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ERROR ANALYSIS OF THE INERTIAL MEASUREMENT UNIT

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Abstract. The paper describes the results of error analysis of the new inertial measurement unit ADIS16364 produced by Analog Devices. This error analysis concerns stochastic sensor errors identified by the Allan variance method. In order to improve the performance of the inertial sensors, the users are keen to know more details about the noise components in each axis for a better modelling of the stochastic parts to improve the navigation solution. The main contribution of this paper is to present data necessary for further inertial sensors signal processing by means of Kalman filtering.

Keywords: inertial measurement unit, error analysis, Allan variance, inertial sensor errors.

1. Introduction

The Inertial Measurement Unit (IMU) contains a three-axis accelerometer and a three-axis gyroscope. The IMU typically provides an output of the vehicle's acceleration (measured by accelerometers) and angular rate (measured by gyroscopes), which are then integrated to obtain the vehicle's position, velocity, and attitude (Cizmar *et al.* 2008). Basically, the inertial sensors (accelerometers and gyroscopes) have different error characteristics (Sotak *et al.* 2006). The overall analysis of IMU output signal errors contains analysis of a systematic part and a stochastic part. This paper presents only analysis of the stochastic part. The requirements for accurate estimation of navigation information require accurate modelling of the sensors' noise components (Chatys, *et al.* 2005; Kopecki *et al.* 2010). Several methods have been devised for stochastic modelling of inertial sensor noise (adaptive Kalman filtering, power spectral density, and autocorrelation function). Variance techniques are basically very similar and primarily differ only in various signal processing, by way of weighting functions, window functions, wavelet analysis, etc. (Sotak 2008) The Allan variance technique provides several significant advantages over the others (IEEE… 1997; Lawrence *et al.* 1997). Traditional approaches, such as computing the sampled mean and variance from a measurement set, do not reveal underlying error sources. Although the combined power spectral density (PSD) and autocorrelation function (ACF) approach provides a complete description of error sources, the results are still difficult to interpret. PSD is ideal for identifying either narrowband harmonic components or broadband sources in general; however, extracting other contributing components such as bias instability, random walk, and quantization error is complicated (Lawrence *et al.* 1997). David Allan proposed a simple variance analysis method for the study of oscillator stability that is the Allan variance method. After its introduction, this method was widely adopted by the time and frequency standards community for the characterisation of phase and frequency instability of precision oscillators (Hou 2004). It can be used to determine the character of the underlying random processes that give rise to data noise. As such, it helps identify the source of a given noise term in the data. Allan variance is a method of representing root mean square (rms) random drift error as a function of average time (IEEE… 1997). It is simple to compute, much better than having a single rms drift number to apply to a system error analysis, and relatively simple to interpret and understand. The Allan variance method can be used to determine the character of the underlying random processes that give rise to data noise (Hou 2004). This technique can be used to characterise various types of noise terms in the inertial sensor data by performing certain operations on the entire length of data. Its most useful application is in the specification and estimation of random drift coefficients in a previously formulated model equation. In the Allan variance method of data analysis, the uncertainty in the data is assumed to be generated by noise sources of specific character. The magnitude of each noise source covariance is then estimated from the data (Lawrence *et al.* 1997). Typical Allan variance sample plots are shown in figure 1. The error sources of interest have slopes between ±1, and these slopes identify the different contributing sources of the accelerometer or angular rate sensor noise (Sotak 2008). Each component is given by a typical correlation time according to appropriate scales. It is important to mention that the error sources considered as the most important ones are in practice usually only the random walk, the bias instability, and the correlated noise (Sotak 2009). Therefore, the parameters of these error sources are sufficient output of the Allan variance analysis. Allan variance analysis is a time domain technique that has been accepted as an IEEE standard (IEEE… 1997).

