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PROACTIVE PRICING STRATEGIES FOR ON-STREET PARKING MANAGEMENT WITH PHYSICS-INFORMED NEURAL NETWORKS

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Abstract. Effective pricing is important for on-street parking management and proactive parking pricing is an innovative strategy to achieve optimal parking utilization. For proactive parking pricing, accurately predicting parking occupancy and deriving the price elasticity of parking demand are necessary. In recent years, there have been an increasing number of studies applying big data technology for parking-occupancy prediction. However, existing research has not incorporated economic knowledge into modeling, thus preventing application of the price elasticity of parking demand. In this study, proactive pricing strategies are proposed to adjust on-street parking prices which involve a parking-occupancy prediction model and a price-optimization method. Physics-informed neural networks are employed to achieve accurate prediction of parking occupancy and calculation of parking price elasticity. An elasticity-occupancy parking-management strategy is proposed for on-street parking management which leverages parking occupancy and price elasticity to guide pricing interventions. A case study shows that the parking-occupancy prediction model can make accurate predictions and derive the price elasticity of parking demand. Proactive parking pricing enables drivers to plan their trips in advance, allowing parking occupancy within an optimal range.

Keywords: on-street parking management, parking pricing, parking-occupancy prediction, physics-informed neural networks, price elasticity, proactive pricing.

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1. Introduction

More and more large cities are facing the problem of insufficient supply of parking spaces (Saharan et al., 2020a). Studies have shown that it takes between 3.5 and 14 minutes to find a parking space, and that between 8% and 74% of the traffic is cruising for parking (Shoup, 2006). This situation further deteriorates the operation of road traffic, causing air pollution and the waste of fuel resources (Yang et al., 2019). In this context, many parking-related smart services have been developed and applied, for example, parking-occupancy predictions, parking-recommendation systems, and dynamic parking pricing, which are important parts of intelligent transportation systems (Saharan et al., 2020b; Yuan et al., 2020). Parking pricing is important for effective parking management. Excessively high parking prices can lead to low parking demand, resulting in wasted parking resources, or may encourage illegal parking if the cost of violating parking rules is lower than the parking fees. On the other hand, low prices can lead to an insufficient supply of parking spaces, causing further issues with illegal parking. Therefore, it is important to study parking pricing to implement reasonable parking price, thereby improving the utilization efficiency of parking resources. To achieve this goal, some cities have made attempts to achieve demand-responsive parking pricing. For instance, the San Francisco government has launched the SF Park project. In this project, parking prices are adjusted every 1–4 months. For on-street parking, the hourly price increases by $0.25 when parking occupancy is in the range of 80–100%, remains the same at 60–80% of parking occupancy, decreases by $0.25 at 30–60% of parking occupancy, and decreases by $0.50 when parking occupancy is less than 30%. Research indicates that since the implementation of the SF Park project, there has been an overall reduction in parking demand, a 15% reduction in cruising time in urban centers, and a 12% reduction in cruising distance (Alemi et al., 2018).

Previous parking-pricing strategies typically rely on reactive approaches, where pricing optimization is based on observed parking demand. However, proactive pricing strategies involve adjusting prices by predicting parking occupancy (Hong et al., 2022), which allows drivers to plan and manage their parking in advance. It is important to accurately predict parking occupancy and to derive the price elasticity of parking demand in proactive parking pricing. For parking-occupancy prediction, the primary objective of prediction models is to enhance the accuracy of parking-occupancy prediction. But there are very few prediction models that incorporate the parking price, a significant factor affecting parking choice. The relationship between parking price and parking demand constitutes economic knowledge. But existing studies have not incorporated this important economic knowledge into their modeling. The price elasticity of demand has been studied for a long time. Methods for deriving price elasticity include market experiments, historical data analysis, and questionnaires. Market experiments and questionnaires have obvious limitations due to the lack of actual parking scenes and the high costs of time and money for experiments. Historical data analysis mainly focuses on extracting the average data before and after a price change, which may ignore the details and complex relationships at the micro level.

A parking-occupancy prediction model and a price-optimization method are proposed to solve the above problems and improve the effectiveness of proactive parking pricing. Based on physics-informed neural networks (PINNs), the model integrates multivariate data and economic knowledge into the training process, which not only predicts the parking occupancy but also derives the price elasticity of parking demand. An elasticity-occupancy parking-management strategy is proposed based on different levels of parking occupancy and price elasticity. On this basis, proactive parking pricing is carried out. The main contributions of this paper are as follows: 1) This paper proposes a proactive parking pricing method that predicts parking occupancy and incorporates parking price elasticity into pricing decisions. Proactive pricing allows drivers to be informed of estimated parking costs in advance, enabling them to plan their trips and choose parking options accordingly. Reasonable pricing can bring parking occupancy rates closer to ideal levels, guiding parking behavior through price adjustments, thereby improving the spatial and temporal distribution of parking demand and optimizing the utilization of parking resources. Additionally, the advance estimation of parking prices aligns with drivers' expectations, making dynamic pricing adjustments more acceptable. 2) The paper employs a method based on PINNs in the parking availability prediction model. This method leverages existing data on parking occupancy and prices, embedding the economic relationship between these two variables into the model training process. This approach not only enhances the prediction accuracy of the model but also simultaneously calculates the price elasticity of parking demand. The PINNs method can address the issue of data scarcity by making full use of limited data through the integration of economic knowledge.

The rest of the paper is organized as follows. Section 2 summarizes the related work. Sections 3 introduce the research method, including the parking-occupancy prediction model and parking management. Section 4 presents the results of the parking-occupancy prediction for the SF Park dataset and discusses price optimization. Section 5 presents the conclusion.

