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EXTERNAL BALLISTICS TASK MODELLING FEATURES

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Abstract. Based on the solutions of an external ballistics task, tables are made for firearms and tables or computer control programs for heavy weapons. If you shoot in a mountainous area where the height difference between gunner and target can be about a hundred meters, defined skills for firing are necessary. We can make specific tables of firing with firearms in mountainous areas. Mortars are relatively simple and inexpensive weapons, and therefore their management has never been computerised. We have shown that this deficiency can be corrected. The programs described above calculate the required parameters very quickly.

Keywords: modelling, military operation, management of mortar, geographical information system.

1. Introduction

External ballistics describes the movement of the fired bullet or explosive in the air, including any factors affecting it. The main task is to find out what the angle of the weapon tube should be to ensure that the bullet drops at the intended place. It is necessary to take into account changing conditions and to assess the impact of random factors on movement (Stankūnas, Suzdalev 2011; Dołega, Rogalski 2009). Based on the solutions of the external ballistics task, tables are made for firearms and tables or computer control programs for mortars.

2. Movement path of sub-machine gun bullet and its elements

A bullet fired with the initial speed V_0 continues to move under its own momentum. It is involved in two independent movements: it goes upwards (component V_{y0} of the initial velocity V_0) and then moves in a horizontal direction at the speed V_{x0} (horizontal component V_0, Fig. 1). Two forces have a material impact on the moving bullet (Fig. 1): the force of gravity, *mg*, and the environmental resistance force, F_p. Its component, F_{py}, stops the bullet from going upwards, and the component F_{px} stops the bullet from moving in the direction of the *x* axis, i.e. towards the target.

The movement of the bullet is described by two second order differential equations systems:

$$\begin{cases} m\dfrac{d^2x}{dt^2} = -F_{px}, \\ m\dfrac{d^2y}{dt^2} = -mg - F_{py}, \end{cases} \tag{1}$$

with the initial conditions

$$x(0)=0,\ x_t'(0)=V_0\cos\alpha,\ y(0)=0,\ y_t'(0)=V_0\sin\alpha,$$

where m is bullet mass and t is time. Other notations come from Fig. 1.

Movements according to the x and y axes happen independently, and we describe each of them using Newton's second law. The environmental resistance force F_p depends on the bullet caliber, form, and other conditions. It is described by the empirical formula (Jančiauskas, Venskus 1999):

$$F_p = \frac{1000kd^2}{g} H(y) F_v, \tag{2}$$

where k – form factor (it is close to 0.5), d – calibre (in meters), g = 9.81 (free fall acceleration), and $H(y) = \dfrac{20000-y}{20000+y}$ – modification due to the change in environmental parameters when moving up and down. The environmental resistance force F_v expression depends on the speed of the bullet. If bullet velocity exceeds the speed of sound in the air, $(V > 330\ m/s)$, the term is recorded in the form of this empirical formula:

$$F_v = \frac{V}{3} - 80, \tag{3a}$$

and if bullet velocity is slower than the speed of sound in the air, then

$$F_v = 1.21 \cdot 10^{-4} V^2. \tag{3b}$$

When shooting on a plain, where the height difference between the shooter and the target does not exceed a few dozen meters, the velocity component V_{0y} is limited. The bullet goes up no more than a few meters above the aiming line and in such a case the component of the environmental resistance force of F_{py} may be excluded. Even when a gun is fired in a mountainous area, the aiming angle is relatively small; the velocity component $V_{0y} < 330$ m/s and the system of equations describing the movement of the projectiles is as follows:

$$\begin{cases} m\dfrac{d^2x}{dt^2} = -\dfrac{1000kd^2}{g}\left(\dfrac{V_{0x}}{3} - 80\right), \\ m\dfrac{d^2y}{dt^2} = -mg - 1.21\cdot 10^{-4} V_{0y}^2. \end{cases} \tag{1a}$$

The MAPLE program provides the solutions of the equation system and reflects them in a graphic form. When shooting on a plain (the $1.21\cdot 10^{-4} V_{0y}^2$ of the term may be excluded), paths of up to 500 m in height shall not exceed 80 cm (Fig. 2). When the opponent is closer than 400 m, the sight may remain unchanged.

