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DAMAGE IDENTIFICATION IN A PLATE-LIKE STRUCTURE USING MODAL DATA

Sandris Ručevskis [1], Andris Chate [2]

Institute of Materials and Structures, Riga Technical University, Riga, Latvia
E-mails: [1]sandris.rucevskis@rtu.lv (corresponding author); [2]and_cate@latnet.lv

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Sandris RUČEVSKIS, *Dr, Sc. Eng.*
Date of birth: 20 March 1979.
Education: 2010 – PhD in mechanical engineering, Riga Technical University.
Affiliation and function: researcher at the Institute of Materials and Structures, Riga Technical University.
Research interests: mechanics of composite materials, dynamics of structures, damage detection and structural health monitoring, computer analysis of structures.
Publications: author of over 10 scientific publications.

Andris CHATE, *Dr, Sc. Eng.*
Date of birth: 5 July 1952.
Education: 1981 – PhD in mechanical engineering, Riga Technical University.
Affiliation and function: professor and head of the Institute of Materials and Structures, Riga Technical University.
Research interests: strength and dynamics of structures, damping analysis, mechanics of composite materials, computer analysis of structures, fracture mechanics.
Publications: author of more than 140 different scientific publications.

Abstract. In this paper, an on-going research effort aimed at detecting and localising damage in plate-like structures by using mode shape curvature–based damage detection algorithms is described. Two alternative damage indexes are examined. The first one uses exclusively mode shape curvature data from the damaged structure. This method was originally developed for beam-like structures. In this paper, the method is generalised to plate-like structures that are characterised by two-dimensional mode shape curvature. To examine limitations of the method, several sets of simulated data are applied and damage detection results are compared to the damage identification method that requires mode shape information from both the undamaged and the damaged states of the structure. The modal frequencies and the corresponding mode shapes for the first 15 modes of a plate are obtained via finite element models. Simulated test cases include damage of various levels of severity. In order to ascertain the sensitivity of the proposed method to noisy experimental data, numerical mode shapes are corrupted with different levels of random noise.

Keywords: damage detection, modal analysis, plate, mode shape curvature, structural health monitoring.

1. Introduction

Modern civil, transport and aerospace engineering structures are becoming more complex and multifunctional and are expected to be fully functional under severe environmental conditions. Their failure can lead to tragic consequences, and therefore structures have regular costly inspections (Hall, Conquest 1999; Nechval *et al.* 2004). Effective non-destructive evaluation techniques for damage detection and structural health monitoring are particularly important for maintaining the integrity and safety of modern aerospace vehicles. Damage identification at the earliest possible stage can increase safety, extend serviceability, reduce maintenance costs, and define reduced operating limits for structures.

In recent years, significant efforts have been undertaken in the area of vibration-based damage detection methods. These methods are based on the fact that dynamic characteristics, i.e. natural frequencies, mode shapes and modal damping, are directly related to the stiffness of the structure. Therefore, a change in natural frequencies or a change in mode shapes will indicate a loss of stiffness. Extensive literature reviews of the state of the art in methods for detecting, localising and characterising damage by examining changes in the dynamic response of a structure can be found in (Doebling *et al.* 1996; Xia 2002; Fan, Qiao 2011; Bayissaa *et al.* 2008). Many studies (Ho, Ewins 2000; Stubbs, Kim 1996; Yuen 1985; Pandey *et al.* 1991; Wahab, Roeck 1999; Maia *et al.* 2003) have shown that mode shapes and corresponding mode shape curvatures are highly sensitive to damage and can be used for detecting and quantifying damage. In these papers, the absolute difference in mode shape data between the undamaged and the damaged state of a structure is defined as damage index, and the maximum value indicates the location and size of damage. The drawback of those methods, a need for the data of the undamaged structure, is solved by either using the finite element model to simulate the dynamic response of the undamaged structure or by employing gapped smoothing techniques to generate a smoothed surface of mode shape curvature obtained from the damaged structure, thus simulating the undamaged state of a structure (Wu, Law 2004; Ratcliffe, Bagaria 1998; Gherlone *et al.* 2005). Previous investigations conducted by the authors of this paper show that exclusively employing mode shape curvature data from the damaged structure could assess damage location and size. The mode shape curvature square magnitude (MSCSM) damage index was successfully used for the identification of the location and size of mill-cut damage at a single location (Rucevskis, Weselowski 2010), as well as for the identification of mill-cut damage at multiple locations (Rucevskis *et al.* 2009a) in a beam structure. In (Rucevskis *et al.* 2009b) the effectiveness and robustness of the proposed method was demonstrated on composite beams subjected to low-velocity impacts.

