1)$

Recipient

Source

	HK	China	US	UK
HK		1808.6641	126.0475	666.2039
China	1113.3489		0.3558	284.5160
US	1.3729	371.4827		1150.4225
UK	517.9354	290.0070	198.3384	

The result of the cokurtosis test is shown in Table 4. From Table 4, we can see that the cokurtosis test has some similar features to the coskewness test. Firstly, the overall significance of contagion is much larger compared to the Forbes-Rigobon test. Using the 5% critical value of 3.84, there are 10 entries in each table greater than this critical value, so additional channels of contagion can be detected. In particular, the contagion effect between Hong Kong and China (in both directions) is the most significant. Secondly, the $CK_1(i\to j;\ r_i^1, r_j^3)$ and $CK_1(i\to j;\ r_i^3, r_j^1)$ statistics show opposite directions of contagion, just like the $CS_1(i\to j;\ r_i^1, r_j^2)$ and $CS_1(i\to j;\ r_i^2, r_j^1)$ statistics do. The reason for this is similar to that for the coskewness test. Comparing the cokurtosis test with the coskewness test, we can find that for each entry in the table of $CS_1(i\to j;\ r_i^1, r_j^2)$ (or $CS_1(i\to j;\ r_i^2, r_j^1)$), the corresponding entry in the table of $CK_1(i\to j;\ r_i^1, r_j^3)$ (or $CK_1(i\to j;\ r_i^3, r_j^1)$) is generally much larger in value. From this result, we can see that the cokurtosis test shows a much greater effect of contagion than the coskewness test does. Moreover, the cokurtosis test can detect further additional channels of contagion that the coskewness test fails to show. For example, the $CS_1(i\to j;\ r_i^1, r_j^2)$ and $CS_1(i\to j;\ r_i^2, r_j^1)$ statistics show that there is no significant evidence of contagion between China and U.S. at 5% significance level, but the $CK_1(i\to j;\ r_i^1, r_j^3)$ statistic shows that there is significant contagion from China to U.S., while the $CK_1(i\to j;\ r_i^3, r_j^1)$ statistic shows significant contagion in the opposite direction. Furthermore, we can see that for both coskewness and cokurtosis tests, the contagion effect between Hong Kong and China, and between U.S. and U.K., is more significant. On the other hand, the contagion effect between Hong Kong/China and U.S./U.K. is relatively smaller.

6. CONCLUSION AND DISCUSSION

In this paper, we use three different tests to investigate contagion across the real estate markets of four different countries during the financial tsunami. From the above section, the following results can be summarized:

1) The Forbes-Rigobon test shows that there is no significance evidence of contagion between any two countries.

2) The coskewness test can show additional channels of contagion that the Forces-Rigobon test fails to show.
3) The cokurtosis test can show further additional channels of contagion. The effect of contagion is much greater than that the coskewness test shows.
4) The coskewness and cokurtosis tests show that the contagion effect between Hong Kong and China, and between U.S. and U.K., is greater than that between Hong Kong/China and U.S./U.K..

From the results, we can see that the Forbes-Rigobon test shows no contagion between any two countries. This test shows the least significance evidence of contagion. The Forbes-Rigobon test has a deficiency that it only tests the interaction between the first moments of the joint distribution of returns, so it often fails to show the whole picture of the contagion pattern, and this deficiency is reflected in our results. The other two tests, which are Lagrange multiplier tests derived from the generalized exponential class, shows additional channels of contagion. In particular, the cokurtosis test shows further additional channels of contagion. In fact, our intention of extending Fry et al. (2006, 2008, 2010)'s method to higher order moments is to get a more complete picture of contagion. From the true result of the cokurtosis test, this goal is really achieved. Our results are similar to Fry et al. (2006), who showed that when testing contagion across real estate and equity markets during the East Asian crisis in 1997-98, the coskewness test showed additional channels of contagion which the Forbes-Rigobon test failed to show. Now we show that the cokurtosis test can explore further additional channels of contagion. In future research of contagion, it is desirable to use the coskewness and cokurtosis tests together.