2. Literature review

2.1. Parking-occupancy prediction

Early traffic-prediction methods were mainly based on statistical models that calculated future states based on selected variables and corresponding coefficients, including the historical average model (HA) (Kamarianakis & Prastacos, 2003), autoregressive moving average model (ARIMA) (Zhang, 2003), and least absolute shrinkage and selection operator (Lasso) (Tibshirani, 1997). Some other prediction methods employed traditional mathematical models such as the Markov M/M/C/C queuing model (Xiao et al., 2018). While traditional models offer the benefits of simple calculation methods and quick solution speeds, they struggle to capture complex features, which results in the inability to reflect data uncertainty and nonlinearity, leading to poor prediction accuracy. Currently, machine learning and neural network models are increasingly used for prediction, including regression tree, support vector regression (SVR) (Smola & Schölkopf, 2004), random forest (Jelen et al., 2021), recurrent neural network (RNN) (Elman, 1990), typical long short-term memory network (LSTM) (Hochreiter & Schmidhuber, 1997) and gated recurrent unit (GRU) models (Chung et al., 2014). These models can automatically learn previous experiences from data samples and approximate the function that best describes the regularity of the sample data. Therefore, machine learning and neural network models are particularly good at solving complex nonlinear problems, allowing them to achieve higher accuracy than previous methods. Some studies have utilized feed-forward neural networks, which contain multiple hidden layers, for parking-occupancy prediction (Vlahogianni et al., 2016; Ismail et al., 2021). This is also known as multilayer perception. Apart from these models, there are variations based on the foundational models mentioned above. A novel multistep LSTM RNN model (Fan et al., 2022) is proposed to predict parking occupancy. A parking-occupancy prediction model named Du-parking successfully integrates LSTM and linear layer outputs, and has been successfully applied to the Baidu Maps app to provide commercial services for large cities (Rong et al., 2018). While these models can effectively capture long- and short-term dependencies in time-series data, they ignore spatial correlations. For spatial and temporal parking-occupancy prediction models, convolutional neural networks are applied to capture the spatial correlations of nodes. Feng et al. (2022) used two parallel convolutional LSTM models to capture temporal and spatial dependencies while utilizing dense convolutional networks to further improve feature propagation and reuse. A spatio-temporal model with a convolutional structure, STGCN, achieved good results on several tasks (Yu et al., 2018).

The inherent "black box" nature of machine learning often makes it challenging to interpret machine-learning results within the complex framework of prediction. To address this issue, PINNs (Raissi et al., 2019) are proposed, which combine deep learning and physics constraints for solving partial differential equations (PDEs) and other physics problems. PINNs utilize the capabilities of deep neural networks as general function approximators by adding the physical equations of the PDEs as constraints to the training process of the neural network (Hornik et al., 1989). This integration allows the model to solve complex scientific problems with high accuracy, even when data is limited.

2.2. Parking management

Parking-demand management compensates for the external costs of congestion and can serve the purposes of optimizing parking activity and adjusting parking demand. The main strategies are strict parking enforcement, parking pricing policies, and changes in parking supply. Parking pricing is considered the most economical way to manage parking demand and has been widely applied. Parking resources cannot be stored. The incremental cost of selling additional parking facilities is close to zero and the benefit of adding an additional unit is very large if the capacity is fully utilized. Parking price can affect parking choice. Research shows that drivers tend to respond to parking pricing policies by moving to different parking facilities rather than transitioning to alternative transportation modes. This tendency is particularly evident in countries like the United States, where gas and parking prices are relatively low. For certain individuals, driving a private car significantly reduces time and is very convenient, making it more attractive than taking public transportation (Yan et al., 2019). Therefore, reasonable parking pricing is necessary (Friesen & Mingardo, 2020). Parking pricing strategies primarily aim to achieve two objectives: maximizing economic benefits and ensuring equitable utilization of parking facilities. Maximizing economic benefits involves three aspects: drivers, parking facility operators, and society. Achieving equilibrium in parking facility utilization often involves optimizing parking occupancy, typically targeted within the range of 60% to 80% (Millard-Ball et al., 2014) or sometimes aiming for an ideal value around 85% (Shoup, 2006). Another optimization goal involves ensuring uniform spatial distribution of parking facility utilization. The main constraints include the range of parking prices and parking facility capacity (Fabusuyi & Hampshire, 2018), etc.

Parking price optimization based on parking occupancy is an effective approach to maintaining parking occupancy within an optimal range (Maternini et al., 2017). Parking pricing techniques can be categorized into optimization-based techniques (Qian & Rajagopal, 2013; Kotb et al., 2016), queuing theory-based techniques (Larson & Sasanuma, 2010), and machine learning-based techniques (Saharan et al., 2020a; Hong et al., 2022). Parking pricing methods encompass both reactive and proactive strategies. Reactive strategies rely on observed parking demand for price optimization, while proactive strategies involve the implementation of prediction-based optimization models that predict parking occupancy and optimize prices based on the prediction results (Hong et al., 2022). Some cities have implemented pilot projects for parking price adjustments, such as San Francisco and Seattle, which have adopted reactive pricing strategies. These projects adjust prices by observing the average parking occupancy over a certain period of time. Real-time parking pricing studies also employ reactive pricing strategies. A study developed a dynamic non-cooperative bilevel model, known as the Stackelberg leader-follower game, to enable real-time adjustment of parking prices (Mackowski et al., 2015). For proactive pricing strategies, a study utilized a proactive prediction-driven optimization framework to adjust parking prices. The framework employs neural ordinary differential equations to predict parking occupancy based on historical occupancy and price information. A "one-shot" pricing-optimization method has also been devised (Hong et al., 2022). Additionally, a two-stage panel data regression and optimization model was proposed to adjust parking prices by calculating the price elasticity of parking demand (Fabusuyi & Hampshire, 2018).

3. Methodology

In this section, the experimental methods used in this study will be introduced. This section consists of two main parts: the parking-occupancy prediction and parking management. The parking-occupancy prediction part includes a parking-occupancy prediction model, which not only predicts the parking occupancy but also calculates the price elasticity of parking demand. Based on the output of the parking-occupancy prediction model, the parking management part uses a price optimization method to adjust parking prices.

3.1. Parking-occupancy prediction

This parking-occupancy prediction model incorporates PINNs to enhance prediction accuracy and derive the price elasticity of parking demand during the training process. The overall structure of the prediction model is shown in Figure 1. The model consists of three modules: a data-enhancement module, a neural network module, and a PINNs module. The data-enhancement module performs data enhancement by decomposing original parking occupancy data into trend and cycle features, and combining this with data on parking prices to improve prediction accuracy. The neural network module predicts parking occupancy by merging three inputs and putting them into neural networks for training. The PINNs module computes partial derivatives of the model's predicted outcomes and integrates them as components of the loss function.