In this paper, an on-going research effort aimed to extend this method to plate-like structures is described. The proposed method is generalised to two-dimensional space and is then applicable to plate-like structures. To examine the limitations of the method, several sets of simulated data are applied, and damage detection results are compared to the damage identification method that requires mode shape information from both the undamaged and the damaged states of the structure.

2. Damage detection algorithms

2.1. Mode shape curvature square magnitude (MSCSM) damage index

The idea of the proposed technique is based on the relationship between mode shape curvature and the flexural stiffness of a structure. Damage-induced reduction in the flexural stiffness of a structure subsequently causes an increase in the magnitude of the mode shape curvature square. The increase in the magnitude of the curvature square is local in nature, and thus the mode shape curvature square may be considered an indicator for damage location. The damage index generalised to two-dimensional space for the n^{th} mode at grid point (i,j) is expressed as follows:

$$MSCSM_{i,j}^{n}=\left(\frac{\partial^2 w_n}{\partial x^2}\right)_{(i,j)}^{2}+\left(\frac{\partial^2 w_n}{\partial y^2}\right)_{(i,j)}^{2}, \tag{1}$$

where w_n is transverse displacement of the structure, n is a mode number, and i and j are numbers of grid point in x and y direction, respectively. The mode shape curvatures are computed from mode shapes by using central difference approximation. In practice, experimentally measured mode shapes are inevitably corrupted by measurement noise causing local perturbations in the mode shape, which can lead to peaks in the mode shape curvature square profiles. These peaks could be mistakenly interpreted as damage or they could mask the peaks induced by real damage and lead to false or missed detection of damage. To overcome this problem, summarising the results for all modes is proposed. The damage index is then defined as the average summation of damage indices for all modes normalised with respect to the largest value of each mode.

$$MSCSM_{i,j}=\frac{1}{N}\sum_{n=1}^{N}\frac{MSCSM_{(i,j)}^{n}}{MSCSM_{\max}^{n}}. \tag{2}$$

2.2. Mode shape curvature square (MSCS) damage index

For comparison purposes, the method that uses mode shape information from both the undamaged and the damaged states of the structure is applied. The damage index for one-dimensional space was defined by Y. K. Ho and D. J. Ewans (Ho, Ewens 2000). In this study, the damage index is generalised to two-dimensional space, and for the n^{th} mode at grid point (i,j) is expressed as follows:

$$MSCS_{i,j}^{n}=\left|\left(\frac{\partial^2 w_n^d}{\partial x^2}\right)_{(i,j)}^{2}-\left(\frac{\partial^2 w_n^h}{\partial x^2}\right)_{(i,j)}^{2}\right|+ \left|\left(\frac{\partial^2 w_n^d}{\partial y^2}\right)_{(i,j)}^{2}-\left(\frac{\partial^2 w_n^h}{\partial y^2}\right)_{(i,j)}^{2}\right|, \tag{3}$$

where w_n^d and w_n^h are transverse displacements of the damaged and the undamaged structure, respectively. The location of damage is assessed by the computed absolute difference in mode shape curvature between the undamaged and the damaged structure. The sum of the damage indices from each mode is defined by

$$MSCS_{i,j} = \frac{1}{N}\sum_{n=1}^{N}\frac{MSCS_{(i,j)}^{n}}{MSCS_{\max}^{n}}. \quad (4)$$