In addition, from the results of the coskewness and cokurtosis tests, the main pattern of contagion is between Hong Kong and China, and between U.S. and U.K. The effect of contagion between Hong Kong/China and U.S./U.K. is relatively smaller. This can be explained as follows: Hong Kong is a Special Administrative Region (SAR) of China, so it has closer economic ties with China. Therefore, they have larger correlation between their market returns. During crisis periods, there is even stronger comovement between their markets. Moreover, with loosening restrictions, Mainland capital can enter or exit Hong Kong more freely than before. For example, during the financial tsunami, in order to reduce loss, many Mainland investors withdrawn a larger amount of capital from Hong Kong. Therefore, the effect of contagion between China and Hong Kong's markets is very strong. U.S. and U.K. are the leading financial centres in the world and have close market relationships for long, so the contagion effect between their markets is also large. In comparison, the contagion effect between China/Hong Kong and U.S./U.K., which belong to two different groups of markets, is not so strong. From this result, we can divide the four countries into two groups: Hong Kong/China and U.S./U.K. The contagion effect is stronger between countries in the same group than that between two countries of different groups. Hence, for effective diversification, we should choose countries of different groups to invest in. This is applicable to real life situation. Investors and fund managers are advised to include markets of different regions in the world in their portfolio to reduce their risk. They should also regularly monitor the indicators of contagion (e.g. coskewness and cokurtosis statistics). If there is significant evidence of contagion from one country to another in a type of asset market, then investors should avoid holding that type of asset of both countries together since their asset prices would tend to move together in the same direction. The same strategies apply for real estate investment. Property practitioners should diversify their real estate investment over different regions in the world, and monitor the indicators of contagion. This will result in better strategic property management of their portfolio.

For academics working on contagion, there is still plenty of scope of future research. One important issue is the interrelation between the equity and the real estate markets. Fry et al. (2006) applied the coskewness test to investigate contagion between equity and real estate markets of Hong Kong, Japan and Singapore during the Asian financial crisis in 1997-98. Their results showed that there existed contagion between the equity and real estate markets. We can use the cokurtosis test described in this paper to find out whether there are additional channels of contagion between these two types of markets.

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APPENDIX. Derivation of the statistics for the coskewness and cokurtosis tests

Here we show the steps of deriving the statistics for the coskewness and cokurtosis tests.

Theorem 1 (Fry et al., 2006, 2008, 2010) *Let* r *be an iid random variable of dimension* K *with the generalized exponential distribution*

$$f(r) = \exp(h - \eta)\,, \tag{13}$$

where: $h = \sum_{i=1}^{M} \theta_i g_i(r)$, θ *is a M vector of parameters summarizing the moments of the distribution, and*

$$\eta = \ln \int \cdots \int \exp h dr_1 \ldots\, dr_K\ . \tag{14}$$

The information matrix is given by

$$I = T\left(E\left(\frac{\partial h}{\partial \theta} \frac{\partial h}{\partial \theta'} \right) - E\left(\frac{\partial h}{\partial \theta} \right) E\left(\frac{\partial h}{\partial \theta'} \right) \right), \tag{15}$$

where: T denotes the sample size.

With the information matrix derived, the Lagrange multiplier statistic is given by

$$LM = G' I^{-1} G\,, \tag{16}$$

where:

$$G = \left. \frac{\partial \ln L}{\partial \theta} \right|_{\theta = \theta_0}\,, \tag{17}$$

$$L = \prod_{t=i}^{T} L_t = \prod_{t=1}^{T} f(r_t)\,. \tag{18}$$

For the coskewness test, at observation *t*, the generalized exponential distribution is given by

$$f(r_{1,t}, r_{2,t}) = \exp(h_t - \eta)\,, \tag{19}$$

where: η is given by (14) (with *h* replaced by h_t), and

$$h_t = -\frac{1}{2}\left(\frac{1}{1-\rho^2} \right) \left(\left(\frac{r_{1,t} - \mu_1}{\sigma_1} \right)^2 + \left(\frac{r_{2,t} - \mu_2}{\sigma_2} \right)^2 - 2\rho \left(\frac{r_{1,t} - \mu_1}{\sigma_1} \right) \left(\frac{r_{2,t} - \mu_2}{\sigma_2} \right) \right) + \varphi \left(\frac{r_{1,t} - \mu_1}{\sigma_1} \right) \left(\frac{r_{2,t} - \mu_2}{\sigma_2} \right)^2 . \tag{20}$$

Under the null hypothesis

$$H_0 : \varphi = 0\,, \tag{21}$$

f follows a bivariate normal distribution where the maximum likelihood estimators of the unknown parameters are

$$\hat{\mu}_i = \frac{1}{T} \sum_t r_{i,t}\ ;\ \hat{\sigma}_i^2 = \frac{1}{T} \sum_t (r_{i,t} - \hat{\mu}_i)^2 ;$$

$$\hat{\rho} = \frac{1}{T} \sum_t \left(\frac{r_{1,t} - \hat{\mu}_1}{\hat{\sigma}_1} \right) \left(\frac{r_{2,t} - \hat{\mu}_2}{\hat{\sigma}_2} \right). \tag{22}$$

The Lagrange multiplier statistic is (for details, please refer to Fry et al., 2008)

$$LM = \frac{T}{4\hat{\rho}^2 + 2} \times. \left(\frac{1}{T} \sum_{t=1}^{T} \left(\frac{r_{1,t} - \hat{\mu}_1}{\hat{\sigma}_1} \right) \left(\frac{r_{2,t} - \hat{\mu}_2}{\hat{\sigma}_2} \right)^2 \right)^2 \tag{23}$$