In Figure 1, $\mathbf{T}(t_r)$, $\mathbf{C}(t_f)$, and $\mathbf{P}(t)$ represent the three inputs of trend feature, cycle feature, and parking price. Trend feature and cycle feature are derived through a time-series decomposition (TSD) of time-series parking-occupancy data. The model sets up a residual-like connection between the first and second layers of the network, which refers to splicing the trend feature and the output

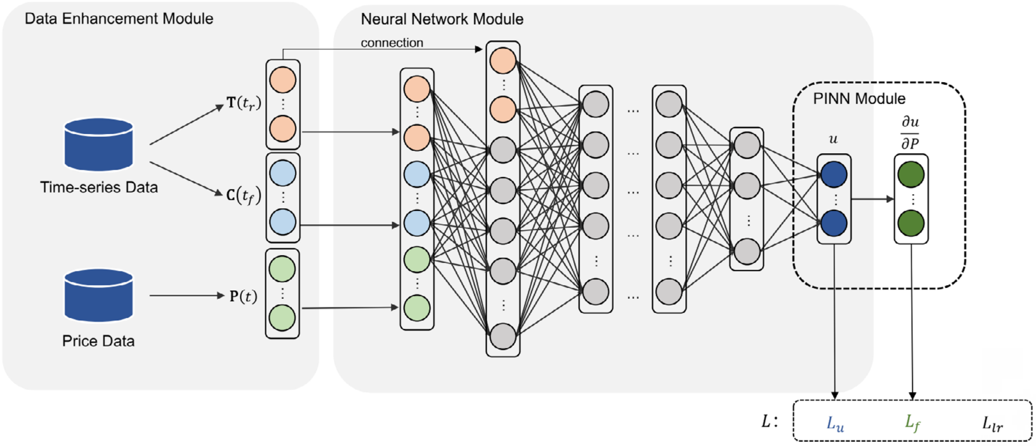

Figure 1. Overall structure of parking occupancy prediction model

result of the first layer and passing this as input to the second layer in the network. In the PINNs module, u is the output result of the prediction model and $\partial u/\partial P$ means the partial derivative of the prediction result and price. The L is the loss function during the training process, which consists of three parts, L_u, L_f and L_{lr}. L_u represents the difference between the predicted value and the true value, L_f represents economic knowledge, and L_{lr} represents the regular term.

3.1.1. Data-enhancement module

Traffic data exhibits periodic characteristics, and recent studies have shown that TSD can improve prediction accuracy (Li et al., 2023; Taylor & Letham, 2018). Therefore, the parking-occupancy prediction model incorporates a data enhancement module that introduces TSD methods. The parking-occupancy data is put into a TSD of the trend feature, which represents the current impact, and the cycle feature, which captures the cyclical changes in traffic data. The model takes three features as input: trend feature, cycle feature, and parking price:

$$\mathbf{T}(t_r) + \mathbf{C}(t_f) + \mathbf{P}(t) \xrightarrow{f} \hat{y}(t), \tag{1}$$

where: $\hat{y}(t)$ is the predicted value at moment t; $\mathbf{T}(t_r)$ represents the trend feature measured by the continuous change in the lookback period t_r; $\mathbf{C}(t_f)$ is the periodic change in predicted period t_f; and $\mathbf{P}(t)$ denotes the parking price at time t.

The trend feature represents the short-term impact. As the predicted values are highly influenced by recent data, the trend feature plays a pivotal role in the prediction. Consequently, the trend feature is represented by the parking occupancy within the lookback period:

$$\mathbf{T}(t_r) = \left[y(t-\lambda), y(t-\lambda+1), \dots, y(t-1) \right], \tag{2}$$

where: y represents the actual values of parking occupancy; λ is the lookback window size.

The cycle feature captures the cyclic characteristics and long-term patterns within the data. In this experiment, the standard Fourier series (Harvey & Shephard, 1993) is used for fitting the periodicity of the parking-occupancy data. The Fourier series constitutes a method to represent periodic functions by utilizing an infinite series of sine and cosine functions. The main idea is that any arbitrary periodic function can be approximated by a composite of multiple sine and cosine functions. As the number of terms included in the series increases, the accuracy of the approximation also increases. Assuming that the function $f(x)$ with a period T, then its Fourier series can be expressed as follows:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx), \tag{3}$$

where: a_0 is a constant coefficient; and a_n and b_n are a series of coefficients that can be computed by integration.

Historical data is used (i.e., the training set) for fitting the Fourier series during training, ensuring zero information leakage. In a specific time context denoted by $\hat{t}$, with a given number of signals N and a constant T, Equation (3) can be reformulated as Equation (4) and the cycle feature can be defined by Equation (5):

$$c(\hat{t}) = \frac{a_0}{2} + \sum_{n=1}^{N} (a_n \cos(\frac{2\pi n\hat{t}}{T}) + b_n \sin(\frac{2\pi n\hat{t}}{T})); \tag{4}$$

$$\mathbf{C}(t_f) = \left[c(t-\lambda), c(t-\lambda+1), \dots, c(t-1) \right], \tag{5}$$

where c represents the values of the cycle feature.

The parking price is an important factor that affects parking choice (Shoup, 2018). In order to enhance the applicability of our research findings for pricing strategy implementation, it is important to take into account the influence of price uncertainty on drivers' behavior. In proactive pricing, the parking price is predetermined before the driver parks. A specific data-processing method is employed to achieve this goal (Figure 2). In general, prediction tasks rely on using data from the lookback period. However, our method uses data from the expected future moment as input. To ensure consistent input data length, the future moment parking data at time t is replicated, extending it to match the length of the lookback period. The data is utilized as parking price data $\mathbf{P}(t)$, serving as one of the inputs to the model:

$$\mathbf{P}(t) = \left[P(t), P(t), ..., P(t)\right], \tag{6}$$

where $P(t)$ represents the parking price data at time t.

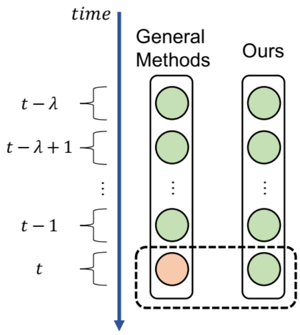

Figure 2. Processing of price data

3.1.2. Neural network module

A fully connected neural network is designed to predict parking occupancy in the neural network module. The network consists of five layers. The input layer includes the trend feature, cycle feature, and price data. These three sets of data are horizontally concatenated. To effectively avoid the gradient disappearance problem and to converge quickly, a series of rectified linear unit (ReLU) activation functions is used from the first to the third layers. Subsequently, a hyperbolic tangent (Tanh) activation function is introduced in the fourth layer. The output layer is composed of a fully connected structure responsible for outputting the prediction results. To enhance the learning process of the network, a residual-like structure is integrated within the input of the second layer. This architectural choice serves to simplify the learning task for the network while retaining more information and optimizing gradient propagation, which can engender a more efficient transfer of information within the network.

In order to clearly express the composition of the neural network, the settings of the network are shown in Table 1. Among them, x(1) and x(1)' are the same, but they are distinguished as input and output. The same applies to x(2) and x(2)', x(3) and x(3)', and x(4) and x(4)'. For the first layer, the input includes the trend feature, cycle feature, and price data, with 45 dims. After passing through a fully connected layer and the ReLU function, the output x(1) with 100 dims is obtained. The output of the first layer is then horizontally concatenated with the trend feature from the input to form the input for the second layer. After passing through another fully connected layer and ReLU function, the dims count increases from 109 to 120. The output of the second layer is denoted x(2). The output of the second layer is used as the input for the third layer, where, after passing through a fully connected layer and ReLU function, the dims decrease from 120 to 100. In the fourth layer, the output x(3) from the third layer serves as input and, after passing through a fully connected layer with Tanh as the activation function, the output x(4) is obtained with the dims decreasing from 100 to 50. In the fifth layer, the input is x(4)' and the output corresponds to the ultimate output u of the model. The fifth layer consists of a fully connected layer, with the dims transitioning from 50 to 1.