2. Description of Allan variance analysis

Let us take n measured data by inertial measurement unit (for simplicity consider only one inertial sensor for instance gyroscope X; let us denote it by the symbol ω^x where ω represents the part of angular rate vector of the body frame with respect to the inertial frame projected to the x axis of body frame). Data was taken at a rate of f_s samples per seconds. Denote it with $\omega^{x[1]},\omega^{x[2]},\ldots,\omega^{x[n]}$ and choose m samples from the measured data. This set of samples will be called cluster and denoted as k, where number of all clusters is $K=n/m$. It can be written by

$$\underbrace{\omega^{x[1]},\omega^{x[2]},\cdots,\omega^{x[m]}}_{k=1},\underbrace{\omega^{x[m+1]},\cdots,\omega^{x[2m]}}_{k=2},\underbrace{\omega^{x[n-m]},\cdots,\omega^{x[n]}}_{k=K} \tag{1}$$

Next calculate the average for each cluster

$$\bar{\omega}^{x[k]}\left(m\right)=\frac{1}{m}\sum_{i=1}^{m}\omega^{x[(k-1)m+i]} \tag{2}$$

where $k=1,2,3,\ldots K$
and then calculate Allan variance from the cluster averages

$$\sigma^2\left(\tau_m\right)\equiv\frac{1}{2}\left\langle\left(\bar{\omega}^{x[k+1]}\left(m\right)-\bar{\omega}^{x[k]}\left(m\right)\right)^2\right\rangle \tag{3}$$

$$\sigma^2\left(\tau_m\right)\cong\frac{1}{2\left(K-1\right)}\sum_{k=1}^{K-1}\left(\bar{\omega}^{x[k+1]}\left(m\right)-\bar{\omega}^{x[k]}\left(m\right)\right)^2 \tag{4}$$

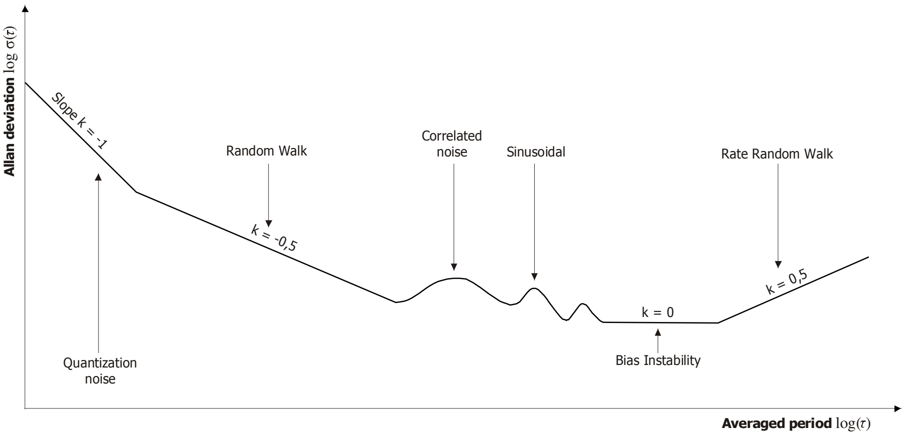

Fig. 1. Sample plot of Allan variance analysis results (Sotak 2009; Sotak 2008)

where $k=1,2,3,\ldots K$, where $\langle\ \rangle$ denotes ensemble average and $\tau_m = {}^{m}\!/_{f_s}$ is averaged period (or specified correlation time) for which the value of the Allan variance has been calculated (Lawrence *et al.* 1997).

The accuracy in the estimate of the Allan deviation (it means square root of the Allan variance) increases with an additional number of cluster averages K. The accuracy of the calculation of Allan deviation (1 σ) for K cluster averages is given by (Lawrence *et al.* 1997; Allan *et al.* 1997).

$$error\ \% = \frac{100}{\sqrt{2\left(K-1\right)}} \tag{5}$$

In order to show the relation of Allan variance and noise source characterization, it is necessary to express the

Allan variance in the frequency domain (IEEE… 1997). The proof can be summarised as:

$$\sigma_\Omega^2(\tau) = 4\int_0^\infty S_\Omega(f)\frac{\sin^4(\pi f \tau)}{(\pi f \tau)^2}df \tag{6}$$

where $S_\Omega(f)$ is PSD of the measured angular rate or acceleration noise data and f is the frequency.