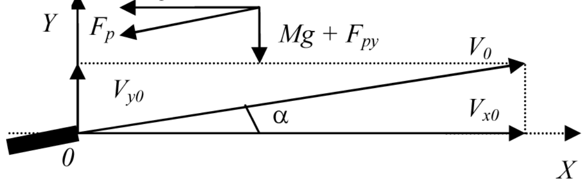

Fig. 1. Forces influencing a bullet moving through the air, components of its velocity. Aiming line (straight line running from the shooter's eye through the sight towards the aiming point) shown in dots and coinciding with the *x* axis, α – aiming angle, i.e., angle between the elevation (direction of the barrel) and the aiming lines

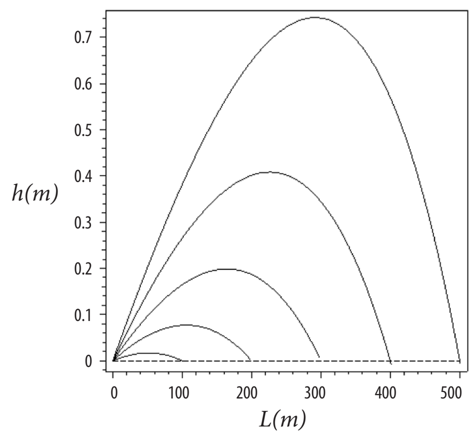

Fig. 2. AK74 bullet paths. The aiming line is reflected in hyphens

If you shoot in a mountainous area where the height difference Δh comes up even to 100 m, the calculation method for the firing parameters can vary. When one is shooting at objects located in a lower position, the aiming angles are equal to several degrees (Fig. 3).

If one is firing at an object that is significantly higher, the term $1.21 \cdot 10^{-4} V_{0y}^2$ has to be taken into account. The aiming angles are large (Fig. 4), and when fighting under such conditions specific skills are necessary.

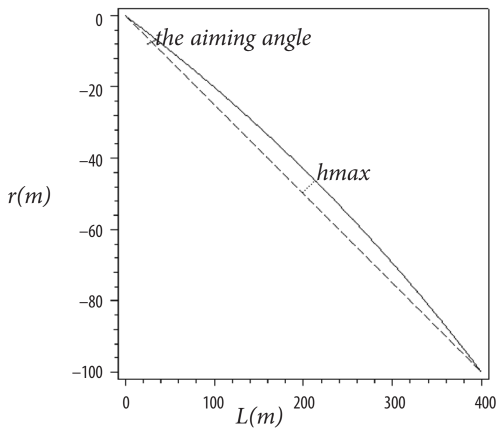

Fig. 3. Path of a bullet fired in a mountainous area (downward direction). The aiming line is reflected in hyphens. The maximum height of the path (dotted line) h_{max}= 6.8 m, the aiming angle is 3° (sight 50 thousand)

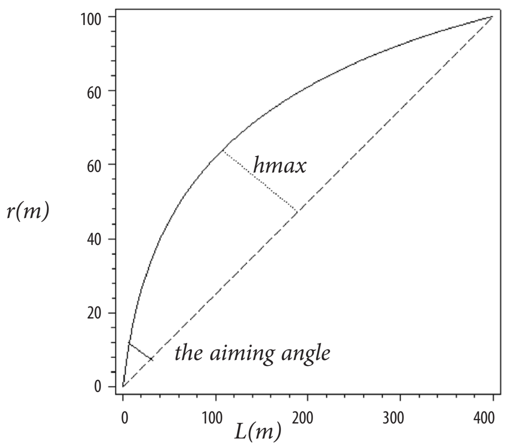

Fig. 4. Path of a bullet fired in a mountainous area (upwards direction). The aiming line is reflected in hyphens. The maximum height of the path (dotted line) h_{max}= 34.5 m, the aiming angle is 50°

3. Solution of external ballistics task of describing the movement of the mine

A mortar is a close combat weapon that fires a mine in a high-arcing ballistic trajectory (α = 45° – 85°) and is used as mobile, powerful infantry artillery.