3.1.3. PINNs module

Parking occupancy is predicted and the price elasticity of parking demand is calculated through the PINNs module. Before defining the loss function, it is necessary to introduce the price elasticity of demand and define a function that incorporates both price and parking occupancy as economic knowledge. The price elasticity of demand is used to measure the sensitivity of demand for goods or services to changes in price. It is typically defined as the percentage change in quantity to the percentage change in price. The general equation can be expressed as:

$$\varepsilon = \frac{\partial q}{\partial p} \cdot \frac{p}{q}, \tag{7}$$

where: ε represents the price elasticity of demand; q represents the demand; and p represents the price.

There usually exists an inverse relationship between price and demand, resulting in a negative value for the price elasticity of demand. The absolute value is often used to indicate the magnitude of price elasticity. The value of the price elasticity of demand can be categorized into three situations: when the price elasticity of demand value is greater than 1, it is referred to as elastic demand; when it falls between 0 and 1, it is called inelastic demand; and when it equals 0, it signifies perfectly inelastic demand. The price elasticity of parking demand value often falls within the range of 0 to 1, indicating inelastic demand. Since the price elasticity of parking demand is

Table 1. The settings of the neural network

Layer number	Input	Input dims	Activation function	Output	Output dims
1	input	45	ReLU	x(1)	100
2	||T, x(1) ' ||	109	ReLU	x(2)	120
3	x(2) '	120	ReLU	x(3)	100
4	x(3) '	100	Tanh	x(4)	50
5	x(4) '	50	–	u	1

Note: || || indicates "horizontally concatenated" operation.

consistently negative, its absolute value is utilized instead in subsequent analyses.

Equation (8) is defined based on Equation (7) and subsequently utilized in the computation of the loss function. Equation (8) facilitates the calculation of the parking price elasticity, which incorporates the economic knowledge into the training of the network:

$$f(t) = \frac{\partial u}{\partial P} - \frac{u}{P} \cdot \varepsilon^*, \tag{8}$$

where: P is the initial parking price; and ε^* is the price elasticity of parking demand, which is set as a trainable parameter.

3.1.4. Loss function module

Three functions are defined representing different meanings. These functions are appropriately weighted and combined, culminating in the computation of model loss:

$$L = \frac{L_u}{L_u + L_f} \cdot L_u + \frac{L_f}{L_u + L_f} \cdot L_f + \alpha L_{lr}, \tag{9}$$

where: L means the loss function; L_u is the real loss value and represents the difference between the predicted value and the real value; L_f stands for the economic knowledge; and L_{lr} is a regular term which stands for the empirical value to tell the model a priori knowledge so that the model does not deviate too much from a certain value when training the parameters; α is the weight of L_{lr} within the loss function.

Each loss function mentioned above can be calculated by the following equations:

$$L_u = \frac{1}{N} \sum_{t=1}^{N} (\hat{y}(t) - y(t))^2 ; \tag{9a}$$

$$L_f = \frac{1}{N} \sum_{t=1}^{N} (f(t) - 0)^2 ; \tag{9b}$$

$$L_{lr} = \frac{1}{N} \sum_{t=1}^{N} (\varepsilon^* - \lambda)^2, \tag{9c}$$

where: $f(t)$ is calculated by Equation (8); and λ denotes an empirical value.

The weights between the three functions are important. One of our innovations is that we implement a useful trick to automatically adjust the weights between L_u and L_f. During training, the ratio is set to match the proportion of their values relative to the sum. This means that the component with the larger value will have a greater training weight, allowing more "effort" to train the larger weighted component, so that there is a certain constraint on the training speed between L_u and L_f.

The loss function introduces L_f representing economic knowledge, ensuring that the model is constrained by the price elasticity equation (Equation (8)) during training. This modification to the loss function embeds economic knowledge into the model training process, enabling the trained model to align as closely as possible with economic principles and real-world conditions.

3.2. Parking management

The parking-occupancy prediction model mentioned above can predict parking occupancy and derive parking price elasticity, but it does not involve price adjustment. In this section, the output results of the parking-occupancy prediction model are used to formulate an elasticity-occupancy parking-management strategy and price optimization.

3.2.1. Parking-management strategy

The price elasticity of parking demand is important for formulating our parking-management strategy. Parking facilities can be classified into four types based on the level of peak-hour parking occupancy and the magnitude of price elasticity: high price elasticity-high occupancy, high price elasticity-low occupancy, low price elasticity-high occupancy, and low price elasticity-low occupancy. Building upon this typology, this study formulates four corresponding on-street parking-management strategies, called the elasticity-occupancy parking-management strategy (Table 2). For parking facilities with high price elasticity, employing pricing measures is preferable for parking demand management. When peak-hour parking occupancy surpasses the upper limit of the ideal parking occupancy, raising parking prices is a viable approach. Conversely, if facilities experience overall parking occupancy below the lower limit of ideal values, lowering prices may be considered in order to alleviate parking pressure on surrounding facilities. Therefore, adjusting prices for different types of parking facilities can alleviate the spatial and temporal imbalance distribution of parking demand, ultimately improving parking facility utilization. However, parking facilities with low price elasticity can be managed through non-price measures, because the impact of pricing strategies is minimal. For example, in the case of peak-hour parking occupancy, implementing restrictions on parking duration may be a useful approach. Therefore, the following parking price optimization only focuses on parking facilities with high price elasticity.

Table 2. The elasticity-occupancy parking management strategy

	High occupancy	Low occupancy
High price elasticity	Increase parking price	Decrease parking price
Low price elasticity	Other non-pricing measures	–

3.2.2. Price-optimization method

According to the above elasticity-occupancy parking-management strategy, we focus on parking facilities with high price elasticity of parking demand and adjust their prices using a price optimization method to control occupancy within an ideal range as much as possible. The price-optimization method consists of an objective function and some constraints. The objective function aims to minimize deviations from the established goals. The constraints include price policy constraints, price change constraints, and non-negativity constraints. The decision variable is the parking price, modified to affect parking occupancy. Because different parking facilities have different price elasticities of parking demand, price optimization is conducted individually for each parking facility, that is, on a facility-by-facility basis. Detailed information is provided below.