3. Stochastic sensor error analysis

The proposed Allan variance method was applied to real data collected from the new IMU ADIS16364. The ADIS16364 iSensor is a complete inertial system that includes a tri-axis gyroscope and tri-axis accelerometer (ADIS16364…). Each sensor in the ADIS16364 combines industry-leading iMEMS technology with signal conditioning that optimises dynamic performance. The factory calibration characterises each sensor for sensitivity, bias, alignment, and linear acceleration (gyro bias). As a result, each sensor has its own dynamic compensation formulas that provide accurate sensor measurements over a temperature range of −20°C to +70°C. The ADIS16364 provides a simple, cost-effective method for integrating accurate, multi-axis, inertial sensing into industrial systems, especially when compared with the complexity and investment associated with discrete designs.

All necessary motion testing and calibration are part of the production process at the factory, greatly reducing system integration time. Tight orthogonal alignment simplifies inertial frame alignment in navigation systems. An improved SPI interface and register structure provide faster data collection and configuration control. This compact module is approximately 23 mm × 23 mm × 23 mm and provides a flexible connector interface, which enables multiple mounting orientation options, see figure 2. A functional block diagram of ADIS16364 is illustrated in figure 3.

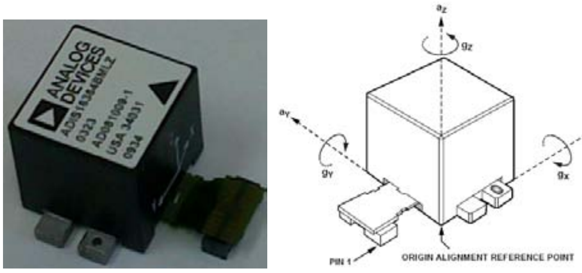

Fig. 2. ADIS16364BMLZ

BASIC SPECIFICATIONS OF ADIS16364

GYROSCOPES

Dynamic Range	±350°/sec
or	±150°/sec
or	±75°/sec
Initial Sensitivity:	
for range ±300°/sec	0.05 °/sec/LSB
for range ±150°/sec	0.025 °/sec/LSB
for range ±75°/sec	0.0125 °/sec/LSB
Initial Bias Error (±1σ)	±3 °/sec
In-Run Bias Stability (1 σ)	0.007 °/sec
Angular Random Walk (1 σ)	2.0 °/√ hr
Output Noise ±300°/sec Range, no Filtering	0.9 °/sec rms
Rate Noise Density (±300°/sec, no filtering)	0.05 °/sec/√ Hz rms

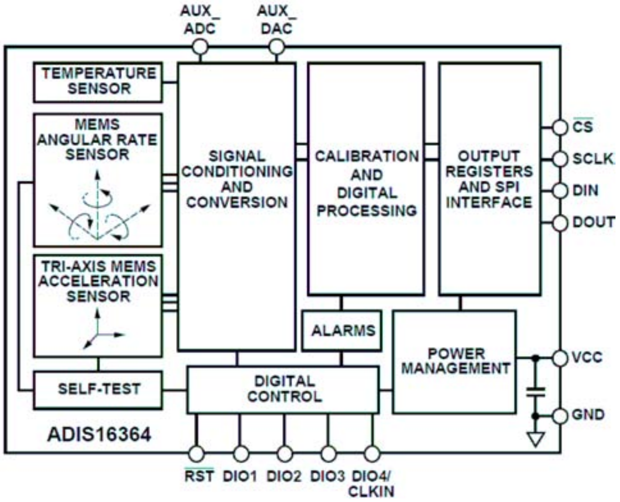

Fig. 3. Functional Block Diagram of ADIS16364 (ADIS16364…)