Under the null hypothesis, *LM* is asymptotically distributed as

$$LM \xrightarrow{d} \chi_1^2\,. \tag{24}$$

Hence the statistics for the coskewness tests are

$$CS_1(i \to j; r_i^1, r_j^2) = \left(\frac{\hat{\psi}_y(r_i^1, r_j^2) - \hat{\psi}_x(r_i^1, r_j^2)}{\sqrt{\dfrac{4\hat{v}_{y/x}^2 + 2}{T_y} + \dfrac{4\hat{\rho}_x^2 + 2}{T_x}}} \right)^2 , \tag{25}$$

$$CS_1(i \to j; r_i^2, r_j^1) = \left(\frac{\hat{\psi}_y(r_i^2, r_j^1) - \hat{\psi}_x(r_i^2, r_j^1)}{\sqrt{\dfrac{4\hat{v}_{y/x}^2 + 2}{T_y} + \dfrac{4\hat{\rho}_x^2 + 2}{T_x}}} \right)^2 , \tag{26}$$

where:

$$\hat{\psi}_y(r_i^m, r_j^n) = \frac{1}{T_y} \sum_{t=1}^{T} \left(\frac{r_{i,t} - \hat{\mu}_i}{\hat{\sigma}_{y,i}} \right)^m \left(\frac{r_{j,t} - \hat{\mu}_j}{\hat{\sigma}_{y,j}} \right)^n , \tag{27}$$

$$\hat{\psi}_x(r_i^m, r_j^n) = \frac{1}{T_x} \sum_{t=1}^{T} \left(\frac{r_{i,t} - \hat{\mu}_i}{\hat{\sigma}_{x,i}} \right)^m \left(\frac{r_{j,t} - \hat{\mu}_j}{\hat{\sigma}_{x,j}} \right)^n . \tag{28}$$

Under the null hypothesis of no contagion, the above testing statistics are asymptotically distributed as χ_1^2.

The method of deriving the statistics for the cokurtosis test generally follows the one used by Fry et al. (2008). For the cokurtosis test, the expression of h_t is

$$h_t = -\frac{1}{2}\left(\frac{1}{1-\rho^2}\right)\left(\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2 + \left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 - 2\rho\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)\right) + \varphi\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^3 . \tag{29}$$

Let the parameters be

$$\theta = \left\{\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho, \varphi\right\}. \tag{30}$$

The log-likelihood at time t is

$$\ln L_t = h_t - \eta , \tag{31}$$

where:

$$\eta = \ln \iint \exp h_t dr_1 dr_2 . \tag{32}$$

The first derivatives of h_t are

$$\frac{\partial h_t}{\partial \mu_1} = \frac{1}{\sigma_1}\left(\frac{1}{1-\rho^2}\left(\frac{r_{1,t}-\mu_1}{\sigma_1} - \rho\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)\right) - \varphi\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^3\right) \tag{33}$$

$$\frac{\partial h_t}{\partial \mu_2} = \frac{1}{\sigma_2}\left(\frac{1}{1-\rho^2}\left(\frac{r_{2,t}-\mu_2}{\sigma_2} - \rho\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\right) - 3\varphi\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2\right) \tag{34}$$

$$\frac{\partial h_t}{\partial \sigma_1^2} = \frac{1}{2\sigma_1^2}\left(\frac{1}{1-\rho^2}\left(\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2 - \rho\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)\right) - \varphi\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^3\right) \tag{35}$$

$$\frac{\partial h_t}{\partial \sigma_2^2} = \frac{1}{2\sigma_2^2}\left(\frac{1}{1-\rho^2}\left(\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 - \rho\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)\right) - 3\varphi\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^3\right) \tag{36}$$

$$\frac{\partial h_t}{\partial \rho} = \frac{1}{(1-\rho^2)^2}\left((1+\rho^2)\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right) \times \left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right) - \rho\left(\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2 + \left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2\right)\right) \tag{37}$$

$$\frac{\partial h_t}{\partial \varphi} = \left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^3 \tag{38}$$