3. Finite element model and modal analysis

To evaluate the capability of the damage algorithms introduced above, a square aluminium plate 1000×1000 mm of $h=5$ mm thickness is considered. The elastic material properties are taken as follows: Young's modulus $E=69$ GPa, Poisson's ratio $\nu=0.31$, and mass density $\rho=2708$ kg/m^3. Numerical modal analysis was carried out by using the commercial FE software ANSYS. The finite element model of the plate consists of eight-node shear-deformable shell elements. Each node has six degrees of freedom, namely three displacements and three rotations. The plate is uniformly divided into 50×50 elements. For the undamaged plate, a constant flexural stiffness, $D=Eh^3/12(1-\nu^2)$, is assumed for all elements, and the damaged plate is modelled by reducing the flexural stiffness of the selected elements. Reduction of flexural stiffness is achieved by decreasing thickness h_1 of the elements in the damaged region of the plate, thus simulating a cut on one side of the plate. The plate is given clamped boundary conditions at all four edges. Several sets of simulated data were used to investigate the effectiveness and applicability of the proposed algorithms. Six different levels of damage severity were introduced in the region $240\le x\le320$ and $720\le y\le780$, representing 0.48% of the plate area (Fig. 1).

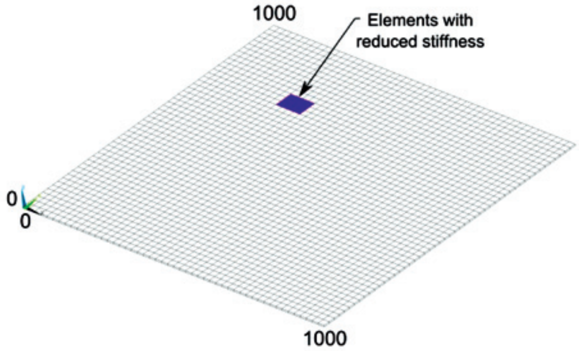

Fig. 1. Finite element model for a square plate with an area of reduced stiffness

Since in practice noise is always present in experimental data, it was proposed to ascertain the sensitivity of the proposed algorithms to noisy experimental data by corrupting numerical mode shapes with different levels of random noise. In order to simulate the effect of measurement noise, an artificial white Gaussian noise (WGN) is added to the numerical mode shapes to generate the noise-contaminated mode shapes

$$w_n = w'_n(1+\delta(2r-1)), \quad (5)$$

where w'_n is the noise-free transverse displacement of the structure, r is the uniformly distributed random values in the interval $(0,1)$, and δ is the random noise level. Simulated test cases are given in Table 1. The modal frequencies and corresponding mode shapes for the first 15 modes are extracted from all the 51×51 nodes in both the undamaged and the damaged plate models.

Table 1. Simulated test cases

Cut depth $h-h_1$ mm	Reduction in flexural stiffness %	Noise level $\delta=0.5\%$	Noise level $\delta=0.1\%$	Noise level $\delta=0.01\%$	Noise level $\delta=0.001\%$
0.5	24.4	Case 1.1	Case 1.2	Case 1.3	Case 1.4
1.0	39.2	Case 2.1	Case 2.2	Case 2.3	Case 2.4
1.5	46.8	Case 3.1	Case 3.2	Case 3.3	Case 3.4
2.0	49.6	Case 4.1	Case 4.2	Case 4.3	Case 4.4
2.5	50.0	Case 5.1	Case 5.2	Case 5.3	Case 5.4
3.0	50.4	Case 6.1	Case 6.2	Case 6.3	Case 6.4