The advantages of these weapons are the possibility to destroy enemy troops and positions of fire hidden in hills and villages. They are characterised by simple structure, easy maintenance, low weight, and mobility (they can be transported by jeeps or carried by troops). Because of these properties, mortars can be used effectively in mountainous areas, and they have the potential of shock effect.

Because these weapons are of interest to our troops, we will discuss the improvement of their management capabilities. In order to ensure that a mine finds its way to its target, we have to deal with trajectory control problems. It is necessary to identify what the aiming angle must be with the selected initial speed of the ammunition (for the chosen charge), specific weather conditions, and so on. The trajectory of the mine will coincide in principal with the one shown in Fig. 1, expect that air resistance will be described in a much more complicated way compared with the movement of the gun bullet. Moving in a straight course, the mine goes up to several kilometres in height, which significantly changes the air density, temperature, and pressure. The resistance force F_p is mostly affected by friction with the environment (viscosity), because the wings on the back stabilize the movement of the mine along a set trajectory. We assume that the resistance force F_p is proportional to the speed of the mine (i.e. the first derivative according to time x_t', y_t'). Then, the movement of the mine is described by the system of equations (Pincevičius *et al.* 2001; Bekešienė *et al.* 2009):

$$\begin{cases} m\dfrac{d^2x}{dt^2} = -k\dfrac{dx}{dt}, \\ m\dfrac{d^2y}{dt^2} = -k\dfrac{dy}{dt} - mg, \end{cases} \tag{4}$$

with the initial conditions (when time $t = 0$):

$$\begin{aligned} &x(0) = 0, \ \ x_t'(0) = V_0 \cos\alpha, \\ &y(0) = 0, \ \ y_t'(0) = V_0 \sin\alpha, \end{aligned} \tag{5}$$

where m – mine mass, g – free fall acceleration, k – resistance coefficient, V_0 – initial speed (mine speed after leaving the tube), and α – aiming angle.

In the system of equations (4), there is a member that is difficult to select or determine experimentally, i.e. the resistance coefficient k. This value depends on the form of the mine and weather conditions: atmospheric pressure, air temperature, and humidity trajectory at a point. Many of these parameters change during movement, because as we said, the mine goes up to great heights. We suggest selecting the coefficient k by *combining the results of calculations and actual experiments*, which are summarised in the appropriate firing tables (Pincevičius *et al.* 2001; Bekešienė *et al.* 2009; Minosvaidžių [Mortar]…1997). For the chosen distance of the shooting target, we find the cartridge number on the tables (we choose the initial speed of a mine), and solve equations system (4) with initial conditions (5). By changing the coefficient k, we reach the point when the aiming attitude in the firing tables and in the domains of equations coincides.

From the available results, we can obtain the analytical dependence of resistance coefficient k from the target range x (the calculated k values approximate by polynomial, for example for mortar 120 mm *ČM*-120 (6)):

$$k = -0.3463203463\ 10^{-10}\, x^2 + 0.00001582x + 0.07091125541. \tag{6}$$

Using the MAPLE possibilities, we can easily obtain the analytical solutions of the system of equations (4) with the initial conditions (5), describing the movement of the mine according to the x axis direction (xs) and y axis direction (ys) (7):

$$\begin{cases} \dfrac{V_0 \cos(\alpha) m}{k}\left(1 - e^{-\frac{kt}{m}}\right) = L, \\ \dfrac{(mg + kV_0\cos(\alpha))m}{k^2}\left(1 + e^{-\frac{kt}{m}}\right) - \dfrac{mgt}{k} = 0. \end{cases} \tag{7}$$

After the inclusion of the numerical values of parameters m, g, V_0, and k into these solutions, they become functions of the launching angle α and time t. If you require the mine to fall in a fixed location, i.e. condition is met where $ys = 0$, when $xs = L$ (L – the target range), we will get the algebraic equations system with variables t and α. Having decided on the latter, we can find all the parameters of the task that are of interest to us and plot the chart (Fig. 5):

Firing trajectories and parameters are calculated: target range 4800 m, aiming attitude 803 thousandths, initial velocity 256 m/s, ending velocity 215 m/s, trajectory height 2069 m, flight time 41 *s*, falling angle 64°.