By Theorem 1, the information matrix under the null hypothesis $(\varphi = 0)$ is

$$I = T\left(E\left(\frac{\partial h_t}{\partial \theta} \frac{\partial h_t}{\partial \theta'} \right) - E\left(\frac{\partial h_t}{\partial \theta} \right) E\left(\frac{\partial h_t}{\partial \theta'} \right) \right) \Bigg|_{\varphi=0} =$$

$$T\begin{bmatrix} \frac{1}{\sigma_1^2(1-\rho^2)} & -\frac{\rho}{\sigma_1\sigma_2(1-\rho^2)} & 0 & 0 & 0 & 0 \\ -\frac{\rho}{\sigma_1\sigma_2(1-\rho^2)} & \frac{1}{\sigma_2^2(1-\rho^2)} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2-\rho^2}{4\sigma_1^4(1-\rho^2)} & -\frac{\rho^2}{4\sigma_1^2\sigma_2^2(1-\rho^2)} & -\frac{\rho}{2\sigma_1^2(1-\rho^2)} & \frac{3\rho}{2\sigma_1^2} \\ 0 & 0 & -\frac{\rho^2}{4\sigma_1^2\sigma_2^2(1-\rho^2)} & \frac{2-\rho^2}{4\sigma_2^4(1-\rho^2)} & -\frac{\rho}{2\sigma_2^2(1-\rho^2)} & \frac{9\rho}{2\sigma_2^2} \\ 0 & 0 & -\frac{\rho}{2\sigma_1^2(1-\rho^2)} & -\frac{\rho}{2\sigma_2^2(1-\rho^2)} & \frac{1+\rho^2}{(1-\rho^2)^2} & 1 \\ 0 & 0 & \frac{3\rho}{2\sigma_1^2} & \frac{9\rho}{2\sigma_2^2} & 1 & 15+81\rho^2 \end{bmatrix} \qquad (39)$$

Taking the gradient for φ under the null gives

$$\frac{\partial \ln L}{\partial \varphi} = \sum_{t=1}^{T} \frac{\partial h_t}{\partial \varphi} - T\frac{\partial \eta}{\partial \varphi} =$$

$$\sum_{t=1}^{T} \left(\frac{r_{1,t}-\mu_1}{\sigma_1} \right) \left(\frac{r_{2,t}-\mu_2}{\sigma_2} \right)^3 - T\left(E\left(\frac{\partial h_t}{\partial \varphi} \right) \right) =$$

$$\sum_{t=1}^{T} \left(\frac{r_{1,t}-\mu_1}{\sigma_1} \right) \left(\frac{r_{2,t}-\mu_2}{\sigma_2} \right)^3 . \qquad (40)$$

The gradient vector under the null is

$$G = \left[0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \sum_{t=1}^{T} \left(\frac{r_{1,t}-\mu_1}{\sigma_1} \right) \left(\times^{2,} \qquad {}^{2} \right) - 3\rho \right.$$

$$\left. \left(\frac{r_{2,t}-\mu_2}{\sigma_2} \right)^3 \right]' . \qquad (41)$$

The M statistic is obtained by substituting (39) and (41) into (16), and replacing the unknown parameters by the maximum likelihood estimators in (22). Thus

$$LM = G' I\hat{\ }(-1) G =$$

$$\frac{1}{2T(7+13\hat{\rho}^2-8\hat{\rho}^4)} \times$$

$$\left(\sum_{t=1}^{T} \left(\frac{r_{1,t}-\hat{\mu}_1}{\hat{\sigma}_1} \right) \left(\frac{r_{2,t}-\hat{\mu}_2}{\hat{\sigma}_2} \right)^3 \right)^2 =$$

$$\frac{1}{2(7+13\hat{\rho}^2-8\hat{\rho}^4)} \times$$

$$\left(\frac{1}{T} \sum_{t=1}^{T} \left(\frac{r_{1,t}-\hat{\mu}_1}{\hat{\sigma}_1} \right) \left(\frac{r_{2,t}-\hat{\mu}_2}{\hat{\sigma}_2} \right)^3 \right)^2 , \qquad (42)$$

which is asymptotically distributed as χ_1^2 under the null hypothesis.

Hence the statistics for the cokurtosis tests are

$$CK_1(i \to j; r_i^1, r_j^3) = \left(\frac{\hat{\psi}_y(r_i^1, r_j^3) - \hat{\psi}_x(r_i^1, r_j^3)}{\sqrt{\frac{2(7 + 13\hat{v}^2_{y|x} - 8\hat{v}^4_{y|x})}{T_y} + \frac{2(7 + 13\hat{\rho}^2_x - 8\hat{\rho}^4_x)}{T_x}}} \right)^2, \tag{43}$$

$$CK_1(i \to j; r_i^3, r_j^1) = \left(\frac{\hat{\psi}_y(r_i^3, r_j^1) - \hat{\psi}_x(r_i^3, r_j^1)}{\sqrt{\frac{2(7 + 13\hat{v}^2_{y/x} - 8\hat{v}^4_{y/x})}{T_y} + \frac{2(7 + 13\hat{\rho}^2_x - 8\hat{\rho}^4_x)}{T_x}}} \right)^2, \tag{44}$$

where: $\hat{\psi}_y(r_i^m, r_j^n)$ and $\hat{\psi}_x(r_i^m, r_j^n)$ are defined by (27) and (28) respectively. Under the null hypothesis of no contagion, the above testing statistics are asymptotically distributed as χ_1^2.