4. Results of damage detection

To evaluate the proposed damage detection algorithms, the concept of the statistical hypothesis testing technique (Bayissaa *et al.* 2008) is used. For this reason, the damage indices determined for each node are standardised and the concept of statistical hypothesis testing is used to classify damaged and undamaged elements and to localize damage depending on the pre-defined damage threshold value. The standardised damage index Z_{ij} at grid point (i, j) is obtained as follows:

$$Z_{ij}^{MSCSM} = \frac{MSCSM_{ij} - \mu_{MSCSM}}{\sigma_{MSCSM}}, \qquad (6)$$

where μ_{MSCSM} and σ_{MSCSM} are the mean and standard deviation of the damage indices, respectively. The decision for the localisation of damage is established based on the level of significance used in the hypothesis test, which can be determined from a pre-assigned classification criterion. The typical damage threshold values for the standardised damage index Z_{ij} widely used in literature include 1.28, 2 and 3 for 90%, 95% and 99% confidence levels for the presence of damage. To analyse the influence of the measurement noise and damage severity on the effectiveness of the proposed damage detection algorithms, a damage index ratio R representing the relationship between the standardised damage index Z_{xy} value at the pre-determined damage location and the average value $\overline{Z}$ for the whole plate is introduced.

$$R_{MSCSM} = \frac{Z_{xy}^{MSCSM}}{\overline{Z}^{MSCSCM}}. \qquad (6)$$

Similarly, the standardised damage index Z_{ij} and the damage index ratio R are obtained for the MSCS damage index. Calculated damage index ratios for the simulated test cases for both damage detection algorithms are summarised in table 2 and shown in figures 2 and 3. It must be noted that damage index ratios are included in the table of results only for those simulated test cases where standardised damage indices indicated peak value at the pre-determined damage location and passed the standardised damage index threshold value for the 99% confidence level for the presence of damage. In general, the capability of the proposed damage detection algorithms is found to be dependent on both the level of noise introduced and severity of damage. From figure 2 it is seen that the level of noise introduced affects the capability of the MSCSM damage index to localise damage but does not affect the damage index ratio R value at the pre-determined damage location. Figure 3 shows that for the MSCS damage index both the capability to localize damage and the damage index ratio R are influenced by the level of noise introduced.

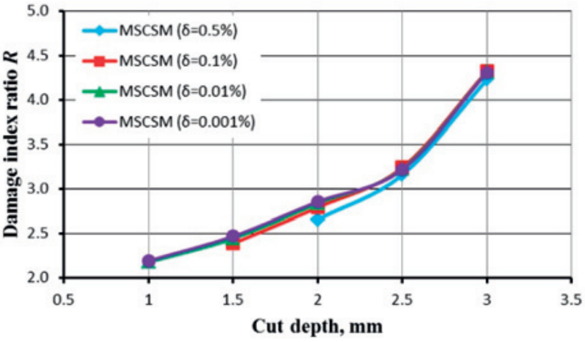

Fig. 2. Damage index ratios for the MSCSM damage index

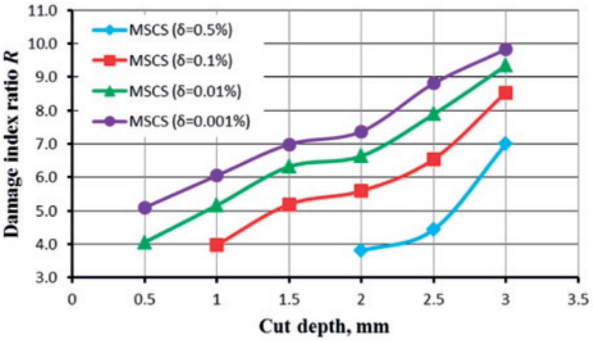

Fig. 3. Damage index ratios for the MSCS damage index

It is also noted that as expected the damage index ratio increases as damage severity increases and decreases as noise level is increased.