Mine dispersion is inevitable: slight changes in powder charge weight, mine weight, charge temperature, wind, etc. Changing the settings for the shooting parameters and counterbalancing the average effect of random factors can reduce errors. Meteorological conditions are set using the meteorological bulletin. Discrepancy in the initial velocity or mine mass, as well as range (due to changing cartridge temperature) corrections, is included in the program by replacing the original data. Corrections due to atmospheric pressure, air temperature, or relative humidity non-compliance with normal conditions (750 mm Hg, 15.9° C, 50%) are entered by changing the resistance coefficient k value.

The system of equations (4) with initial conditions can be solved when random values of initial velocity and other parameters are provided:

$$v0 + random[normal]\_\Delta v0\ ,\ k + random[normal]\_\ \Delta k, \tag{8}$$

where *random[normal]* is a random value distributed according to standard normal distribution $N(0, 1)$; $\Delta v0$ and Δk are maximum errors of initial velocity $v0$; and the resistance quotient is k.

For example, the possible difference in the initial velocity of a 120 mm mortar *ČM*-120 mine in tables Minosvaidžių [Mortar]…1997) is indicated as 0.5%. While the mortar is firing, the barrel is not completely still and this error increases. Accidental fluctuations in atmospheric pressure and temperature determine approximately a 1% difference in the resistance quotient k. If we compute 10,000 shots at a target within the chosen distance with random parameter values, we will be able to rather accurately estimate mean standard deviations σ_x and σ_y. A group target (the adversary in the assembly area of 150×200 m) is usually fired upon with a three-mortar group. Because of firing errors, explosives though aimed at the same point [xi, yi] fall at ever-different spots. If one knows σ_x and σ_y, deviations can be estimated:

$x_i = x_i + \sigma_x \times random[normal]$,

$y_i = y_i + \sigma_y \times random[normal]$.

A random dispersion of mines with the three-mortar battery having fired 10 salvos (ten shots to each point of aiming) is represented in Fig. 6.

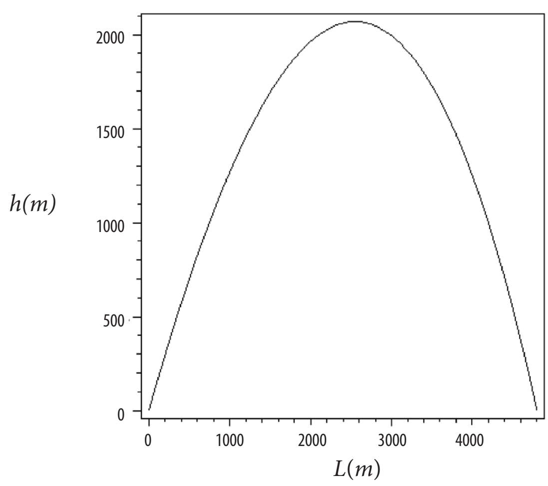

Fig. 5. Flight trajectory of mortar mine (L = 4800 m)

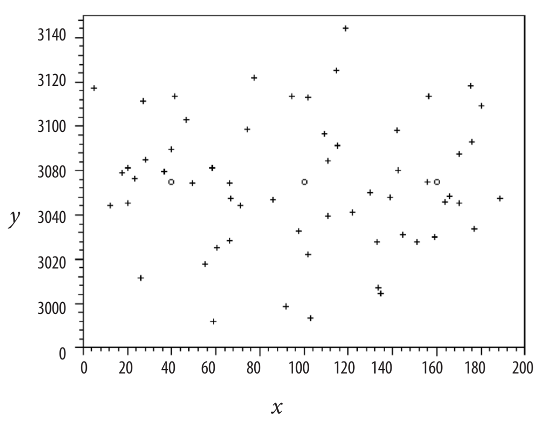

Fig. 6. Scheme of targets when firing at a group target and mine dispersion after 10 salvos. L = 3050 m, σ_x = 26 m, σ_y = 18 m. 'o' is points of aiming, '+' is points where mines fell