The objective of this experiment is to achieve parking occupancy close to the optimal range by adjusting parking prices. Therefore, in the objective function, the decision variable is the parking price. By varying the price and leveraging the elasticity of parking demand, adjustments are made to minimize the deviation between the actual parking occupancy rate and the optimal range of parking occupancy rates. The objective function for parking price optimization is defined as follows. Let d_y^+ represent the upward deviation of the parking occupancy from the upper limit of the ideal parking occupancy y_+^*, and let d_y^- represent the downward deviation from the lower limit y_-^*. The optimization method minimizes the objective function Z by adjusting the parking price:

$$\begin{aligned}\arg\min_{\mathbf{P}^*} Z &= f(\mathbf{d}_y^+, \mathbf{d}_y^-) \\ &= \sum \left\| \hat{y} - y^* \right\|_1 \\ &= \sum \varphi \left\| \hat{y} - y_+^* \right\|_1 + \gamma \left\| \hat{y} - y_-^* \right\|_1 . \end{aligned} \quad (10)$$

The values of φ and γ can be expressed as follows:

$$\varphi = \begin{cases} 1, \text{ if } \hat{y} > y_+^*; \\ 0, \text{ else} \end{cases} \quad (10a)$$

$$\gamma = \begin{cases} 1, \text{ if } \hat{y} < y_-^*, \\ 0, \text{ else} \end{cases} \quad (10b)$$

where: $\mathbf{P}^*$ represents the set of all parking facilities prices after optimization; y^* represents the optimal parking occupancy; and $f(\mathbf{d}_y^+, \mathbf{d}_y^-)$ represents the difference between the predicted parking occupancy and the optimal parking occupancy.

The relationship between parking price and occupancy can be expressed as follows:

$$\Delta P = \frac{\left\| \hat{y} - y^* \right\|_1}{\mathring{a}^*} = P - P^*, \quad (11)$$

where: ΔP is the price change; P^* represents the optimal parking price of a parking facility; and all of the optimal price P^* values constitute $\mathbf{P}^*$.

The constraints are as follows. Parking price policies often come with upper and lower limits on prices. To comply with the price policy requirements, parking prices should be limited to between $p_{\min}$ and $p_{\max}$ per hour:

$$p_{\min} \le P^* \le p_{\max}. \quad (12)$$

Considering drivers' feelings, the price changes should be integer multiples of g. Let the price change amount be c times, where c is a constant. The price change constraint is as follows:

$$P^* = P + cg. \quad (13)$$

At the same time, make sure that the optimized price does not deviate significantly from the previous moment. δ is set to achieve the purpose and can be expressed as follows:

$$\left| P^* - P \right| \le \delta. \quad (14)$$

Other constraints include the non-negativity constraint on the change in parking occupancy and the requirement that the change in parking occupancy, either upward or downward, should be at least zero for one of them:

$$d_y^+, d_y^- \ge 0; \quad (15)$$

$$d_y^+ \cdot d_y^- = 0. \quad (16)$$

4. Results and findings

Experiments were conducted on the dataset of the SF Park project. This section describes the experiments on parking-occupancy prediction and the experiments on parking pricing. The parking-occupancy prediction experiments predict parking occupancy and derive the price elasticity of parking demand for each parking facility. The parking pricing experiments utilize the results from the parking-occupancy prediction experiments to establish reasonable parking prices.

4.1. Experiments on parking-occupancy prediction

4.1.1. Data description

The SF Park project is a parking pricing program in San Francisco, initiated and managed by the San Francisco Municipal Transportation Agency (SFMTA). The project started in 2008 and covers on-street and off-street parking in several areas of San Francisco, almost covering the entire city. The goal is to optimize parking management, increase parking utilization, reduce traffic congestion, and improve the urban traffic environment through the introduction of modern technologies and data analytics.

This experiment used data collected during the SF Park pilot program available on the SFMTA website. The dataset contains hourly on-street parking occupancy and meter price data for each block (parking facility) in seven parking districts from April 2011 to July 2013, involving a total of 10 price adjustments. This study focuses on the pilot area of the SF Park project in San Francisco and the subject of the study is on-street parking. We selected data from weekdays, spanning 9 AM to 5 PM, over a period from August 8, 2011 to December 7, 2012, encompassing a total of 70 weeks. The study included 192 parking facilities, with each undergoing 8 adjustments within this timeframe. A total of 604,800 parking records were used.

4.1.2. Experimental setup

Three standard evaluation metrics were employed to assess the model's performance: mean absolute error (MAE), root mean square error (RMSE), and coefficient of determination (R^2). By applying these metrics, a comprehensive evaluation of the effectiveness of the prediction model against the established baselines was obtained:

$$\mathrm{MAE} = \frac{1}{N}\sum_{t=1}^{N}\left|\hat{y}(t) - y(t)\right|; \quad (17)$$

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(\hat{y}(t) - y(t))^2}; \quad (18)$$

$$R^2 = 1 - \frac{\sum_{t=1}^{N}(\hat{y}(t) - y(t))^2}{\sum_{t=1}^{N}(y(t) - \bar{y}(t))^2}, \quad (19)$$

where: $\hat{y}(t)$ and $y(t)$ denote the predicted and actual values at time t; $\bar{y}(t)$ is the mean value of the sample; and N is the total number of samples.

Eight models were selected as comparison models: HA (Kamarianakis & Prastacos, 2003), ARIMA (Zhang, 2003), SVR (Smola & Schölkopf, 2004), Lasso (Tibshirani, 1997), fully convolutional neural network (FCNN), LSTM (Hochreiter & Schmidhuber, 1997), Du-parking (Rong et al., 2018), and GRU (Chung et al., 2014).

The dataset was split into training and testing sets, with a ratio of 7:3. All datasets were collected hourly. To predict data for one hour, the model utilized data from the previous nine continuous hours as input. For training, the Adam optimizer was used with a learning rate of 1×10^{-3} and a weight decay of 1×10^{-5}. The training process spanned 3000 epochs. For the prediction model parameters, the input for the model's initial parameters ε^* was a matrix of size $1\times1\times192$ filled with the value of 0.5. Building upon previous research findings (Millard-Ball et al., 2013; Pierce & Shoup, 2013), λ was designated 0.5 in Equation (9c) and α was set to 0.01 in Equation (9). In the case of the RNN-based model, there is an RNN layer and a fully connected layer. The hidden layer was set to 192 and the num layer was 1.