For the illustration of the effectiveness of the proposed algorithms, damage detection results of selected

Table 2. Damage detection results

Cut depth $h-h_1$, mm	Noise level $\delta = 0.5\%$		Noise level $\delta = 0.1\%$		Noise level $\delta = 0.01\%$		Noise level $\delta = 0.001\%$	
	R_{MSCSM}	R_{MSCS}	R_{MSCSM}	R_{MSCS}	R_{MSCSM}	R_{MSCS}	R_{MSCSM}	R_{MSCS}
0.5	–	–	–	–	–	4.1	–	5.1
1.0	–	–	–	4.0	2.2	5.2	2.2	6.0
1.5	–	–	2.4	5.2	2.4	6.3	2.5	7.0
2.0	2.7	3.8	2.8	5.6	2.8	6.6	2.9	7.4
2.5	3.2	4.4	3.2	6.5	3.2	7.9	3.2	8.8
3.0	4.2	7.0	4.3	8.5	4.3	9.3	4.3	9.8

test cases are given in figures 4–9. In figure 4a the standardised MSCSM damage indices for test case 6.1 are presented. Although the peak values occur at the pre-determined damage location, one can see that large values also emerge at the boundaries of the plate and some smaller peaks are present at other areas of the plate where no damage was introduced. The boundary distortion problem is caused by discontinuity of mode shapes at their ends due to clamped boundary conditions. In some cases those extreme values can mask the peaks induced by real damage and lead to false or missed detection of damage.

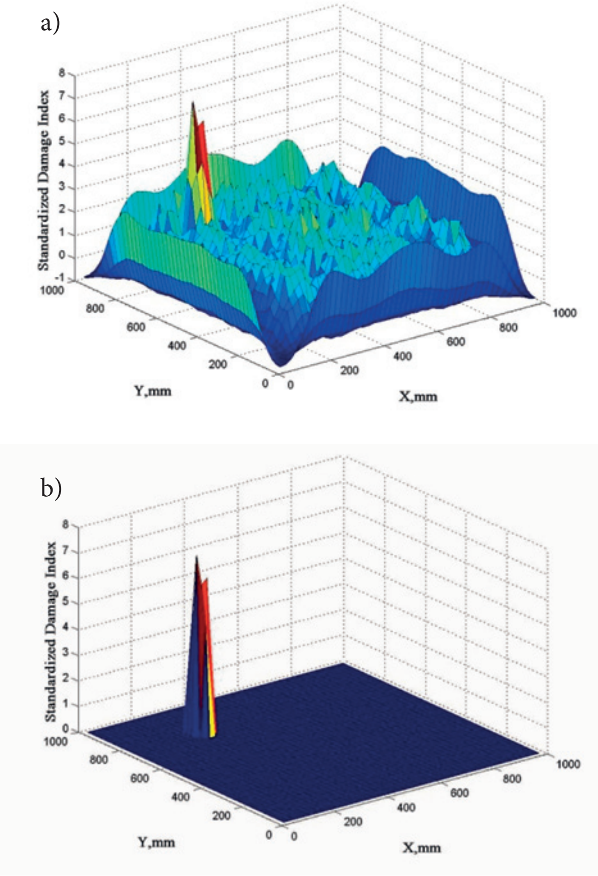

Fig. 4. Standardized MSCSM damage index for test case 6.1: (a) before truncation and (b) after truncation

There are commonly two methods to reduce the boundary effect (Fan, Qiao 2009). One method is to extend the mode shape data beyond the original boundary by cubic spline extrapolation based on points near the boundaries. The other method is simply to ignore those values near the boundaries by cutting them off or setting them to zeros. In this study, the latter method is adapted. It should be noted that neither the extrapolation method nor the *set-to-zero* method is capable of detecting damage close to the boundaries, since both of them smooth out the information near the boundaries. To deal with the problem of smaller peaks, it is proposed to truncate the values of damage indices smaller than three according to the standardised damage index threshold value for the 99% confidence level for the presence of damage.