ACCELEROMETERS

Dynamic Range	±5.25g
Initial Sensitivity	1.00 m*g*/LSB
Initial Bias Error (±1 σ)	8 m*g*
In-Run Bias Stability (1 σ)	0.1 m*g*
Velocity Random Walk (1 σ)	0.12 m/sec/√ hr
Output Noise (no filtering)	5 m*g* rms
Noise Density (no filtering)	0.27 m*g*/√ Hz rms

TEMPERATURE SENSOR

Scale Factor Output = 0x0000 @ +25°C (±5°C)	0.14 °C/LSB

ADC INPUT

Resolution	12 Bits
Input Range	0 – +3.3 V
Input Capacitance	20pF

DAC OUTPUT

Resolution	12 Bits
Output Range	0 – 3.3V
Output Impedance	2 Ω

POWER SUPPLY

Operating Voltage Range VCC	5.0 V
Power Supply Current (max)	49 mA

To assess the performance of the ADIS16364, a static test was conducted. The test was performed in the laboratory where the temperature was 21°C. The measured data were the outputs of accelerometers (axes X, Y, Z) and the outputs of gyroscopes (axes X, Y, Z). The data-sampling rate was 100 Hz and twelve hours of static data were collected. Then, the entire data set was analysed. A log-log plot of the ADIS16364's three axis gyros' and three axis accelerometers' Allan standard deviation versus averaged time are shown in figures 4 and 5.

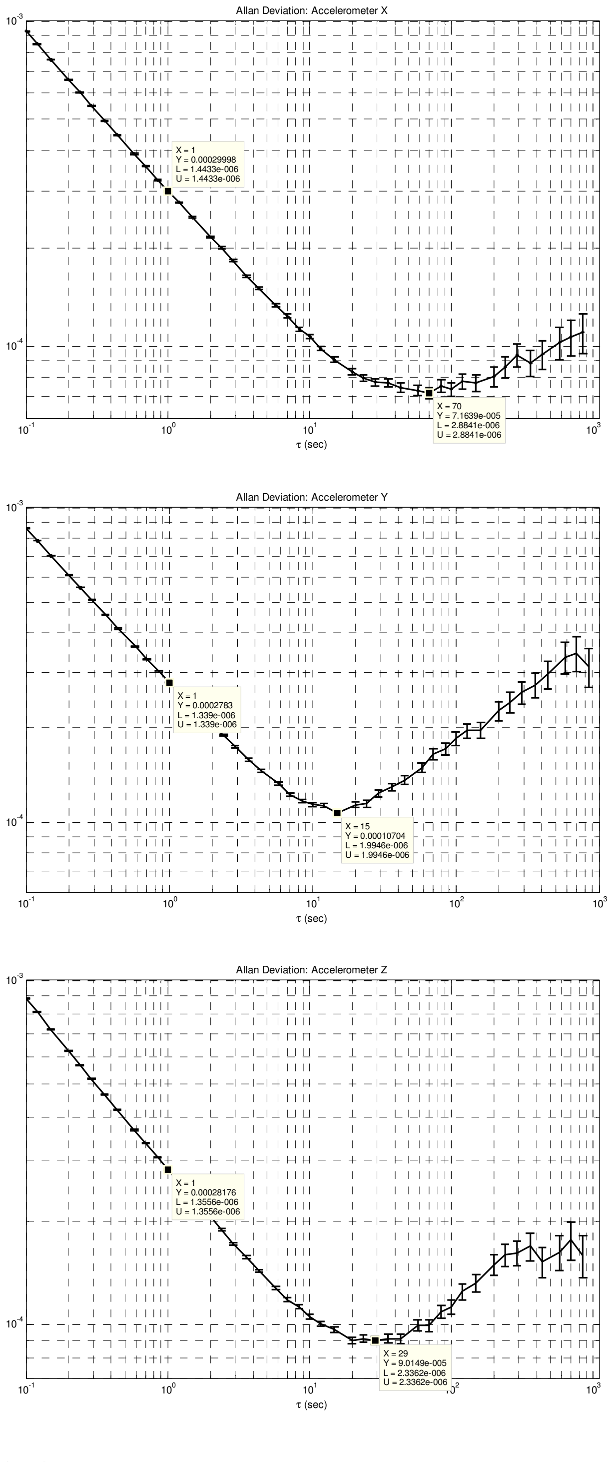

Fig. 4. Error analysis of accelerometers (X – averaged time, Y – Allan deviation, L – lower sigma boundary, U – upper sigma boundary)