4.1.3. Comparison experimental results

Comparison experiments were conducted to verify the superiority of the proposed model and the results are shown in Table 3. The experiments utilized the SF Park dataset. In prediction error, the proposed model outperformed the baseline models in all three metrics. The mean value of RMSE was 0.1131, the mean value of MAE was 0.0813, and the mean value of R^2 was 51.42%. Compared with HA, ARIMA, SVR, Lasso, FCNN, LSTM, GRU, and Du-parking, the proposed model increased the prediction accuracy by 26.54%, 38.79%, 10.91%, 5.07%, 10.91%, 21.39%, 17.06%, and 11.03% respectively, highlighting its potential in prediction.

4.1.4. Price elasticity of parking demand

Comparison experiments took the average value of five random experiments for comparison. Therefore, after five training sessions, five sets of price elasticity values of parking demand were obtained. The resulting price elasticity values demonstrated a consistent trend of change, with only minor differences between values. To determine the final price elasticity values for each parking facility, the price elasticity values obtained from the five sessions were averaged. And to better show the distribution of parking price elasticity values and standard deviation, all the price elasticity values were divided into 10 intervals and the numbers of each interval counted to create a statistical

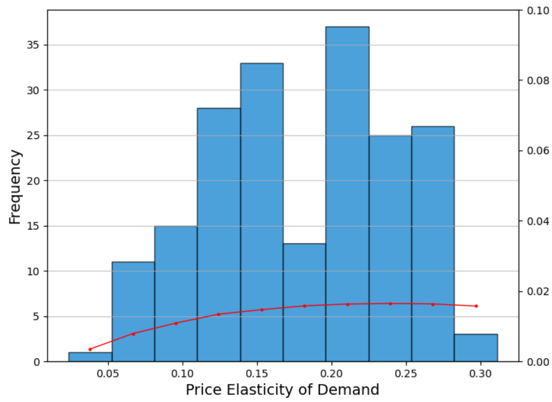

Figure 3. Distribution of parking price elasticity values and standard deviation

Table 3. Comparison experimental results

Model	RMSE ($\times10^{-2}$)	MAE ($\times10^{-2}$)	R^2 (%)
Ours	*11.31±0.02*	*8.13±0.04*	*51.42%±0.23%*
HA	15.23±0.00	11.19±0.00	14.56%±0.00%
ARIMA	17.51±0.00	14.06±0.00	−24.62%±0.00%
SVR	11.46±0.00	8.48±0.00	44.92%±0.00%
Lasso	11.56±0.00	8.83±0.00	43.84%±0.00%
FCNN	12.67±0.00	9.15±0.00	39.13%±0.00%
LSTM	14.16±0.05	10.51±0.07	23.49%±0.92%
GRU	13.43±0.01	9.96±0.04	30.33%±0.50%
Du-parking	12.66±0.03	9.18±0.03	39.15%±0.30%

Note: Table shows the mean and standard deviation of the prediction results.

histogram. Based on this, the mean value of the standard deviation was calculated for each interval and a line chart was constructed (Figure 3).

The horizontal coordinate is the price elasticity value. The vertical coordinate on the left is the frequency and the vertical coordinate on the right is the value of the standard deviation. It can be seen that the price elasticity of parking demand is primarily distributed between 0.10 and 0.30. The highest price elasticity values are distributed in the range of 0.20–0.23, with 37 out of the 192 parking facilities in this range, followed by the range of 0.14–0.17, with 33 parking facilities in this range, and the lowest values are distributed in the range of 0.02–0.05, with only one parking facility. The value of the standard deviation is very small, which indicates that the parameter values obtained by the results of five training sessions were very similar, i.e., the results of the parameter obtained in this way were less affected by stochasticity, enhancing the credibility of the results. Through calculation, the results show that the average price elasticity value for the 192 parking facilities is 0.18. Combined with previous studies, the derived price elasticity corresponds well with research findings regarding the price elasticity of parking demand (Table 4).

Table 4. Studies on the price elasticity of parking demand

Study	Price elasticity of parking demand
Vaca and Kuzmyak (2005)	Typically between 0.1 and 0.6, with a common value of 0.3
Kelly and Clinch (2009)	0.29 for on-street parking in Dublin, Ireland
Ostermeijer et al. (2022)	A value of 0.19 for parking in Amsterdam
Concas and Nayak (2012)	An average value of 0.39 in the United States, along with 0.86 in non-U.S. countries

The derived price elasticity values have been visualized on San Francisco road network maps (Figure 4). To facilitate visualization and achieve better results, each segment of on-street parking facility is represented as a point on the map. The 192 parking facilities are divided into five levels according to the magnitude of price elasticity.