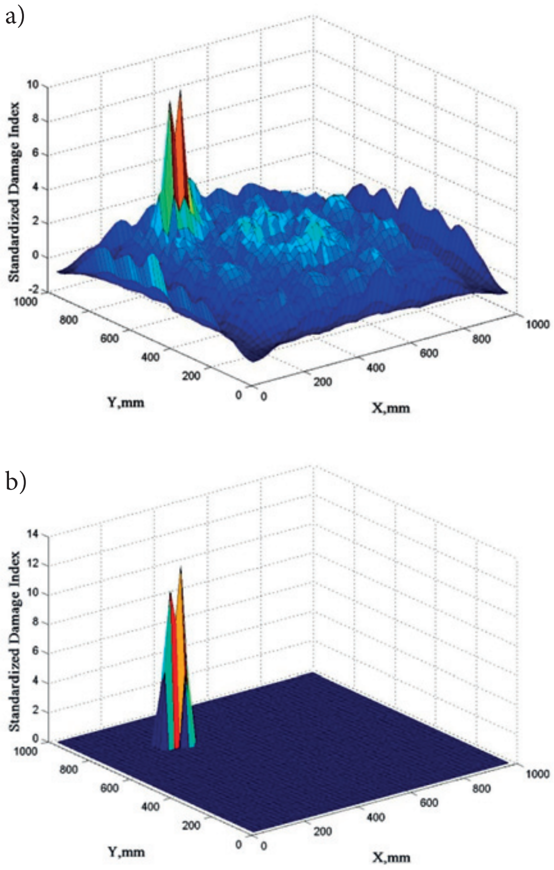

Fig. 5. Standardized MSCS damage index for test case 1.3: (a) before truncation and (b) after truncation

Now the damage indices clearly reveal the pre-determined damage location as shown in figure 4b. Similarly, plots for the standardised MSCS damage indices are altered (Fig. 5). The results obtained and plots presented show that in general the peak values for the standardised MSCS damage index and damage index ratios at the pre-determined damage location are larger than they are for the standardised MSCSM damage index. This can be explained by the MSCS damage index using mode shape information from both the undamaged and the damaged states of the structure and therefore being more sensitive to any changes in mode shape. On the other hand, for real applications where the location of damage is not known in advance, this damage index has to be used carefully if the accuracy of the measurement data is low. For example, in figure 6 one can see that besides the peak values at the pre-determined damage location one more peak has passed the pre-assigned damage index threshold and could be classified as damage. Opposite to this, in figure 7 the standardised MSCS damage indices for test case 3.3 are presented. In this case, damage severity is less than in the previous test case (cut depth – 1.5 mm / 2.5 mm), but measurement accuracy is higher (level of noise introduced – $\delta = 0.01\%$ / $\delta = 0.1\%$), which results in the peak values present only at the pre-determined damage location.