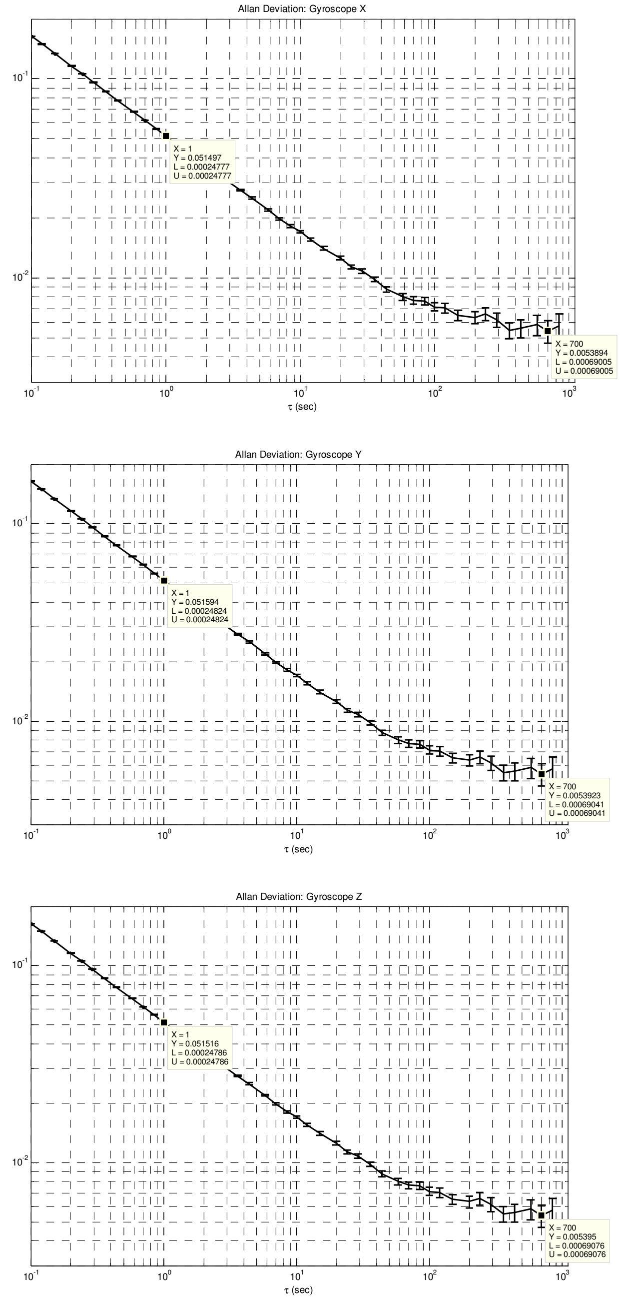

Fig. 5. Error analysis of gyroscopes (X – averaged time, Y – Allan deviation, L – lower sigma boundary, U – upper sigma boundary)

4. Results

The magnitude of each stochastic error can be determined from the data by the Allan deviation analysis.

The random walk is the dominant noise for short averaged periods. It can be shown how to obtain the random walk coefficients from the Allan deviation log-log plot result. For accelerometer X, a straight line with slope of –0.5 is fitted to the long averaged time part of the plot and meets the τ=1 second line at a value of 0.00029 (IEEE… 1997; Sotak *et al.* 2009; Reinstein *et al.* 2009).

The almost flat part of the curve of the long averaged part is indicative of the low frequency noise that determines the bias variations of the run (bias instability). The origin of this noise is the electronics or other components susceptible to random flickering (Hou 2004; Stockwell 2008). The zero slope line, which is fitted to the bottom of the curve, determines the upper limit of bias instability. Such a line meets the ordinate axis at a value of 7.16e^{-5} and dividing this by 0,664 yields the maximum bias instability value of 0.107 m*g* (IEEE… 1997; Sotak *et al.* 2009; Reinstein *et al.* 2009). We can determine the same parameters for other accelerometers and gyroscopes.