Given that each mine destroys enemy soldiers within a radius of 15 m, it is possible to estimate what portion of the enemy soldiers will be destroyed. Since mine dispersion is random, this process, that is the computation of losses caused by ten salvos, must be performed 1000 and more times (1000 realisations). Having counted the average of the results of the realisations, we will get a true-to-life result. The number of salvos is increased until the desired level of group target destruction is reached. It is possible to change the arrangement of group target aiming points and optimize artillery fighting against a group target.

4. Representation of results. Information interface (glyph)

We shall discuss the concept of information interface (glyph) (Pincevičius *et al.* 2006; Stankūnas, Suzdalev 2011; Zakarevičius *et al.* 2010). By employing the possibilities of Geographical Information Systems (GIS), we created a 'tool' able to supply additional information about the capabilities of an object in the future. The possibilities of the future actions of this object are analysed in relation to the real position of the object. The position of the object is visualised not just by estimating its spatial position (point on a map). The program estimates the capabilities of an object for a certain action and represents them on a map (what the object or its parameters will look like after a certain action). The 'tool' that was created makes it possible to represent group target destruction results on the map. Having activated the program describing the destruction of a group target, we receive percentage values associated with a concrete place (coordinates of the location and matrix elements that indicate destruction probability during the foreseen firing).

Fig. 7 represents the area covered by enemy fortifications. When losses increase, the colour darkens (it is possible to indicate destruction percentage on the colour scale). **X** is points of aiming as shown in Fig. 6. The distance between the points is 10 m horizontally and 16 m vertically.

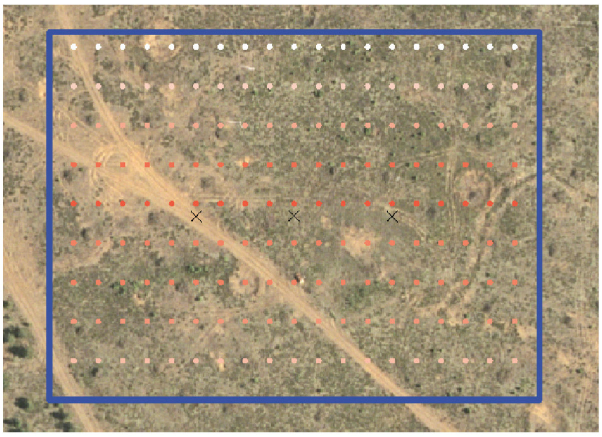

Fig. 7. The area covered by enemy fortifications

5. Conclusions

A mortar is a relatively simple and inexpensive weapon, and therefore its management is never computerised. We have shown that this deficiency can be corrected. The programs described above in a few dozen seconds calculate the required parameters. We can also make specific firing tables with firearms in mountainous areas. Such programs are important for preparing cadets because they can be introduced to specific military issues.

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IŠORINĖS BALISTIKOS UŽDAVINIO MODELIAVIMO YPATUMAI

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Santrauka. Remiantis išorinės balistikos uždavinio sprendimais, šaulių ginklams sudaromos lentelės, o sunkiesiems ginklams – lentelės arba kompiuterinės valdymo programos. Jeigu šaudoma kalnuotoje vietovėje, kur aukščių skirtumas tarp šaulio ir taikinio siekia net šimtą metrų, reikia konkrečių įgūdžių. Galima sudaryti specifines šaudymo šaulių ginklais kalnuose lenteles. Minosvaidis yra santykinai paprastas ginklas, todėl jo valdymas nebūna kompiuterizuotas. Parodyta, kad šį trūkumą galima pataisyti. Aprašytos programos per kelias dešimtis sekundžių suskaičiuoja reikiamus parametrus.

Reikšminiai žodžiai: modeliavimas, karinės operacijos, minosvaidžio valdymas, geografinės informacinės sistemos.