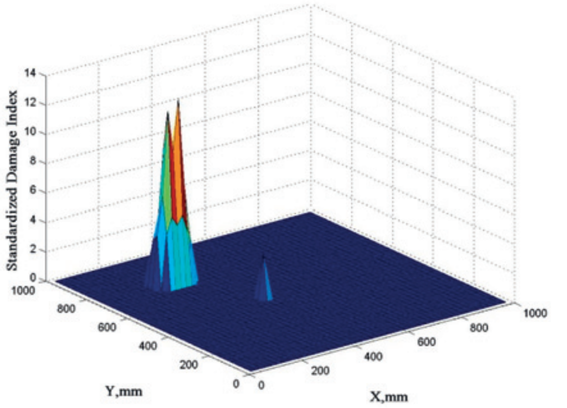

Fig. 6. Standardized MSCS damage index for test case 5.2 after truncation

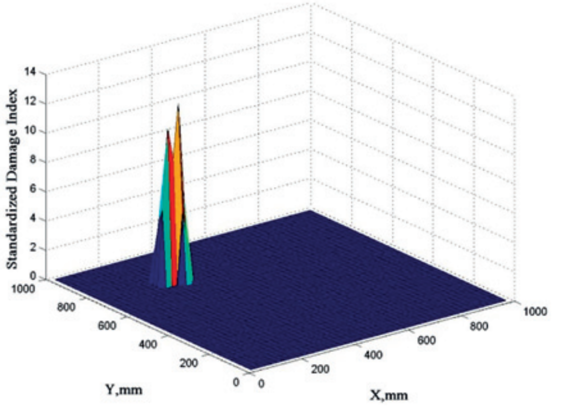

Fig. 7. Standardized MSCS damage index for test case 3.3 after truncation

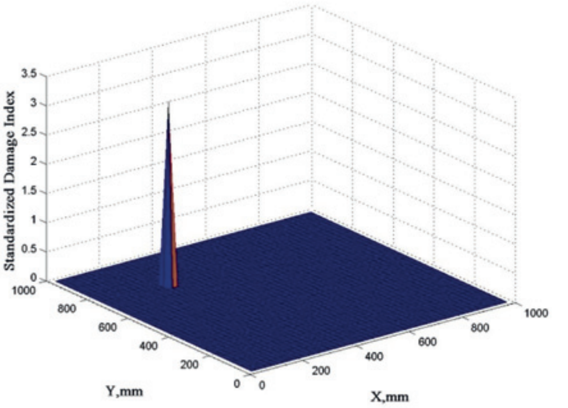

Fig. 8. Standardised MSCSM damage index for test case 2.4 after truncation

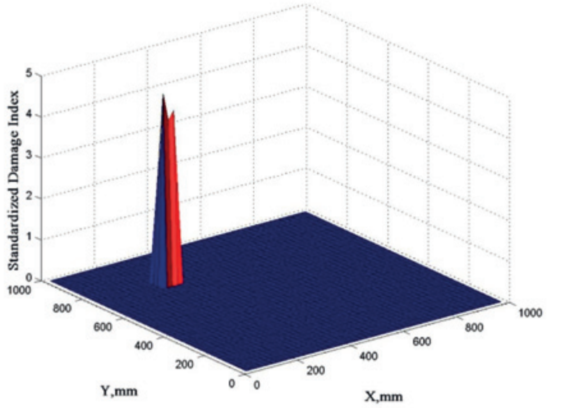

Fig. 9. Standardized MSCSM damage index for test case 4.2 after truncation

5. Conclusions

In this paper a numerical study on the applicability of mode shape curvature-based damage detection algorithms for detection and localisation of damage in plate-like structures is presented. The proposal was made to generalise the MSCSM damage index, which previously has been successfully used to identify damage in one-dimensional structures, to two-dimensional plate-like structures. The advantage of the proposed method is that it requires mode shape information only from the damaged state of the structure. For comparison purposes, the damage location is also assessed by the MSCS damage index, which requires mode shape information from both the undamaged and the damaged states of the structure. The results obtained show that the MSCSM damage index provides reliable information about the location of the damage when there is medium severe damage and relatively accurate measurement data. From the point of view of practical structural applications, measurement noise would not be an issue if a scanning laser vibrometer (SLV) system is used. The latest SLV systems allow transverse displacement measurements with a low degree of measurement noise $\left(\delta = 0.1 \ldots 0.01\%\right)$. In this case the major drawback of the application of the MSCSM damage index for real practical applications is that the severity of damage has to be relatively high for successful detection and localisation of damage in plate-like structures (the damage index was not able to detect the location of damage when flexural stiffness was reduced 24%). To overcome this shortcoming, the authors of the paper suggest using the MSCS damage index. Results show that this damage index is very sensitive to damage-induced changes in mode shape data and can also be successfully applied if severity of damage is relatively low. Moreover, this approach does not necessarily require mode shape information from the undamaged state of a structure if proper algorithms, for example gapped smoothing techniques, are used.

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