RESULTS FOR ADIS16364

Random Walk	*measured*	*datasheet*
Accelerometer X	0.29 m*g*/√ s	0.2 m*g*/√ s
Accelerometer Y	0.27 m*g*/√ s	0.2 m*g*/√ s
Accelerometer Z	0.28 m*g*/√ s	0.2 m*g*/√ s

Bias Instability		
Accelerometer X	0.107 m*g*	0.1 m*g*
Accelerometer Y	0.161 m*g*	0.1 m*g*
Accelerometer Z	0.135 m*g*	0.1 m*g*

Random Walk	*measured*	*datasheet*
Gyroscope X	0.05°/√ s	0.03 °/√ s
Gyroscope Y	0.05°/√ s	0.03 °/√ s
Gyroscope Z	0.05°/√ s	0.03 °/√ s

Bias Instability		
Gyroscope X	0.008 °/sec	0.007 °/sec
Gyroscope Y	0.008 °/sec	0.007 °/sec
Gyroscope Z	0.008 °/sec	0.007 °/sec

Bias	*measured (average of data)*
Accelerometer X	0.0273 m*g*
Accelerometer Y	-0.0211 m*g*
Accelerometer Z	-0.9986 m*g (including gravity)*

Output noise (rms)	*measured*	*datasheet*
Accelerometer X	3 m*g*	5 m*g*
Accelerometer Y	2.8 m*g*	5 m*g*
Accelerometer Z	2.9 m*g*	5 m*g*

Bias	*measured (average of data)*
Gyroscope X	0.331°/s
Gyroscope Y	-0.104°/s
Gyroscope Z	-0.099°/s

Output noise (rms)	*measured*	*datasheet*
Gyroscope X	0.52 °/sec	0.8 °/sec
Gyroscope Y	0.51 °/sec	0.8 °/sec
Gyroscope Z	0.52 °/sec	0.8 °/sec

5. Conclusions

The paper describes the crucial importance of the identification of inertial sensor error parameters. The random walk process and sensor bias instability were considered as the most important and hence determined for the tested new IMU ADIS16364. Comparing the results obtained from sensor error analysis using the Allan variance method and sensor errors obtained from the datasheet it is clear that sensor errors are very similar and are different for each sensor. For long averaged periods the Allan variance curves of the accelerometers show the presence of correlated noise. For determining the correlated noise parameters it is necessary to have more static data. Using obtained error parameters users can better model sensor performance according to the existing noise terms within the sensor output. Random walk is an important noise term and can be used to evaluate the sensor noise intensity. In the Kalman filter design, the amplitude of random walk coefficients can be directly used in the process noise covariance matrix with respect to the appropriate sensor. Therefore, the error analysis can be widely used in inertial sensor stochastic modelling.

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INERCINIO MATAVIMŲ BLOKO KLAIDŲ ANALIZĖ

F. Adamčik

Santrauka

Straipsnyje pateikiami naujojo inercinio matavimų bloko ADIS16364 klaidų analizės rezultatai. Aprašyta klaidų analizė yra susijusi su stochastinio jutiklio klaidomis, nustatytomis Allano variacijos metodu. Siekiant pagerinti inercinių jutiklių našumą, naudotojai yra linkę daugiau sužinoti apie kiekvienoje ašyje esančius komponentus, kad būtų pagerintas stochastinių dalių modeliavimas bei rasti pažangesni navigacijos sprendimai. Šiuo darbu siekiama pristatyti duomenis, kurie yra reikalingi tolimesniam inercinių jutiklių signalų apdorojimui panaudojant Kalmano filtravimą.

Reikšminiai žodžiai: inercinis matavimo blokas, klaidų analizė, Allano variacija, inercinio jutiklio klaidos.