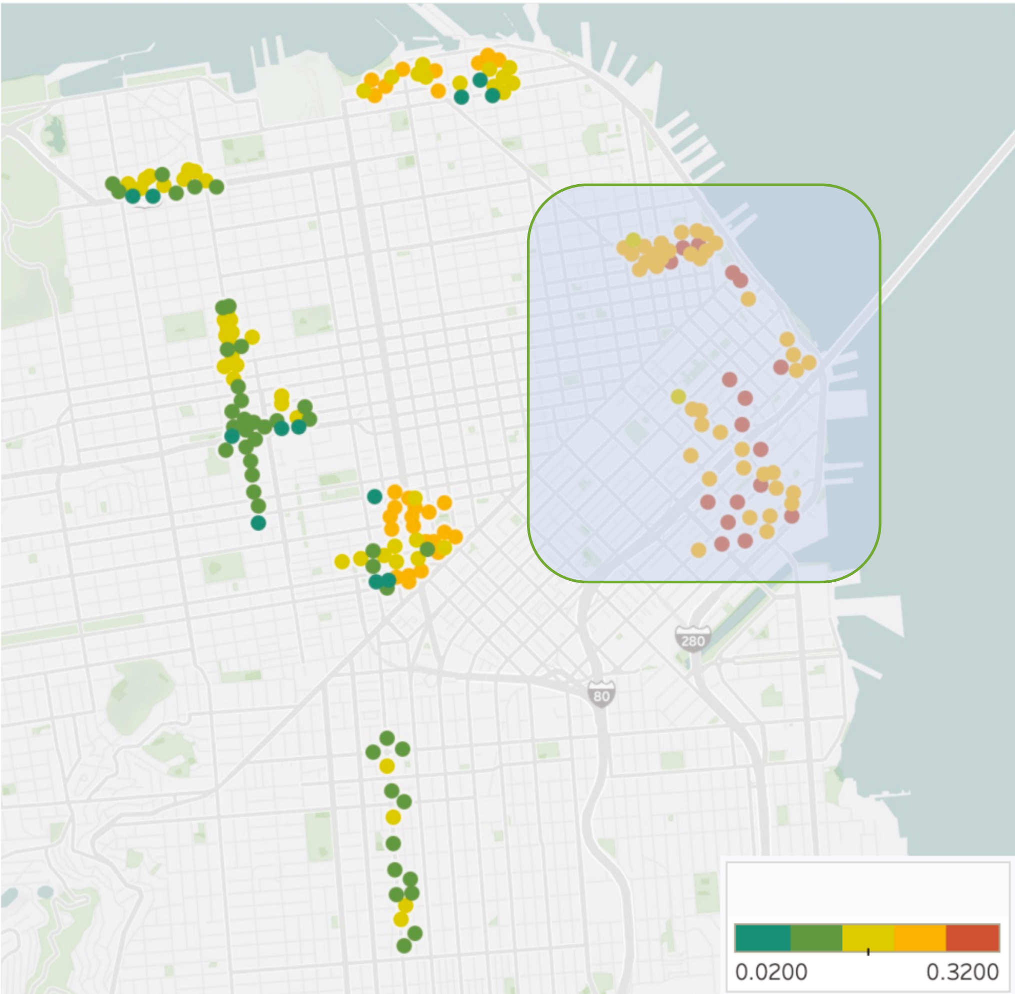

Figure 4. Price elasticity for 192 parking facilities in SF Park

The specific range of price elasticity values and the corresponding color size can be referred to in Figure 4.

Figure 4 shows that high price elasticity values are indicated on the right side, moderate values in the middle section, and low values in the remaining areas. Certain patterns can be observed. Firstly, the clustering effect is apparent, where neighboring areas exhibit similar price elasticity values. This suggests that parking demand elasticity is influenced by localized factors such as local business density and types of land use. This also indicates that parking demand elasticity varies across different areas. This variation potentially reflects distinct parking behavior patterns in various areas. Understanding these patterns is important for developing pricing strategies. For instance, in high elasticity areas, small price adjustments could significantly impact parking behavior, thus optimizing occupancy and potentially reducing congestion. In contrast, in low elasticity areas, price changes might be less effective in influencing demand, necessitating alternative management strategies. The derived elasticity can be used to determine optimal parking prices and achieve optimal occupancy, which can help us understand how on-street parking demand in a specific block responds to pricing. Overall, the acquisition of these price elasticity values provided strong support to develop smarter and more accurate parking pricing strategies in the following experiments.

4.2. Experiments on parking management

4.2.1. Experimental setup

Parking pricing experiments are conducted on the results of the parking-occupancy prediction experiments. According to the elasticity-occupancy parking management strategy, areas with high price elasticity were selected for the parking pricing experiments. The area is demarcated by green line boxes in Figure 4. There are 59 parking facilities in this area and the price elasticity of parking demand ranges from 0.19 to 0.32, with an average value of 0.25.

The experimental parameters were set as follows. Based on existing research (Millard-Ball et al., 2014; Shoup, 2006), we set the upper occupancy limit y_{+}^{*} and the lower limit y_{-}^{*} in Equation (10), (10a), and (10b) to 0.85 and 0.60, respectively. According to the SF Park project, the values for the price constraints p_{min} and p_{max} in Equation (12) were determined as 0.25 and 6, respectively. In Equation (13), g was set to 0.25, indicating that price adjustments are made in integer multiples of 0.25. This aligns with the pricing adjustment pattern in the SF Park project. The value of δ in Equation (14) was set to 2, which means that, for each parking facility, the price expansion in every two adjacent hours does not exceed \$2. This setting avoids significant fluctuations in prices.

4.2.2. Price-optimization results

Based on the experimental results given in Section 4.1, the parking price optimization experiment was carried out in the selected areas. The experimental results for price optimization are shown in Figure 5. The experiment simulated one day's price adjustment in the study area. The price adjustment experiment was carried out every hour and adjustments were made facility by facility. The figure illustrates the hourly prices and changes in prices from 9:00 to 17:00. The size of the circles represents the magnitude of the prices and different colors indicate whether the prices increased or decreased from the original values.

Figure 5 shows variations in price adjustments at different times of a day. There is still room for price improvement in the SF Park project. Some parking facilities need to increase their prices, while other parking facilities need to lower their prices. Price adjustment strategies should vary at different times throughout the day. For instance, during the morning peak hours (e.g., 9:00 to 11:00), prices at many facilities increase significantly, likely due to the higher parking demand from commuters. In the afternoon (e.g., 15:00 to 17:00), price adjustments are more dispersed. Some facilities experiencing price decreases, possibly due to reduced parking demand or lower utilization rates at certain facilities. The figure also highlights the price adjustment patterns across different areas. Some parking facilities show larger price adjustments, while others have relatively smaller changes. This reflects the differences in parking demand and supply across regions, as well as the varying sensitivity of each area to price changes. The overall pricing levels are consistent across areas. However, there is variance in parking prices between different areas, which reflects the differences in the attractiveness of surrounding facilities in different areas. Moreover, within the same area there are still differences in pricing. Thus, the proposed parking pricing strategies can make uniform the uneven spatial and temporal distribution of parking demand by adjusting prices. By comparing the price adjustments across different time periods and areas, we can assess the effectiveness of the optimization strategy. Price increases during peak hours effectively alleviate excessive parking demand, while price decreases during lower demand periods attract more drivers, enhancing facility utilization. This dynamic adjustment helps achieve round-the-clock optimization of parking resources, improving parking efficiency.

During the price optimization based on the derived price elasticity, it was shown that for some parking facilities, even free parking prices could not achieve the expected parking occupancy. This finding suggests that while price is important in parking choice, it is not the only factor influencing parking demand. In fact, it may not even be the primary factor influencing demand. The core determinant of parking demand lies in the attractiveness of an area and this attractiveness is largely dependent on the various activity opportunities offered in that area (Ottosson et al., 2013).

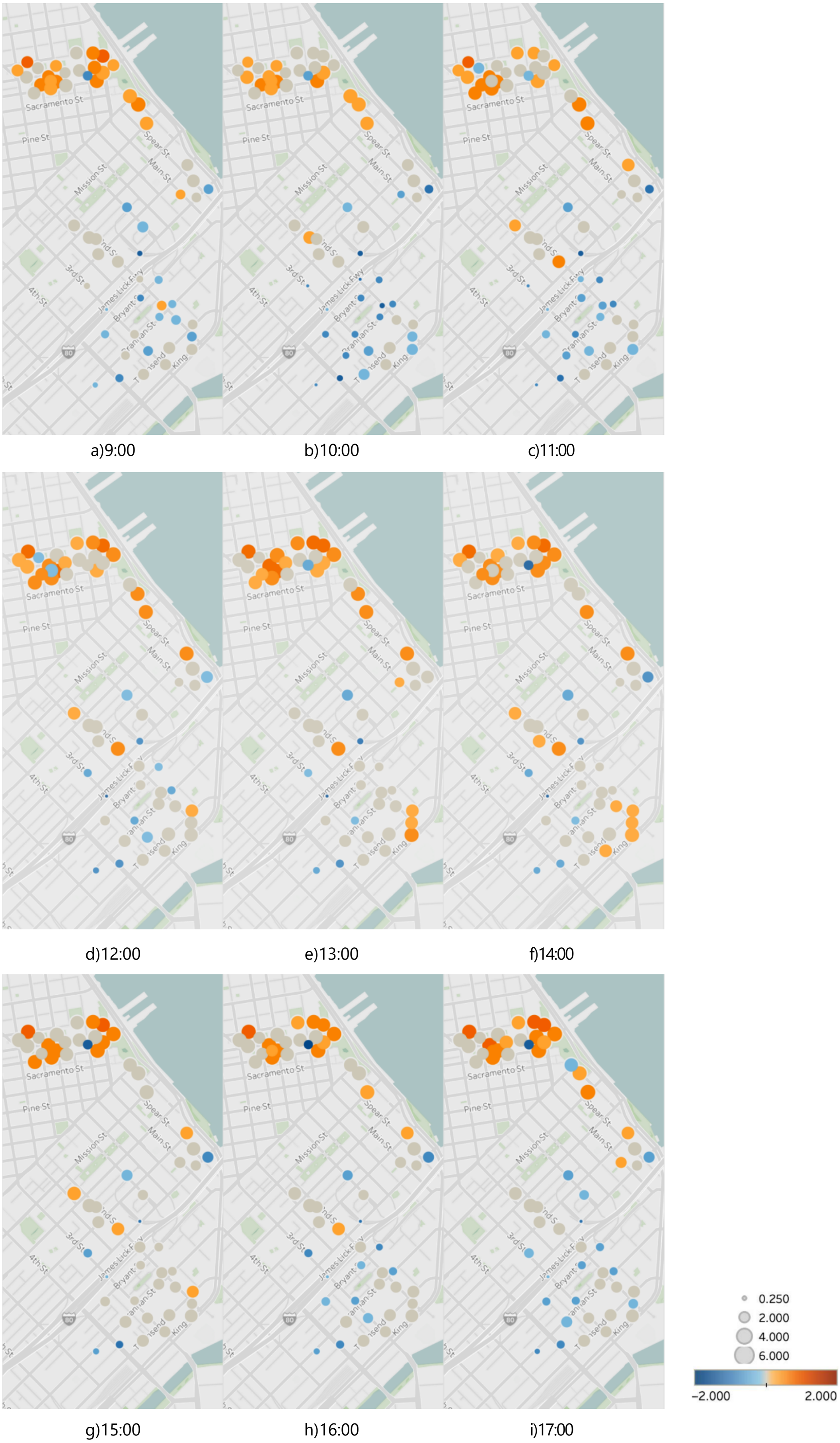

Figure 5. Price-optimization results

5. Conclusions

The proposed proactive pricing strategies involve price adjustments facility by facility. By considering different price elasticity and parking occupancy characteristics, an elasticity-occupancy parking management strategy is proposed, leading to more efficient parking management. Proactive parking pricing provides drivers with advanced information on parking prices, enabling them to plan their trips rationally. Compared with reactive pricing, proactive pricing aligns better with drivers' psychological expectations. Moreover, it has the potential to control parking occupancy within an ideal range and to make uniform uneven spatial and temporal distribution of parking demand. This will contribute to alleviating congestion, improving air quality, and enhancing the economic vitality of an area.

This study has investigated proactive pricing strategies for on-street parking by predicting parking occupancy and deriving the price elasticity of parking demand. This included a parking-occupancy prediction model and a price-optimization method. The parking-occupancy prediction model employed a PINNs approach which incorporated economic knowledge of parking prices and occupancy into the neural network training process. This approach enhanced prediction accuracy and derived the price elasticity of parking demand. Based on the derived price elasticity values and the differences in parking occupancy characteristics, parking facilities were divided into four types: high price elasticity-high occupancy, high price elasticity-low occupancy, low price elasticity-high occupancy, and low price elasticity-low occupancy. This constituted a parking management strategy known as the elasticity-occupancy parking management strategy. In the strategy, since prices show significant effects only on high-price-elasticity parking facilities, the price-optimization method focuses exclusively on adjusting prices for high-price-elasticity parking areas. For the price-optimization method, the objective was set as minimizing the difference between parking occupancy and ideal occupancy (0.60–0.85). The constraints included price policy constraints, price change constraints, and other constraints. By adjusting prices, the model attained its objectives. This method adjusts parking demand by reducing prices during low parking occupancy and increasing prices during high parking occupancy.

The proactive pricing strategies proposed in this study have been applied to the real-world dataset of the SF Park project in San Francisco. Experimental results showed that the parking-occupancy prediction model has good performance, showing an average improvement in prediction accuracy of 17.71% compared to the baseline models. The price elasticity of most parking facilities falls within the range of 0.20 to 0.23, with an average value of 0.18. By visualizing the price elasticity values of each parking facility on road network maps of San Francisco, it has been shown that neighboring facilities exhibit close on-street parking price elasticity values, reflecting real-world experience where neighboring facilities generally share similar levels of attractiveness. Finally, based on the elasticity-occupancy parking management strategy, we selected areas with higher price elasticity for parking pricing research. Through the optimization method and utilization of data from the prediction model, the price of each parking facility for each hour from 9 AM to 5 AM in one day was calculated. The conclusion was drawn that there is still room for optimization in on-street parking prices in the SF Park project.

On-street parking mainly serves as short-term parking, which has a certain impact on road traffic conditions and surrounding commercial activities. So reasonable on-street parking management is very necessary. Proactive pricing strategies for on-street parking can alleviate spatiotemporal disparities in parking occupancy, making them effective tools for parking management. However, parking pricing is particularly effective in areas with high price elasticity of parking demand. For areas with low elasticity, the impact of pricing measures is minimal. Additionally, the relationship between parking price elasticity and the surrounding land-use types is significant. To address these two issues, there is a need for research on how to integrate parking pricing with other methods to enhance parking-management efficiency. Nevertheless, only the spatial differences in price elasticity for different parking facilities have been considered in this study. In the future, temporal variations in price elasticity should also be taken into account.

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Author contributions

Jun LI: Conceptualization, methodology, writing-original draft, supervision. Yijun DONG: Software, formal analysis, writing-original draft, writing-review & editing, visualization. Qiuxuan WANG: Writing-review & editing, visualization. Chunlu LIU: Writing-review & editing, supervision.

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